Reduction of Matter in the Universe to Protons and Electrons via the Lie-isotopic Branch of Hadronic Mechanics

Ruggero Maria Santilli

The Institute for Basic Research, 35246 U. S. 19N, Suite 215, Palm Harbor, FL 34684, USA.
E-mail: research@i-b-r.org

Matter was originally conceived as bound states of the permanently stable protons and electrons because stars initiate their lives as sole aggregates of Hydrogen atoms, and must synthesize neutrons from protons and electrons as a necessary condition to produce light via nuclear fusions. In oblivion to the Einstein-Podolsky-Rosen argument that *quantum mechanics is not a complete theory,* said conception was abandoned despite its plausibility because of the unverified assumption that the exact validity of Heisenberg's uncertainty principle for point-like particles in vacuum was equally valid for extended protons and neutrons under strong nuclear forces, resulting in the assumption that electrons cannot remain within a nuclear structure. In this paper, we review and update: the insufficiencies of quantum mechanics in nuclear physics; the completion of quantum mechanics into the axiom-preserving, Lie-isotopic branch of *hadronic mechanics* for the invariant representation of *extended* protons and neutrons under potential and contact/non-potential interactions; the exact hadronic representation of *all* characteristics of the neutron in its synthesis from the proton and the electron at the non-relativistic and relativistic levels; the completions of Bell's inequalities with ensuing *iso-deterministic principle* for strong interactions. We then present the apparent resolution of the historical objections against the reduction of all stable matter in the universe to protons and electrons and point out a number of open problems whose treatment is beyond the capabilities of quantum mechanics, such as: the cosmological implications of the missing energy in the neutron synthesis, the prediction of negatively charged pseudo-protons, and the possible recycling of radioactive nuclear waste by nuclear power plants via their stimulated decay.

Content

1 Introduction

1.1 Historical notes

As it is well known to historians (see, e.g. [1] [2]), nuclei were originally conceived to be bound states of protons and electrons because stars initiate their lives as aggregates of Hydrogen atoms and they must synthesize neutrons from protons and electrons as a necessary condition to initiate the production of light via nuclear fusions.

The above original conception of the nuclear structure was abandoned in oblivion of the Einstein-Podolsky-Rosen (EPR) argument that *Quantum mechanics is not a complete theory* [3] (see also the recent verifications [4]–[8]), under the experimentally unverified *assumption* that the validity of Heisenberg's uncertainty principle for *point-like* particles in vacuum was also valid for the *extended* protons and neutrons under strong nuclear forces, resulting in the *assumption* that electrons cannot remain within the dense nuclear structure on various grounds, such as:

1.1) The inability for the electron to remain within a nucleus [1]. By recalling the value of the electron mass m_e = 0.511 MeV = 9.1×10^{-31} kg and the nuclear radius R = 10^{-14} m, *Heisenberg's uncertainty principle* [9]

$$\begin{aligned}\Delta r\,\Delta p &= \frac{1}{2}|\langle\psi|[r,p]|\psi\rangle \geq \frac{1}{2}\hbar = \\ &= 5.26548578 \times 10^{-34}\,\mathrm{J\,Hz^{-1}}\,,\end{aligned} \tag{1}$$

would imply the electron to have the superluminal velocity

$$v \geq \frac{\hbar}{\Delta r \times m_e} = 5.79 \times 10^{10}\,\mathrm{m/s}\,. \tag{2}$$

1.2) Under the validity of principle (1), an electron would have the linear momentum uncertainty [10]

$$\Delta p = 1.05 \times 10^{20}\,\mathrm{kg\,m/s}\,, \tag{3}$$

with corresponding energy

$$E = 19.5\,\mathrm{MeV}\,, \tag{4}$$

contrary to the evidence that electrons emitted in Beta decays have a maximum energy of 3 MeV.

1.3) The excessive value for nuclear standards of the magnetic moment of the electron [7]. In fact, expressed in nuclear magnetron μ_N, the magnetic moment of the electron has the value

$$\begin{aligned}\mu_e^{spin} &= -9.284764 \times 10^{-24}\,\mathrm{J/T} \\ &= -9.284764 \times 10^{-24} \times 1.9798907610^{26}\,\mu_N \\ &= -928.4784 \times 1.979890\,\mu_N = 1838.2851\,\mu_N,\end{aligned} \tag{5}$$

which is 961 times the magnetic moment of the neutron $\mu_n = -1.91304\,\mu_N$.

In this paper we show that, thanks to the availability of new mathematics for the time-invariant representation of extended protons and neutrons under strong nuclear forces, and the related completion of quantum into hadronic mechanics, Heisenberg's uncertainty principle for point-like particles in vacuum is replaced by a progressive validity of Einstein's determinism for extended protons and neutron under strong nuclear forces [3]–[8], with ensuing resolution of the historical objections against the reduction of matter to protons and electrons.

1.2 Insufficiencies of quantum mechanics in nuclear physics

By using well known nuclear experimental data [11]–[18], we recall the following, century-old, generally ignored insufficiencies of quantum mechanics in nuclear physics:

Quantum mechanical insufficiency I: Inability to represent the synthesis of the neutron from a proton and an electron in the core of stars [19]. Notwithstanding the extremely big (for particle standards) *attractive* Coulomb force of about 230 Newtons between the (negatively charged) electron and the (positively charged) proton,

$$\begin{aligned}F &= -\frac{e^2}{r^2} = \\ &= -(8.99 \times 10^9)\,\frac{(1.60 \times 10^{-19})^2}{(10^{-15})^2} = -230\,\mathrm{N}\,,\end{aligned} \tag{6}$$

quantum mechanics allows no quantitative representation of the fundamental synthesis of the neutron in the core of stars. This insufficiency was first identified by R. M. Santilli in the 1978 Harvard's Lyman Laboratory of Physics [20] (see also the subsequent 1979 paper from Harvard's Department of Mathematics [22] on grounds that the mass/rest energy of the neutron is 0.782 MeV *bigger* than the sum of the masses/ rest energies of the proton and of the electron

$$\begin{aligned}&E_p = 938.272\,\mathrm{MeV}\,, \quad E_e = 0.511\,\mathrm{MeV}\,, \\ &E_n = 939.565\,\mathrm{MeV}\,, \\ &\Delta E = E_n - (E_p + E_e) = 0.782\,\mathrm{MeV} > 0\,,\end{aligned} \tag{7}$$

by therefore requiring a *positive binding energy* and resulting in a *rest energy excess* for which the Schrödinger equation admits no physically meaningful solutions (for a two-body bound state). A similar case occurs for the Dirac equation, which after achieving an exact relativistic representation of the bound state of a proton and the electron at large mutual distances in the Hydrogen atom, the Dirac equation fails to provide any quantitative representation of the bound state of the same particles at nuclear mutual distances.

By no means the neutron synthesis is an isolated case because as we shall see in Sect. 4.1, the representation of *unstable* leptons, mesons and baryons as generalized bound states of particles and antiparticles generally produced free in their spontaneous decays, permits the numerically exact representation of *all* their characteristics, including the mechanism of their spontaneous decays, which has been impossible to date via quantum mechanics.

Quantum mechanical insufficiency II: Inability to achieve a numerically exact representation of nuclear magnetic moments. In fact, under the use of the tabulated values of the magnetic moments of the proton and of the neutron in vacuum [12]

$$\mu_p = +2.79285\,\mu_N\,, \quad \mu_n = -1.91304\,\mu_N\,, \tag{8}$$

quantum mechanics (qm) predicts that the magnetic moment of the Deuteron is given by

$$\mu_D^{qm} = (2.79285 - 1.91304)\,\mu_N = 0.87981\,\mu_N\,, \qquad (9)$$

while the experimentally measured value is given by

$$\mu_D^{ex} = 0.85647\,\mu_N\,, \qquad (10)$$

resulting in the *deviation* of the quantum mechanical prediction from the experimental value of about 3%, with embarrassing deviations for heavier nuclei such as the zirconium.

Quantum mechanical insufficiency III: Inability to achieve a consistent representation of nuclear spins. According to quantum mechanics, the only stable bound state of two particles with spin 1/2, such as the proton and the neutron, is the *singlet coupling.* Consequently, quantum mechanics predicts that the Deuteron D has the structure

$$D = (p_\uparrow, n_\downarrow)_{qm}\,, \qquad (11)$$

for which the total angular momentum is null, $J_D = 0$, contrary to the experimental value of the spin of the Deuteron $J_D = 1$. As a result of this insufficiency, quantum mechanics represents the spin of the Deuteron via such a *collection of orbital contributions to have the value* $L_D = 1$ (see, e.g. [21]) in clear disagreement with experimental evidence for which *the spin* $S_D = 1$ *has been measured for the Deuteron in its true ground state,* i.e. the state for which $L_D \equiv 0$.

Quantum mechanical insufficiency IV: Inability to represent the nuclear stability despite the natural instability of the neutron. As it is well known, the neutron is naturally unstable with spontaneous decay following 887.7 s [17], at which point nuclei should disintegrate evidently due to the excessive number of positive charges. In view of the inability to represent the neutron synthesis form the proton and the electron, quantum mechanics does not allow a meaningful treatment of the mechanism according to which neutrons become stable when members of a nuclear structure.

Quantum mechanical insufficiency V: Inability to represent the nuclear stability despite strongly repulsive protonic Coulomb forces. As it is well known [11], nuclei contain a number of positively charged protons indicated with the atomic number Z, thus experiencing a *repulsive* Coulomb force of type (6) which is so big to overcome known nuclear forces.

Needless to say, the above insufficiencies also apply to relativistic quantum mechanics, as well as to related space time symmetries and relativities.

1.3 Rudiments of isotopic theories

The indicated insufficiencies of quantum mechanical methods, space time symmetries and relativities for the representation of the synthesis of the neutron from the Hydrogen are primarily due to the *local* character of quantum mechanical methods [3], here referred to the sole dependence of the wave function $\psi(r)$, the potential $V(r)$, and the differential calculus, on a finite number of isolated points r in empty space, as it is the case, e.g. for the linear momentum

$$p\psi(r) = -i\partial_r\psi(r)\,, \qquad (12)$$

of the Schrödinger equation

$$\left[\Sigma_{k=1,2,\ldots,A}\,\frac{1}{2m_k}\,p_k p_k + V(r)\right]\psi(r) = E\psi(r)\,. \qquad (13)$$

Such an approximation of nature has been effective for *atomic structures* due to the large mutual distances between the constituents which allow particles to be approximated as the Newtonian *massive points.* However, the indicated local character of quantum mechanics is excessively approximated for nuclear structures since, according to clear experimental measurements [16]–[18], protons and neutron are *extended charge distributions,* and nuclear volumes are generally *smaller* than the sum of the volumes of their protons and neutrons.

Consequently, *nuclei are generally composed by extended protons and neutrons in condition of partial mutual penetration,* resulting in the expectation that nuclear forces comprise conventional, action-at–a-distance, linear, local and potential interactions (herein called *Hamiltonian interactions*), plus contact, thus zero-range, non-linear, non-local and non-potential interactions (herein called *non-Hamiltonian interactions*).

By noting that a point-like electron cannot possibly be bonded to a point-like proton, we expect that the neutron synthesis requires the representation of the charge distribution of the proton and of the electron wave packet as being extended, with ensuing Hamiltonian and non-Hamiltonian interactions at mutual distances smaller than their size.

Since at the time of the initiation of the studies herein reported (late 1970's), mathematical and physical theories for the time invariant representation of extended particles did not exist, they had to be constructed. In this paper, we adopt *isotopic methods* comprising:

1) The *Lie-isotopic mathematics,* or *iso-mathematics* for short.

2) The *Lie-isotopic branch of hadronic mechanics,* or *iso-mechanics* for short.

3) The non-relativistic and relativistic *iso-symmetries and iso-relativities.*

The above isotopic methods were proposed by R. M. Santilli (when at Harvard University under DOE support) in the 1978 Springer-Verlag monographs [23,24] and they do achieve the needed time invariant representation of extended particles and/or their wave packets, with consequential Hamiltonian and non-Hamiltonian interactions.

As it is well known, the mathematics of quantum mechanics is based on the universal, enveloping, associative algebra $\xi\{A, B, ...; A \times B, I\}$ of operators $A, B, ...$ on a linear space $\mathcal{H}$

with conventional associative product and related (multiplicative) unit

$$AB = A \times B\,, \qquad I:\; IA = AI \equiv A \;\forall A \;\in \xi\,, \tag{14}$$

which envelope allows a rigorous treatment of Lie's theory via algebra L isomorphic to the antisymmetric sub-algebra $L \approx \xi^-$ with the familiar Lie product $[A, B] = AB - BA$, and ensuing mechanics, symmetries and relativities.

Santilli's iso-mathematics is based on the *axiom-preserving,* thus isotopic lifting of the enveloping algebra $\xi\{A, B, ...;$-$A\times B, I\}$ into the universal enveloping *iso-associative algebra* $\hat{\xi}\{\hat{A}, \hat{B}, ...; \hat{A}\hat{\times}\hat{B}, \hat{I}\}$ of iso-operators $\hat{A}, \hat{B}, ...$ on an iso-linear iso-space $\hat{\mathcal{H}}$ with *iso-product* introduced in the 1978 Harvard's paper [20], extended in the 1979 paper [22] and systematically studied in Sect. 5.2, p. 154 on of [24])

$$\hat{A}\hat{\times}\hat{B} = \hat{A} \times \hat{T} \times \hat{B}\,, \tag{15}$$

and related *iso-unit*

$$\hat{I} = 1/\hat{T}:\;\; \hat{I}\hat{\times}\hat{A} = \hat{A}\hat{\times}\hat{I} \equiv \hat{A} \;\forall \hat{A} \;\in \hat{\mathcal{H}}\,. \tag{16}$$

Under the condition that, for consistency, iso-product (15) is applied to the *totality* of the products of the new mathematics, including numbers, functions, operators, etc., the associativity-preserving lifting $\xi \rightarrow \hat{\xi}$ allowed in 1978:

1) The foundations of iso-mathematics, including the Lie-Isotopic theory (nowadays called the *Lie-Santilli iso-theory*) consisting of the step by step isotopic lifting of Lie's theory, including Lie algebras, Lie groups and the transformation theory, with generic N-dimensional iso-algebra $\hat{L}$ of Hermitean operators X_k, $k = 1, ..., N$ and *iso-commutation rules* (Eq. (38c), p 170 of [42])

$$\begin{aligned}[X_i \hat{,} X_j] &== X_i\hat{\times}X_j - X_j\hat{\times}X_i = \\ &= X_i \times \hat{T} \times X_j - X_j \times \hat{T} \times X_i = C^k_{ij} X_k\,.\end{aligned} \tag{17}$$

After leaving Harvard University, Santilli completed the above studies with the 1994 construction of the new *iso-number theory* [89] with iso-unit (16), the 1996 construction of the new *iso-differential calculus* [50] defined for *volumes*, rather than points, and other advances.

2) The foundations of iso-mechanics comprising the *Schrödinger-Santilli iso-equation* (Eq. (14), p. 259 of [24])

$$\begin{aligned}\hat{H}\hat{\times}|\hat{\psi}\rangle &= \left[\Sigma_{k=1,2,...,A}\, \frac{1}{2m_k}\, \hat{p}_k\hat{\times}\hat{p}_k + V(r)\right]\hat{\times}|\hat{\psi}\rangle = \\ &= \hat{E}\hat{\times}|\hat{\psi}\rangle = (E \times \hat{I}) \times \hat{T} \times |\hat{\psi}\rangle = E \times |\hat{\psi}\rangle\,,\end{aligned} \tag{18}$$

and the *Heisenberg-Santilli iso-equation* (Eq. (16), p. 153 of [24]) in its infinitesimal and finite form

$$\begin{aligned} i\,\frac{dA}{dt} &= [\hat{A}, \hat{;} \hat{H}] = \hat{A}\hat{\times}\hat{H} - \hat{H}\hat{\times}\hat{A}\,, \\ A(t) &= e^{\hat{H}\hat{T}ri} \times A(0) \times e^{-it\hat{T}\hat{H}}\,,\end{aligned} \tag{19}$$

thus requiring *two* quantities for the characterization of nuclear structures, the conventional Hamiltonian $H > 0$ for the representation of linear, local and potential interactions, and the isotopic element $\hat{T} > 0$ for the representation of the extended character of particles and their non-linear, non-local and non-potential interactions.

3) The iso-Galilean symmetry and relativity (Chapter 6, p. 199 on of [24]).

Following the above foundations, hadronic mechanics has been studied by various scholars (see monographs [25]–[34] and papers quoted therein) at about thirty workshops and various international conferences (see representative proceedings [35]–[40], comprehensive presentations [41]–[43]) (see also the summary of the various branches of hadronic mechanics [49], the overviews [45]–[49], and the recent summaries [46]–[48]).

Nowadays, hadronic mechanics has various branches of increasing complexity for the description of particles with increasingly complex physical conditions [49].

The above mathematical and theoretical studies, combined with experimental verifications [43], allowed the identification of the following explicit form of *the isotopic element (15) and iso-unit (16) for a two-body hadronic system* [44]

$$\begin{aligned}
&\hat{T} = \Pi_{\alpha=1,2}\,\text{Diag.}\left(\frac{1}{n^2_{1,\alpha}}, \frac{1}{n^2_{2,\alpha}}, \frac{1}{n^2_{3,\alpha}z}, \frac{1}{n^2_{4,\alpha}}\right) \times e^{-\Gamma} \ll 1\,, \\
&\hat{I} = 1/\hat{T} \\
&\;\;= \Pi_{\alpha=1,2}\,\text{Diag.}\left(n^2_{1,\alpha}, n^2_{2,\alpha}, n^2_{3,\alpha}, n^2_{4,\alpha}\right) \times e^{+\Gamma} \gg 1\,, \\
&\Gamma(r, p, a, E, d, \pi, \tau, \psi, ...) > 0\,,\;\; n_{\mu,\alpha} > 0, \\
&\mu = 1, 2, 3, 4,\;\; \alpha = 1, 2\,,
\end{aligned} \tag{20}$$

where $\hat{T}$ is solely restricted by the condition of being positive-definite, but otherwise possess an unrestricted functional dependence (hereon tacitly assumed) on coordinates r, momenta p, accelerations a, energy E, density d, pressure π, temperature τ, wave functions ψ, and any other needed local variable:

1) The representation of the dimension and shape of the individual nucleons is done via semi-axes $n^2_{k,\alpha}$, $k = 1, 2, 3$ (with n_3 parallel to the spin) and normalization for the vacuum $n^2_{k,\alpha} = 1$.

2) The representation of the density is done via the characteristic quantity $n^2_{4,\alpha}$ per individual nucleons with normalization for the vacuum $n^2_{4,\alpha} = 1$.

3) The representation of the non-Hamiltonian interactions between extended nucleons which is achieved by the exponential term $e^{-\Gamma}$.

On pedagogical grounds, it should be indicated that any given quantum mechanical model with point-like nucleons and sole Hamiltonian interactions can be uniquely and unambiguously completed into the covering hadronic model for extended nucleons with Hamiltonian and non-Hamiltonian interactions via the simple *non-unitary transformation* (first proposed in Eq. (11), p. 249 of [24])

$$U \times U^{\dagger} = \hat{I} = 1/\hat{T} > 0\,, \tag{21}$$

provided that, to avoid insidious inconsistencies, it is applied to the totality of the quantum formalism with no exception known to this author. In fact, under transformation (8), the conventional associative product of quantum operators A, B is mapped into the *iso-product* of *iso-operators*

$$\begin{gathered} U \times (A \times B) \times U^{\dagger} = \hat{A}\hat{\times}\hat{B} = \hat{A} \times \hat{T} \times \hat{B}\,, \\ \hat{T} = (UU^{\dagger})^{-1}, \;\; \hat{A} = U \times A \times U^{\dagger}, \;\; \hat{B} = U \times B \times U^{\dagger}\,, \end{gathered} \tag{22}$$

and the same holds for all aspects of iso-mechanics as we shall see in detail in Section 3.

Finally, it is important to indicate from these initial notes that *the representation of the dimensions of particles and their non-Hamiltonian interactions via hadronic mechanics is invariant over time,* of course, not under the *unitary* time evolution of Heisenberg's equations, but under the *iso-unitary* time evolution of the Heisenberg-Santilli iso-equation,

$$\begin{gathered} U = \hat{U}\hat{T}^{1/2}\,, \\ \hat{U}\hat{\times}\hat{U}^{\dagger} = \hat{U}^{\dagger}\hat{\times}\hat{U} = \hat{I}\,, \end{gathered} \tag{23}$$

under which the iso-unit and the isotopic element of hadronic mechanics are *numerically invariant* [51]

$$\begin{gathered} \hat{U}\hat{\times}\hat{I}\hat{\times}\hat{U}^{\dagger} \equiv \hat{I}\,, \\ \hat{U}\hat{\times}(\hat{A}\hat{\times}\hat{B})\hat{\times}\hat{U}^{\dagger} = \hat{A}' \times \hat{T}' \times \hat{B}'\,, \;\; \hat{T}' \equiv \hat{T}\,. \end{gathered} \tag{24}$$

By using a language accessible to the general physics audience, in Section 2 we review half a century of mathematical, theoretical, experimental and industrial studies in the *non-relativistic* synthesis of the neutron from the proton and the electron.

In Section 3, we report the relativistic studies in the synthesis of the neutron with particular reference to the space time iso-symmetries and iso-relativities necessary for their derivation.

In Section 4, we show that all objections against electrons being part of the nuclear structure are resolved by the recent EPR verifications [4]–[8] and more particularly, by the progressive validity of the *iso-deterministic principle under strong interactions* which occurs in the structure of hadrons, nuclei and stars and the full achievement of Einstein's determinism at the limit of the Schwartzschild horizon.

An initial understanding of this paper an be reached via a knowledge of reviews [46]–[48], with the understanding that a technical knowledge of this paper can solely be reached via a technical knowledge of hadronic mechanics according to the general presentations [41]–[43].

2 Non-relativistic representation of the neutron synthesis from the Hydrogen atom

In this section, we shall outline and update one century of studies on the synthesis of the neutron from the Hydrogen atom in the core of stars as well as in laboratory. Needless to say, we can only outline the main aspects of such a vast topic and provide the references for detailed studies.

2.1 Historical notes

As recalled in Sect. 1.1, stars initiate their lives as an aggregate of Hydrogen that grows by accretion during travel in interstellar spaces. At the moment when the temperature in the core of the aggregate reaches a value of the order of 10 MK, E. Rutherford [19] suggested in 1920 that the Hydrogen atom is "compressed" into a new neutral particle which he called the *neutron,*

$$e^- + p^+ \;\rightarrow\; n\,. \tag{25}$$

The existence of the neutron was experimentally established in 1932 by J. Chadwick [52]. In 1933, W. Pauli [53] pointed out that synthesis (11) violates the conservation of angular momentum. Therefore, E. Fermi [54] submitted in 1935 the hypothesis that the synthesis of the neutron occurs with the joint *emission* of a neutral and massless particle ν with spin $1/2$ which he called the *neutrino* (meaning "little neutron" in Italian)

$$e^- + p^+ \;\rightarrow\; n + \nu\,. \tag{26}$$

Subsequent tests (see the recent review [17]) established that *the neutron is naturally unstable* with a mean life of $\tau = 877$ s and spontaneous decay

$$n \rightarrow e^- + p^+ + \bar{\nu}\,, \tag{27}$$

where $\bar{\nu}$ is the *antineutrino.*

Predictably, the synthesis of the neutron from the Hydrogen attracted attention soon following the Chadwick confirmation. According to the historical account [55], Ernest J. Sternglass conducted in 1951 the first test for the laboratory synthesis of the neutron from Hydrogen, followed by tests in 1952 by E. Trounson and others, although none of these initial tests were reported in published papers in view of the incompatibility of the neutron synthesis with quantum mechanics (Insufficiency I) and for other reasons.

To the author's best knowledge, the first published tests on the laboratory synthesis of the neutron from Hydrogen were done in the 1960's by the Italian priest-physicist, Don Carlo Borghi and his associates [56]. In essence the experimentalists constructed a cylindrical metal chamber (called *klystron*) filled up with the Hydrogen gas (at a fraction of 1 bar pressure) kept the gas mostly ionized via an electric arc with about 500 V and 10 mA. Additionally, the gas was traversed by microwaves with the frequency of $10^{-10}\,\mathrm{s}^{-1}$. The experimentalists then placed in the exterior of the Klystron various materials suitable to be activated when exposed to a neutron flux (such as gold or silver). Following exposures over several weeks, the experimentalists reported clear and reproducible nuclear transmutations that can only be due to a neutral hadron emitted from the Klystron. Due to insufficient evidence on neutron emission, the experimentalists conjectured that the detected nuclear transmutations were due to a new neutral particle with the mass of the neutron but spin different than $1/2$ that they called the *neutroid.*

2.2 Santilli's studies on the neutron synthesis

In view of its fundamental character for all quantitative sciences, R. M. Santilli has conducted over the past five decades mathematical, theoretical, experimental and industrial research on the synthesis of the neutron from the Hydrogen atom in the core of stars, as well as in laboratory (see the mathematical studies [20,23,24,41,50] [57]–[67], the physical studies [42] [68]–[73], the experimental studies [43, 74, 80], and the independent studies [25]–[34] [81]–[85].

These studies were initiated in the late 1970's at Harvard University under DOE support with the inapplicability of quantum mechanics for the neutron synthesis [20] (Quantum insufficiency I) followed by the proposal to construct *hadronic mechanics* in monographs [23, 24].

By far the biggest difficulty of the above studies has been the representation of the spin of the neutron $S_n = 1/2$ from two particles each having spin $1/2$, as originally conceived by Rutherford [19]. This problem stimulated the construction of the Lie-Santilli iso-theory (see Sect. 4.4, p. 173 on of [24] and independent work [26]), followed by systematic studies on the isotopies of spacetime symmetry [57]–[67], with particular reference to the isotopies $\widehat{\mathrm{SO}}(3)$ and $\widehat{\mathrm{SU}}(2)$ of the angular momentum and spin symmetries at the classical and operator levels [57]–[60] and then passing to the isotopies of spacetime symmetries [61]–[67].

As a result of these preparatory studies, Santilli was able to achieve a numerically exact and time invariant representation of *all* characteristics of the neutron at the non-relativistic level in the 1990 paper [68], and at the relativistic level in the 1995 paper [72], with additional studies available in monograph [73].

Following, and only following, the achievement of a consistent representation of the neutron synthesis via the Lie-isotopic branch of hadronic mechanics, Santilli initiated in 2007 experimental tests on the laboratory synthesis of the neutron from Hydrogen [74]–[80]. According to these experiments, the neutron synthesis from Hydrogen can be generated by *hadronic reactors* consisting of a metal vessel containing in their interior a commercially available Hydrogen gas at pressure and a pair of submerged carbon electrodes powered by a specially designed (patent pending) DC source with a gap controllable from the outside. During operations (Fig. 1), the DC arc is continuously connected and disconnected because of the consumption of the carbon electrodes. During its activation (left of Fig. 2), the special form of the DC arc ionizes the Hydrogen gas by creating a plasma mostly composed by protons and electrons in its cylindrical surroundings, while during its deactivation (right of Fig. 2), the specially designed DC electric arc compresses the ionized gas from all radial directions toward its symmetry axis.

Interested readers should be aware that commercially available DC electric arcs between carbon electrodes submerged within a Hydrogen gas may synthesize neutroids (Fig. 2) and other unstable hadronic bound states under their big Coulomb attraction, but they are not designed to *compress electrons inside the proton* according to Rutherford's original conception [19].

Experiments [74]–[80] have confirmed: 1) The production of Don Borghi's neutroids (Fig. 2) for DC power of the order of 5 kw, gas pressure of 5 psi and electrode gap of 2 mm. 2) The production of neutrons (Fig. 3) for DC power with at least 50 kw, gas pressure from 10 psi on and electrode gas of at least 5 mm. In particular, the synthesis of neutroids (Fig. 2) resulted to be an unavoidable step prior to the synthesis of the neutron (Fig. 3).

Following, and only following sufficient experimental evidence on the laboratory synthesis of the neutron from a Hydrogen gas, Santilli founded in 2012 the U.S. publicly traded company *Thunder Energies Corporation* (now the privately held Hadronic Technologies Corporation www.hadronictechnologies.com) for the production and sale of a thermal neutron source (see Sect. 4.1).

2.3 Non-relativistic representation of the neutron synthesis

This study was initiated by Santilli with his 1978 Harvard University memoir [20], continued in various works [68]–[73] thanks to the collaboration by various scholars, and reviewed in the 2021 paper [48].

These studies have been conducted under the assumption [20] that the angular momentum of the electron compressed inside the proton is *constrained* to be equal to the spin of the proton as a necessary condition to prevent *extreme resistive forces* caused by the motion of its *extended* wave packet against the *dense* medium in the interior of the proton.

More particularly, when compressed inside the dense proton, the electron e is mutated into a new particle called the *eleton* in Sect. 5.1 of [20] and indicated with the symbol ϵ^- to distinguish it from the electron and the elemenatary charge e, but recently called the *iso-electron*

$$\hat{\epsilon}^- = U \epsilon^- U^\dagger \,, \tag{28}$$

because characterized by the complex lifting of the elementary charge (identified in Sect. 3 as an open problem) generated by the isotopic completion $\hat{\mathcal{G}}(3.1)$ of the Galilean symmetry [87, 88] (see monograph [25] for an extensive inde-

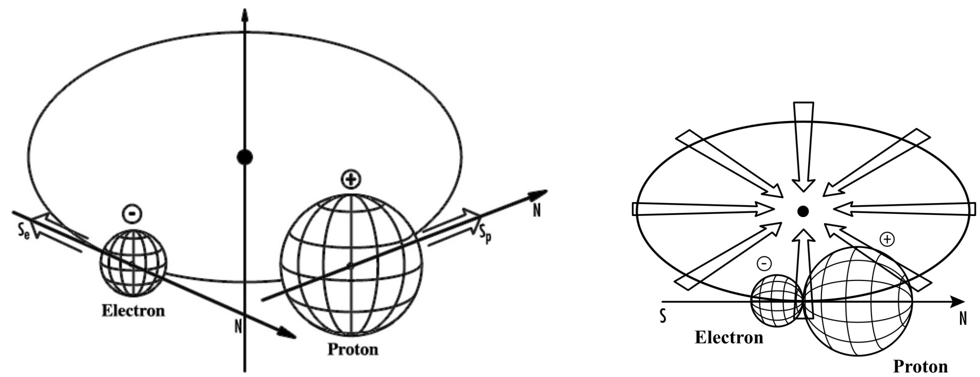

Fig. 1: In this figure, we illustrate the mechanism used by hadronic reactors for the synthesis of neutroids and neutrons via a specially designed (patent pending) DC electric arc between Carbon electrodes submerged within a Hydrogen gas. The mechanism comprises the ionization of Hydrogen atoms into electrons and protons by the activation of the arc (left view) and the compression of the electron within the proton by the de-activation of the arc (right view).

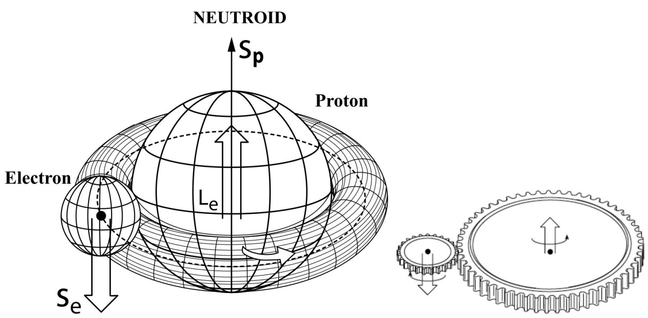

Fig. 2: In the left of this figure, we illustrate the predicted structure of the neutroid in its ground state as a hadronic bond of electrons and protons under their very big Coulomb attraction, Eq. (6), in singlet couplings with null eigenvalues of the angular momentum and of the spin. In the right view, we present a conceptual gear equivalent of the left view to illustrate the reason for the half life of neutroids as being about 10% that of neutrons, i.e. of about 8 s.

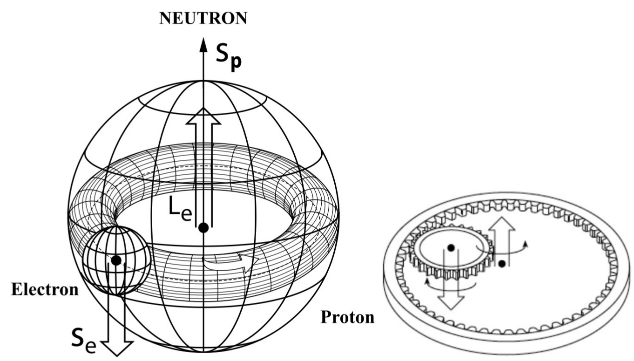

Fig. 3: In the left view, we illustrate the compression of the neutroid of Fig. 2 via the mechanism of Fig. 1, resulting in a constrained hadronic angular momentum of the electron within the dense medium inside the proton that, to avoid extreme resistive forces, has to be equal to the proton spin with ensuing total angular momentum 1/2. In the right of this figure, we provide a conceptual rendering of the left view via coupled gears to illustrate the rather large half life of the neutron of 887 s.

pendent study), with corresponding relativistic extension characterized by the isotopy $\widehat{\mathrm{SO}}(3.1)$ of the Lorentz symmetry SO(3.1) [61] and the isotopy $\hat{\mathcal{P}}(3.1)$ of the spinorial covering of the Poincaré symmetry $\mathcal{P}(3.1)$ characterizing the 20th century notion of *particle*.

By comparison, the proton is assumed in first approximation to be un-mutated, $\hat{p}^+ = p^+$ since the iso-electron is about 1800-times lighter than the proton.

The above assumptions imply the following *structure model of the neutron* as a bound state of a proton p^+ and an iso-electron $\hat{\epsilon}^-$ according to hadronic mechanics (hm)

$$n = \left(\hat{\epsilon}_{\downarrow}^{spin}, \hat{\epsilon}_{\uparrow}^{orb}, p_{\uparrow}^{spin}\right)_{hm}, \tag{29}$$

under the Coulomb *attraction* in the macroscopic value of 230 Newton, Eq. (6).

It should be stressed that, in view of the extremely big value of Coulomb attraction (6), *the numeric value of the mean life of the neutron according to model (29) can be subject to scientific debates, but not its existence.*

2.3.1 Representation of the neutron mass, mean life and charge radius

Let us recall the well known essential elements of the non-relativistic, quantum mechanical representation of the Hydrogen atom as a bound state of a proton p and an electron e, which are given by:

1) The geometric representation on the *Euclidean space* $E(r, \delta, I)$ with relative coordinate $r = r_p - r_e$, metric $\delta = \mathrm{Diag.}(1, 1, 1)$, unit $I = \mathrm{Diag.}(1, 1, 1)$, and invariant

$$r^2 = r^i \times \delta_{ij} \times r^j = r_1^2 + r_2^2 + r_3^2 \,. \tag{30}$$

2) The operator representation on the *Hilbert space* $\mathcal{H}$ over the field of complex numbers C with states $|\psi(r)\rangle$, normalization

$$\langle\psi(r)| \times |\psi(r)\rangle = I\,, \tag{31}$$

and expectation value of a Hermitean operator A

$$\langle A\rangle = \langle\psi(r)| \times A \times |\psi(r)\rangle\,. \tag{32}$$

3) The *Schrödinger representation,* comprising the linear momentum

$$p \times |\psi(r)\rangle = -i \times \hbar \times \partial_r |\psi(r)\rangle\,, \tag{33}$$

the eigenvalue equation

$$\begin{aligned} H(r,p) \times |\psi(r)\rangle &= E_H \times |\psi(r)\rangle \\ &= \left[\Sigma_{k=1,2,3}\, \frac{1}{2\,m} \times p_k \times p_k - \frac{e^2}{r}\right] \times |\psi(r)\rangle\,, \end{aligned} \tag{34}$$

where m is the reduced mass

$$m = \frac{m_e \times m_p}{m_e + m_p}\,, \tag{35}$$

and the canonical commutation rules

$$\begin{aligned} &[r^i, p_j] \times |\psi(r)\rangle = (r^i \times p_j - p_j \times r^i) \times |\psi(r)\rangle = \\ &= -i \times \hbar \times \delta^i_j \times |\psi(r)\rangle\,, \\ &[r^i, r^j] \times |\psi(r)\rangle = [p_i, p_j] \times |\psi(r)\rangle = -0\,. \end{aligned} \tag{36}$$

As it is well known, the above formulation characterizes the Hydrogen atom binding energy

$$E_H = 13.6\ \text{eV}\,, \tag{37}$$

stability, Bohr's radius and all other features.

According to studies first done in the 1990 paper [68], completed in the 1995 monograph [73] and updated in Sect. 2 of the 2021 memoir [47], the non-relativistic hadronic treatment of structure model (29) is given by the following, step-by-step, *non-unitary* transformation of the quantum treatment of the Hydrogen atom

$$U \times U^\dagger = \hat{I} = 1/\hat{T} > 0\,, \tag{38}$$

where, for the non-relativistic treatment, we asume iso-unity (20) with value for the density $n_{4,k} = 1,\ k = 1,2$ whose treatment is done at the relativistic level (Sect. 3.4).

In fact, the geometric treatment of model (29) is done in the *iso-Euclidean iso-space* $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ [41,61] over the iso-real iso-field $\hat{R}(\hat{n}, \hat{\times}, \hat{I})$ [41, 89] (see also monograph [29]) with iso-unit (20), iso-coordinates $\hat{r} = UrU^\dagger = r\hat{I}$, iso-metric $\hat{\delta} = \hat{T} \times \delta$ and *iso-invariant*

$$\begin{aligned} \hat{r}^{\hat{2}} &= Ur^2U^\dagger = U(r^i \times \delta_{ij} r^j)U^\dagger = \\ &= (Ur^iU^\dagger)(UU^\dagger)^{-1}[(U\delta_{ij}U^\dagger)(UU^\dagger)^{-1}](Ur^jU^\dagger) = \\ &= \hat{r}^i \hat{\times} \hat{\delta}_{ij} \hat{\times} \hat{r}^j = \left(\frac{r_1^2}{n_1^2} + \frac{r_1^2}{n_1^2} + \frac{r_1^2}{n_1^2}\right)\hat{I}\,, \end{aligned} \tag{39}$$

where the exponential term of iso-unit (20) has been embedded in the characteristic n-quantities, and one should note the final multiplication by $\hat{I}$ which is necessary for the iso-invariant to be an *iso-scalar,* that is an element of $\hat{R}(\hat{n}, \hat{I})$.

The operator treatment of structure model (29) is done in the *Hilbert-Myung-Santilli isospace* [90] over the iso-field of iso-complex iso-numbers $\hat{C}$ [89] with *iso-states*

$$|\hat{\psi}(\hat{r})\rangle = U(|\psi(r)\rangle)U^\dagger\,, \tag{40}$$

iso-normalization

$$\langle\hat{\psi}(\hat{r})|\hat{\times}|\hat{\psi}(\hat{r})\rangle = \langle\hat{\psi}(\hat{r})| \times \hat{T} \times |\hat{\psi}(\hat{r})\rangle = \hat{T}\,, \tag{41}$$

and *iso-expectation values* of an iso-operator

$$\begin{aligned} \hat{\langle}\hat{A}\hat{\rangle} &= \langle\hat{\psi}(\hat{r})|\hat{\times}\hat{A}\hat{\times}|\hat{\psi}(\hat{r})\rangle = \\ &= \langle\hat{\psi}(\hat{r})| \times \hat{T} \times \hat{A} \times \hat{T} \times |\hat{\psi}(\hat{r})\rangle\,. \end{aligned} \tag{42}$$

The reader should note that iso-normalization (41) is characterized by the isotopic element $\hat{T}$ (rather than the iso-unit $\hat{I}$) for consistency because $\hat{T}$ can be a constant as a particular case, but also because from normalization (31), we expect

$$[|\hat{\psi}(\hat{r})\rangle]^\dagger|\hat{\psi}(\hat{r})\rangle = \langle\hat{\psi}(\hat{r})| \times |\hat{\psi}(\hat{r})\rangle = I\,. \tag{43}$$

Iso-Schrödinger iso-representation (see Chapter 5, p. 182 of [42] for a detailed treatment). It should be indicated that despite considerable efforts reviewed earlier, by the early 1990's the hadronic form of the Schrödinger equation was still unknown due to the inapplicability of the Newton-Leibnitz differential calculus in general and in particular, the inapplicability for hadronic mechanics of the conventional form (33) of the quantum mechanical linear momentum, with ensuing inability to compute the iso-Hamiltonian.

The axiomatic origin of this impasse was the incompatibility between the sole applicability of the differential calculus to *isolated points r* compared to isotopic methods which are entirely devoted to the representation of *volumes* via iso-unit $\hat{I} = \hat{I}(r, ...)$, Eq. (20), iso-coordinates $\hat{r} = r\hat{I}(r, ...)$ and iso-functions $\hat{f}(\hat{r}) = [f(r\hat{I})]\hat{I}$.

This impasse left R. M. Santilli with no other option than that of generalizing the Newton-Leibnitz differential calculus from its sole applicability to isolated points r to volumes $\hat{r}$. This generalization was first achieved in the 1994 paper submitted for the 1996 memoir [50] (see the 1995 general study [41, 42] and systematic independent works from 2014 on [33, 34]) via the introduction of the infinite class of *iso-differentials of an iso-coordinate* $\{\hat{d}\hat{r}\}$ on $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ on $\hat{R}$ solely restricted to admit the conventional differential dr for the particular case $\hat{I} = 1$

$$\{\hat{d}\hat{r}\}_{\hat{I}=1} = dr\,, \tag{44}$$

with selected solution (Eq. (1.27), p. 20 of [50])

$$\hat{d}\hat{r} = \hat{T} d[r\hat{I}(r, ...)] = dr + r\hat{T} d\hat{I}(r, ...)\,, \tag{45}$$

consequential *iso-derivative*

$$\frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I} \times \frac{\partial \hat{f}(\hat{r})}{\partial \hat{r}}\,, \tag{46}$$

and finally, the needed expression for the *iso-linear iso-momentum* of hadronic mechanics, first achieved in Sect. 2.5, p. 52 of [50] and Eq. (3.1.10), p. 82 of [42]

$$\hat{p}\hat{\times}|\hat{\psi}(\hat{r})\rangle = -\hat{i}\hat{\times}\hat{\hbar}\hat{\times}\hat{\partial}_{\hat{r}}|\hat{\psi}(\hat{r})\rangle = -i\hat{I}\partial_{\hat{r}}|\hat{\psi}(\hat{r})\rangle\,. \tag{47}$$

By using non-unitary transformations of the type

$$\begin{aligned}
&U\left[\Sigma_{k=1,2,3}\,\frac{1}{2m}\,p_k p_k - \frac{e^2}{r}\right]|\psi(r)\rangle U^\dagger = \\
&= \left[\Sigma_{k=1,2,3}\,\frac{1}{2m}\,(Up_kU^\dagger)(UU^\dagger)^{-1}(Up_kU^\dagger) - \right.\\
&\left.-(U\frac{e^2}{r}U^\dagger)\right](UU^\dagger)^{-1}(U|\psi(r)\rangle U^\dagger = \\
&= U[E|\psi(r)\rangle]U^\dagger = E[U|\psi(r)\rangle U^\dagger] = E|\hat{\psi}(\hat{r})\rangle\,,\\
&U\left(\frac{e^2}{r}\right)U^\dagger = \frac{e^2}{r}\hat{I} = \frac{\hat{I}^2e^2}{\hat{I}r} = \frac{\hat{e}^{\hat{2}}}{\hat{r}}\,.
\end{aligned} \tag{48}$$

Schrödinger's equation (34) for the Hydrogen atom on $\mathcal{H}$ over C is mapped into the *iso-Schrödinger equation for the neutron on iso-space $\hat{\mathcal{H}}$ over the iso-field $\hat{C}$*

$$\begin{aligned}
&\hat{H}(\hat{r},\hat{p})\hat{\times}|\hat{\psi}(\hat{r})\rangle = \\
&= \left[\Sigma_{k=1,2,3}\,\frac{\hbar^2}{2\,m}\,\hat{p}_k\hat{\times}\hat{p}_k - \frac{\hat{e}^{\hat{2}}}{\hat{r}}\right]\hat{\times}|\hat{\psi}(\hat{r})\rangle = E_n|\hat{\psi}(\hat{r})\rangle\,,
\end{aligned} \tag{49}$$

and the canonical commutation rules (36) are mapped into the *iso-canonical iso-commutation rules*

$$\begin{aligned}
&[\hat{r}^i\hat{,}\hat{p}_j]\hat{\times}|\hat{\psi}(\hat{r})\rangle == (\hat{r}^i\hat{T}\hat{p}_j - \hat{p}_j\hat{T}\hat{r}^i)\hat{T}|\hat{\psi}(\hat{r})\rangle = \\
&= -\hat{i}\hat{\times}\hat{\hbar}\hat{\times}\hat{\delta}^i_j\hat{\times}|\hat{\psi}(\hat{r})\rangle = -i\hbar\delta^i_j|\hat{\psi}(\hat{r})\rangle\,,\\
&[\hat{r}^i\hat{,}\hat{r}^j]\hat{\times}|\hat{\psi}(\hat{r})\rangle = [\hat{p}_i\hat{,}\hat{p}_j]\hat{\times}|\hat{\psi}(\hat{r})\rangle = 0\,.
\end{aligned} \tag{50}$$

As one can see, (49) is formally equivalent to (34) and therefore, it can be solved on the iso-space over the iso-field, yielding the following value of the *neutron binding energy* similar to that for the positronium [14]

$$E_n \;\approx\; 7\,\text{eV}\,, \tag{51}$$

by therefore confirming the expectation, from the high centripetal force of the iso-electrons compressed inside the proton, that *the neutron is a quasi-free hadronic bound state of an (iso-)proton and an iso-electron.*

To identify the impact of the non-Hamiltonian interactions in the neutron structure model (29), it is necessary to assume an explicit realization of the isotopic element and iso-unit of (20). We here assume the original realization of Sect. 5.1, p. 827 on of the 1978 memoir [20], merely reformulated according to iso-mathematics and iso-mechanics with the simplifying assumptions $n_{\mu,\alpha} = 1,\ \mu = 1,2,3,4,\ k = 1,2,$

$$\begin{aligned}
&\hat{I} = 1/\hat{T} = UU^\dagger = e^{+\frac{V_h(r)}{V_c(r)}} \approx 1 + \frac{V_h(r)}{v_c(r)} \;\gg\; 1\,,\\
&\hat{T} = (UU^\dagger)^{-1} = e^{-\frac{V_h(r)}{V_c(r)}} \approx 1 - \frac{V_h(r)}{v_c(r)} \;\ll\; 1\,,
\end{aligned} \tag{52}$$

where $V_h(r)$ is the *Hulten potential* first adopted in Eq. (5.1.6) p. 833 of [20]

$$V_h(r) = K\frac{e^{-br}}{1-e^{-br}}\,, \tag{53}$$

with

$$b = R^{-1} \approx 10^{-13}\,\text{cm}\,, \tag{54}$$

and $V_c(r)$ is the conventional Coulomb potential

$$V_c(r) = \frac{e^2}{r}\,. \tag{55}$$

We consider now the *projection* of iso-equation (49) into the conventional Euclidean and Hilbert spaces. By using isotopic element (52), the needed projection can be written

$$\begin{aligned}
&\left[\Sigma_{1,2,3}\frac{\hbar}{2m}(-i\hat{I}\partial_r)(-i\hat{I}\partial_r) - (UV_c(r)U^\dagger)\right]|\psi(r)\rangle = \\
&= E_n|\psi(r)\rangle\,,
\end{aligned} \tag{56}$$

where, in first approximation,

$$UV_c(r)U^\dagger = V_c(r)\hat{I} \approx V_c(r) + V_h(r)\,. \tag{57}$$

But the Hulten potential behaves at very short distances like the Coulomb potential (Eq. (5.1.5) p. 936 of [20]) by therefore absorbing the latter with a mere re-definition K' of the constant K. Consequently, (56) can be reduced in one space dimension to

$$\left[\frac{1}{2m}\,(-i\hat{I}\partial_r)(-i\hat{I}\partial_r) + K'\,\frac{e^{-br}}{1-e^{-br}}\right]|\psi(r)\rangle = E|\psi(r)\rangle\,, \tag{58}$$

whose radial form

$$\left[\frac{1}{r^2}\left(\frac{d}{dr}r^2\frac{d}{dr}\right) + \bar{m}K'\,\frac{e^{-br}}{1-e^{-br}}\right] = 0\,, \tag{59}$$

has been studied in great details in Sect. 5.1, p. 827 on of the 1978 memoir [20], including its full analytic solution with boundary conditions.

By adding the isotopy of the non-relativistic quantum mechanical mean life, yielding the expression (Eq. (5.1.13), p. 835 of [20])

$$\tau^{-1} = 2\pi\lambda^2|\hat{\psi}(0)|^2\,\frac{\alpha^2E_{\hat{e}}}{\hbar}\,, \tag{60}$$

we reach the *hadronic equations for the mass mean life and charge radius of the neutron according to model (29)* (Eq. (5.1.14), p. 836 of [20])

$$\begin{aligned}
&\left[\frac{1}{r^2}\left(\frac{d}{dr}r^2\frac{d}{dr}\right)+\bar{m}\left(E+K'\,\frac{e^{-br}}{1-e^{-br}}\right)\right]|\psi(r)=0\,,\\
&E_n^{tot}=E_p+E_{\bar{\epsilon}}-E_n-E=939.565\,\mathrm{MeV}\,,\\
&\tau^{-1}=2\pi\lambda^2|\hat{\psi}(0)|^2\,\frac{\alpha^2 E_{\hat{\epsilon}}}{\hbar}=877\,\mathrm{s}\,,\\
&R=b^{-1}-10^{-13}\,\mathrm{cm}\,,
\end{aligned}\tag{61}$$

where $\bar{m}$ is the *iso-renormalized reduced mass* (Eq. (5.1.7), p. 833 of [20]), and the last three equations are subsidiary constraints on the first equation.

The analytic solution of the above equations was reduced to the solution of the following two algebraic equations on the parameters k_1 and k_2 (Eq. (5.1.32), p. 840 of [20])

$$\begin{aligned}
&E_n^{tot}=\frac{2\hbar c k_1}{b}\,[k_1-(k_2-1)^2]=939.37\,\mathrm{MeV}\,,\\
&\tau=\frac{48\times(137)^2}{4\pi bc}\,\frac{k_1}{(k_2-1)^3}=877\,\mathrm{s}\,,
\end{aligned}\tag{62}$$

with numeric solutions (Eq. (2.20), p. 521 of [68])

$$k_1=0.34,\quad k_2=1+0.81\times10^{-8}\,.\tag{63}$$

The energy spectrum results to be the typical *finite* spectrum of the Hulten potential (for $k_2=1$)

$$E=\frac{1}{4R^2\bar{m}}\left(\frac{1}{n}-n\right)^2,\quad n=1,2,3,...\tag{64}$$

whose sole consistent solution occurs for $n=1$, as a result of which *the sole possible value for the binding energy E caused by the Hulten potential is null*

$$E=\frac{1}{4R^2\bar{m}}\left(\frac{1}{n}-n\right)^2=0,\quad n=1\,,\tag{65}$$

because, as expected, *all the excited states of neutron structure (29) are the various states of the Hydrogen atom.* Alternatively, the null value of the binding energy E is expected from the fact that *contact, zero-range interactions have no potential energy by central assumption.*

2.3.2 Representation of the neutron spin

The central assumption of hadronic model (29) requires that, to avoid extreme resistive forces, the hadronic angular momentum of the iso-electron $\hat{\epsilon}^-$ be equal to the spin of the proton, thus having value $\hat{L}_{3,\hat{\epsilon}}=1/2$. The study of this assumption was initiated in the 1984 papers on the isotopies of the rotational symmetry [57, 58] and continued in the 1990 paper [68] via the iso-trigonometric iso-functions (see p. 304 on of [41]), under the use of the Lie-Santilli iso-algebra $\widehat{SO}(3)$. Regrettably, we cannot review these studies to avoid an excessive length.

Fig. 4: In this figure, we illustrate some of the hadronic reactors used for the synthesis of the neutron from Hydrogen (see [80] for a complete presentation).

We here present, apparently for the first time, the non-relativistic representation of the hadronic angular momentum $\hat{L}_3=1/2$ under the assumptions that *the orbit of the extended iso-electron within the dense proton is a perfect circle perpendicular to the proton symmetry axis* with radius $R=10^{-13}$ cm. In fact, deviations from the above assumptions imply instabilities generally preventing a representation of the significant (for particle standards) neutron mean life of 887 s, under which assumptions the acting iso-symmetry is the two-dimensional Lie-Santilli iso-group $\widehat{SO}(2)$ [57, 58] (see also Sect. 6.4, p. 233 on of [42]).

Consider the conventional O(2) symmetry which is classically formulated on the two-dimensional Euclidean space $E(z,\delta,I)$, and quantum mechanically treated on a Hilbert space $\mathcal{H}$ over C. By continuing the construction of hadronic models via a non-unitary transformation of quantum models of the preceding section, we map the entire classical and quantum mechanical formulation of O(2) under the nonunitary transformation

$$\begin{aligned}
&UU^\dagger=\hat{I}=1/\hat{T}=\mathrm{Diag.}(n_1^2,n_2^2)=\\
&=\mathrm{Diag.}(b_1^{-2},b_2^{-2})\,,\quad b_k=1/n_k>0,\quad k=1,2\,,
\end{aligned}\tag{66}$$

and represent the orthogonality condition via Bohm's hidden variable [91]

$$\frac{1}{n_1}=\frac{1}{n_2}=b_1=b_2=\lambda>0\,.\tag{67}$$

Therefore, the iso-representation occurs in the two-dimensional iso-Euclidean iso-space $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ over the isofield $\hat{C}$ with iso-coordinates

$$\hat{r} = r\hat{I} = \{x, y\}\lambda^2 I_{2\times 2}\,, \tag{68}$$

iso-metric

$$\hat{\delta} = \hat{T}\delta = \lambda^2\delta\,, \tag{69}$$

iso-invariant

$$\hat{r}^{\hat{2}} = \lambda^2 r^2\,, \tag{70}$$

and iso-trigonometric representation (Appendix 5C, p. 300 on of [41])

$$\begin{aligned} &x = r\lambda^{-1}\cos\hat{\phi}, \quad y = r\lambda^{-1}\sin\hat{\phi}\,, \\ &\hat{\phi} = T_\phi\phi = n_1 n_2\phi = \lambda^{-2}\phi\,, \quad \hat{T}_\phi = b_1{-}1b_2^{-2} = \lambda^{-2}\,. \end{aligned} \tag{71}$$

The iso-unitary and iso-irreducible iso-representations of $\widehat{\mathrm{SO}}(2)$ is defined on the iso-space $\mathcal{H}$ [90] C with iso-states $|\hat{\psi}(\hat{r})\rangle$, iso-normalization (41), iso-generator $\hat{R}(\hat{\phi})$ and related iso-eigenvalues

$$\begin{aligned} &\hat{R}(\hat{\phi})\hat{\times}|\hat{\psi}\rangle = \hat{e}^{iM\hat{\phi}}\hat{\times}|\hat{\psi}\rangle = (e^{i\hat{M}\hat{\phi}})\hat{I}_\psi\hat{\times}|\hat{\psi}\rangle = (e^{i\lambda^2 M\hat{\phi}})|\hat{\psi}\rangle\,, \\ &\hat{M} = b_1 b_2 M = \frac{1}{n_1 n_2}M\,, \\ &\lambda^2 = b_1 b_2 = \frac{1}{n_1}\,\frac{1}{n_2}\,, \end{aligned} \tag{72}$$

(where $\hat{e}$ is, this time, the iso-exponentiation in the ϕ-plane) with Lie-Santilli iso-group laws

$$\begin{aligned} &\hat{R}(\hat{\phi})\hat{\times}\hat{R}(\hat{\phi}')\hat{\times}|\hat{\psi}\rangle = \hat{R}(\hat{\phi}')\hat{\times}\hat{R}(\hat{\phi})\hat{\times}|\hat{\psi}\rangle = \hat{R}(\hat{\phi} + \hat{\phi}')\hat{\times}|\hat{\psi}\rangle\,, \\ &\hat{R}(\phi)\hat{\times}\hat{R}(-\hat{\phi})\hat{\times}|\hat{\psi}\rangle = \hat{R}(0)\hat{\times}|\hat{\psi}\rangle = |\hat{\psi}\rangle\,. \end{aligned} \tag{73}$$

The iso-eigenvalue of the hadronic angular momentum $\hat{L}$ is given by

$$\hat{L}\hat{\times}|\hat{\psi}\rangle = \hat{M}|\hat{\psi}\rangle = \lambda^2 M|\hat{\psi}\rangle\,. \tag{74}$$

But isotopies preserve original numeric values. Therefore,

$$\hat{M} = \lambda^2 M = 0, 1, 2, 3, \ldots\,. \tag{75}$$

Consequently, the angular momentum measured by the experimentalist in our space is given by

$$M = \frac{\hat{M}}{\lambda^2}\,, \tag{76}$$

and can represent the *constrained* angular momentum of the electron inside the proton for the value

$$M = \frac{\hat{M}}{\lambda^2} = \frac{1}{2}\,, \tag{77}$$

resulting in the *numeric value of Bohm's hidden variable,* here presented for the first time,

$$\lambda = \sqrt{b_1 b_2} = \sqrt{\frac{1}{n_1 n_2}} = \sqrt{2} = 1.4142\,, \tag{78}$$

which should be compared with essentially the double value of Bohm's hidden variable for the representation of the Deuteron spin [7, 8].

The total spin of the neutron is then given by

$$S_n = s_p + s_{\hat{\epsilon}} + L_{\hat{\epsilon}} = \frac{1}{2} - \frac{1}{2} + \frac{1}{2}\,. \tag{79}$$

Hence, according to hadronic structure model (29), *the spin of the neutron coincides with the spin of the proton* as expected. Alternatively, we can say that *the total angular momentum of the iso-electron compressed inside the proton is identically null,* with intriguing applications, e.g. for the exact representation of nuclear spins to be studied in a separate work.

For brevity, we leave to the interested reader the representation of the spin $S = 1/2$ of the iso-electron via the iso-symmetry $\hat{SO}(2)$, which can be derived from the above treatment with the $\widehat{\mathrm{SO}}(2)$ symmetry.

The spin of the neutroid according to Fig. 2 is characterized by the following value for the hadronic angular momentum of the iso-electron

$$M = \frac{\hat{M}}{\lambda} = 0\,, \quad \hat{M} = 0\,, \quad \lambda > 0\,. \tag{80}$$

Consequently, *the spin of the neutroid according to Fig. 2 is predicted to be zero,* by therefore explaining the reason for their lack of detection via commercially available neutron detectors.

2.3.3 Representation of the neutron magnetic moment

The anomalous magnetic moment of the neutron according to model (29) has been first represented in the 1990 original paper [68] via the following *three* contributions

$$\mu_n = \mu_p + \mu_{\hat{\epsilon}}^{spin} + \mu_{\hat{\epsilon}}^{orb}\,. \tag{81}$$

The biggest difficulty for the above representation is that the magnetic moment of the electron $\nu_{\hat{\epsilon}}^{spin}$, Eq. (5), is so big for nuclear standard to prevent a quantum mechanical model of the neutron synthesis as well as to prevent that electrons can be members of nuclear structures (Section 1). These insufficiencies are here resolved, apparently for the first time via the magnetic moment of the *orbital* motion of the iso-electron $\mu_{\hat{e}}^{orb}$ which is *opposite* that of the iso-electron (Fig. 3) and its value is predicted to be [68]

$$\mu_{\hat{\epsilon}}^{orb} = 1833.580\,\mu_N\,. \tag{82}$$

By recalling the known values of the magnetic moments of the proton and the neutron [12] $\mu_p = 2.792\,\mu_N$, $\mu_n = -1.913\,\mu_N$, we reach in this way the *numerically exact and time invariant representation of the anomalous magnetic moment of the neutron* [68]

$$\mu_n = \mu_p + \mu_e^{spin} + \mu_{\hat{e}}^{orb} = \\ 2.792\,\mu_N - 1838.285\,\mu_N + 1833.5801\,\mu_N = -1.913\,\mu_N \tag{83}$$

It should be noted that the assumption of the above orbital contribution of the iso-electron not only allows a representation of the *numeric value* of the anomalous magnetic moment of the neutron, but also of its *negative value*.

3 Relativistic representation of the neutron synthesis from the Hydrogen atom

Recall that the relativistic treatment of the Hydrogen atom is based on the rotational symmetry SO(3), the spin symmetry SU(2), the Lorentz symmetry SO(3.1), the Poincaré symmetry $P(3.1) = SO(3.1) \times \mathcal{T}(3.1)$, the spinorial covering of the Poincaré symmetry $\mathcal{P}(3.1) = SL(2, C) \times \mathcal{T}(3.1)$ and related special relativity.

Immediately following the construction in 1983 of the isotopies of the various branches of Lie's theory (Sect. 5.2 on p. 154 of [24]), Santilli constructed the isotopies of the above symmetries and relativities on iso-spaces over iso-fields as a condition to achieve a relativistic representation of the neutron synthesis from the proton and electron, and prove its compatibility with the non-relativistic treatment [57]–[67], including:

1) The rotational iso-symmetry $\widehat{SO}(3)$ [57]–[59].

2) The spin iso-symmetry $\widehat{SU}(2)$ [60].

3) The Lorentz iso-symmetry $\widehat{SO}(3.1)$ [61, 62].

4) The Poincaré iso-symmetry $\hat{P}(3.1) = \widehat{SO}(3.1) \times \hat{\mathcal{T}}(3.1)$ [63, 64].

5) The spinorial covering of the Poincaré iso-symmetry $\hat{\mathcal{P}}(3.1) = \widehat{SL}(\hat{2}, \hat{C}) \times \hat{\mathcal{T}}(3.1)$ [66, 67].

The use of the above iso-symmetries then allowed Santilli to construct the unique and unambiguous isotopies of special relativity for the description of extended particles and electromagnetic waves propagating within a physical medium, known under the name of *special iso-relativity,* or *iso-relativity* for short, which was first presented in the 1983 Nuovo Cimento paper [61] for the classical part and in the adjoining paper [62] for the operator counterpart, and subsequently treated in the 1991 monographs [92, 93] with 1996 update [41]–[43] and in 2021 overview [44] (see also the review in monograph [25] from Santilli's lecture notes at the ICTP, Trieste, Italy, monographs [28, 32], and papers quoted therein).

Note that the above extended scientific journey was necessary for the *time invariant representation of the size and density of extended particles* without which experimental verifications cannot be consistently formulated.

Note also that *iso-symmetries and iso-relativities coincide at the abstract level with conventional symmetries and relativities.* Therefore, the representation of the dynamics within physical media solely occur in their *projection* on conventional spaces over conventional fields.

Therefore, *the same symmetries and relativities represent, at the abstract level, both the Hydrogen atom and the neutron.* All differences between the two bound states of a proton and an electron solely occur in their *realizations.*

3.1 The main open problem for particle fusions

As indicated in Sect. 5, p. 819 on of the 1978 Harvard University memoir [20], hadronic mechanics was proposed and constructed not only for a more accurate representation of *nuclear fusions,* but also for the representation of *particle fusions* (also called synthesis), beginning with the fusion of the proton and the electron into the neutron. Additionally, the 1978 memoir [20] proposed isotopic methods for the representation of the structure of *unstable* particles as hadronic bound states of lighter particles and antiparticles generally produced free in their spontaneous decays.

While the quantum mechanical point-like abstraction of particles and nuclei has provided a first approximation of nuclear fusions, quantum mechanics is inapplicable for the representation of particle fusions (Sect. 1.2) due to the *mass excess/rest energy excess,* namely, the mass of the synthesized particle is bigger than the sum of the masses of the constituents as it is clearly the case for the neutron synthesis, (7) while by comparison, nuclear fusions cause the well known *mass defect/energy defect.*

Following the identification of the open problem of the neutron synthesis, in Sect. 5.1, p. 827 on of [20], Santilli achieved the first known representation of *all* characteristics of the π^0 meson as the hadronic bound state (i.e. the fusion) of a mutated electron, then called *eleton* ϵ^- (more recently called *iso-electron*) and a mutated positron ϵ^+,

$$\pi^0 = (\epsilon^-_\uparrow, \epsilon^+_\downarrow)_{hm} \,. \tag{84}$$

This proposal was based in the following experimental evidence: 1) The extremely big Coulomb attraction (6) between the $\epsilon^- - \epsilon^+$ constituents. 2) The spontaneous decay of the π^0 into an electron and a positron

$$\pi^0 \;\rightarrow\; e^- + e^+ \,, \quad 7.5 \times 10^{-8} \,, \tag{85}$$

via a process interpreted as a *hadronic tunnel effect of the constituents.* 3) The π^0 primary decay which is evidently due to electron-positron annihilation

$$\pi^0 \;\rightarrow\; \gamma + \gamma \,, \quad 98.5\% \,, \tag{86}$$

which decay allowed the first known identification of the *mechanism* triggering the spontaneous decay of the π^0 and the *exact* representation of its mean life $\tau = 0.828 \times 10^{-16}$ s. The extension of the model to all remaining mesons was also proposed in the same section 5.1 of [20].

Another important aim of Sect. 5.1 of [20] was to show that quantum mechanics is completely inapplicable for any structure model of the *pi*^{0} as a bound state of lighter constituents due to the *rest energy excess* similar to that for the neutron (7) which, for the case of model (84) is given by

$$\pi^0 = (\hat{e}^-_\uparrow, \hat{e}^+_\downarrow)_{hm}\,, \quad \Delta E = -133.954\,\text{MeV}\,. \tag{87}$$

In the author's view, the indicated inapplicability of quantum mechanics for the *structure* of particles, jointly with the unavailability at the time of a suitable covering method, explains (and justifies) the sole studies of particles in the 20th century via *classification* methods, such as *mass spectra* that as such, has never produced a structure equation for any particle.

In the subsequent Sect 5.2, p. 849 on of [20] (see also the recent confirmations [48, 98, 99]), Santilli confirmed the results of Sect. 5.1 by reaching the first known representation of *all* characteristics of the $\mu^\pm$ leptons via the hadronic structure model (i.e. particle fusion)

$$\mu^\pm_\uparrow = (\epsilon^-_\uparrow, \epsilon^\pm_\uparrow, \epsilon^+_\downarrow)_{hm}\,, \tag{88}$$

on the experimental ground that the $\mu^\pm$ leptons decay spontaneously into the indicated constituents via a hadronic tunnel effect

$$\mu^\pm \;\rightarrow\; e^- + e^\pm + e^+\,, \quad 1.0\times 10^{-12}\,, \tag{89}$$

while the electron-positron pair annihilation explains the spontaneous character of the decay and its mean life, which annihilation is experimentally confirmed by the muon decay

$$\mu^\pm \;\rightarrow\; e^\pm + 2\gamma\,, \quad 7.2\times 10^{-11}\,. \tag{90}$$

Santilli concluded Sect. 5.2, p. 849 on of [20] by indicating the complete inapplicability of quantum mechanics for any structure model of the leptons with lighter constituents due to the rest energy excess

$$\mu^\pm_\uparrow \;= (\epsilon^-,_\uparrow \epsilon^\pm_\uparrow, \epsilon^+_\downarrow)_{hm} \;\rightarrow\; \Delta E = -104.636\,\text{MeV}\,. \tag{91}$$

The extension of the model to the remaining (unstable) leptons was also proposed in the same section 5.2 of [20].

The use of hadronic mechanics under the same principles (the physical constituents of unstable particles are produced free in the spontaneous decays) allowed similar structure models of unstable baryons, such as the model for the Λ^0 [48, 100]

$$\Lambda^0_\uparrow = (\hat{p}^+_\uparrow, \hat{\pi}^-)_{hm}\,, \tag{92}$$

(where the "hat" indicates isotopic mutation due to total mutual immersion) based on the primary spontaneous decay

$$\Lambda^0_\uparrow \;\rightarrow\; p^+_\uparrow + \pi^-\,, \quad 20-30\%\,, \tag{93}$$

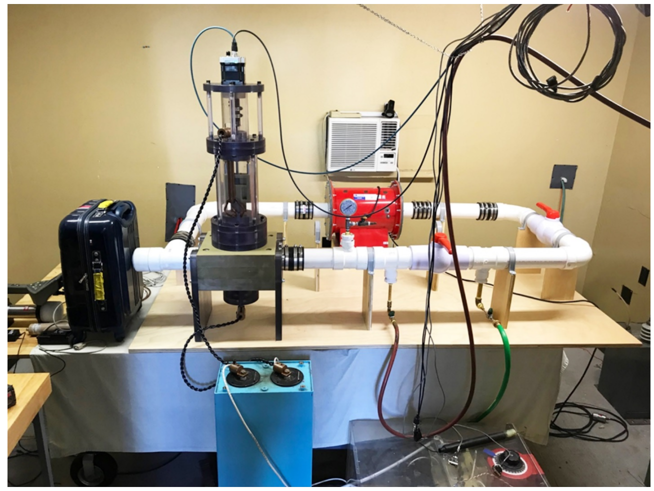

Fig. 5: In this picture, we illustrate the Directional Neutron Source (DNS) produced and sold by Thunder Energies Corporation (now Hadronic Technologies Corporation) generating a flux of thermal neutrons in the desired direction and intensity. The DNS is suggested for the detection of fissile material that may be hidden in baggages, and other applications.

with rest energy excess

$$\Lambda^0_\uparrow = (\hat{p}^+_\uparrow, \hat{\pi}^-)_{hm} \quad \Delta E = -37.812\,\text{MeV}\,. \tag{94}$$

The extension of the above hadronic structure model to the remaining (unstable) baryons is left to the interested reader.

The compatibility of the above *structure* models of unstable particles with their known *classification* was shown to be possible via the *iso-units* of the representations that turned out to be different for different particles (see Fig. 4 for mesons and Fig. 12 for baryons of [48]).

Evidently, the indicated excess energies are *physically* acquired by the constituents. The open problem to be addressed in this section is that we are currently unable to calculate the kinetic energy of an extended particle moving within a dense hadronic medium. Consequently, in this section we shall review and upgrade the isotopic methods used for the *geometric* representation of the excess energy for the neutron, in a form extendable to all other particle fusions.

3.2 Iso-Minkowskian iso-spaces

As it is well known, the Minkowski space in 3 + 1-dimensions provides a geometric representation of the *homogeneity and isotropy of empty space.* By contrast, the primary function of the *Minkowski iso-space* in 3+1-dimensions (first proposed in the 1983 paper [61] and called *Minkowski-Santilli iso-space*) is to provide a geometric representation of the *inhomogeneity and anisotropy of physical media.*

Let $M(x, \eta, I)$ be the conventional Minkowski space over the reals $\mathcal{R}$ with space time coordinates, metric, unit and invariant

$$x = \{x^\mu\} = \{x^1, x^2, x^3, x^4 = ct, \},$$
$$\eta = \text{Diag.}(1,1,1,-1), \quad I = \text{Diag.}(1,1,1,1), \tag{95}$$
$$\mu, \nu = 1,2,3,4,$$

and invariant

$$x^2 = x^\mu \eta_{\mu,\nu} x^\nu = x_1^2 + x_2^2 + x_3^2 - t^2 c^2. \tag{96}$$

Relativistic isotopic methods, including most importantly the Lie-Santilli iso-theory [24] (see also independent studies [25, 26] and review [47]), are uniquely and unambiguously characterized by the conventional Minkowski space $M(x, \eta, I)$ and the infinite family of positive-definite isotopic elements which, for the case of iso-relativities, are assumed to have the simplified form of the general expression (20)

$$\begin{aligned} T &= 1/\hat{I} = \text{Diag.}\left(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2}\right) = \\ &= \text{Diag.}\left(b_1^2, b_2^2, b_3^2, b_4^2\right), \quad n_\mu > 0, \quad b_\mu > 0, \end{aligned} \tag{97}$$

where we have indicated the characteristic quantities $b_\mu = 1/n_\mu$ mostly used in the early literature in the field, and the exponent of isotopic element (20) is embedded in the characteristic quantities to be factored out whenever needed.

Relativistic methods are then formulated on the infinite family of *iso-Minkowski iso-spaces* $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I})$ over the iso-real iso-field $\hat{\mathcal{R}}$ with iso-unit $\hat{I} = 1/\hat{T}$ [89], *iso-coordinates*

$$\hat{x} = x\hat{I} = \left(\frac{x_1}{n_1}, \frac{x_2}{n_2}, \frac{x_3}{n_3}, \frac{x_4}{n_4} = t^2 \frac{c}{n_4}\right), \tag{98}$$

iso-metric

$$\hat{\Omega} = \hat{\eta}\hat{I} = (\hat{T}\eta)\hat{I}, \tag{99}$$

where one should note the final multiplication by $\hat{I}$ as a necessary consistency condition for the *iso-metric to be an iso-matrix* (namely, a matrix whose elements are iso-numbers) [41]), and *iso-invariant*

$$\begin{aligned} &\hat{x}^{\hat{2}} = \hat{x}^\mu \hat{\times} \hat{\Omega}_{\mu,\nu} \hat{\times} x^\nu = \hat{x}_1^2 + \hat{x}_2^2 + \hat{x}_3^2 - t^2 \hat{c}^2 = \\ &\hat{x}^\mu \hat{T} \hat{\Omega}_{\mu,\nu} \hat{T} \hat{x}^\nu = x^\mu \hat{\eta}_{\mu,\nu} x^\nu = \\ &\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{n_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2}, \end{aligned} \tag{100}$$

illustrating the *identity* at the abstract level between the Minkowski invariant (85) and its iso-Minkowskian image in the first line of invariant (88), all differences occurring in the projection of the latter in the conventional Minkowski space.

It should be noted that, as it was the case for isotopic element (20), the iso-metric has the Minkowskian topological structure $(+, +, +, -)$ but an unrestricted functional dependence on local (space time) coordinates x, momenta p, acceleration a, energy E, density d, pressure π, temperature τ, wave function ψ, and any other needed local variable,

$$\hat{\eta}_{\mu\nu} = \hat{\eta}_{\mu\nu}(x, p, a, E, d, \pi, \tau, \psi, ...). \tag{101}$$

Consequently, the *iso-Minkowskian geometry* with iso-invariant (89) (first introduced in the 1996 paper [50] on the iso-differential calculus and treated in more details in the 1998 paper [67]) is the most general possible geometry with a symmetric invariant in (3 + 1)-dimensions, thus including in particular the Minkowskian, Riemannian, Fynslerian and other geometries (see Sect. 3.8 for details).

3.3 The Fundamental theorem on iso-symmetries

The following theorem was first presented in the 1983 paper [61] and upgraded in Section 4.6, page 169 on of [41] as well as in other publications.

3.3.1. FUNDAMENTAl THEOREM ON ISO-SYMMETRIES: Let G be an N-dimensional Lie symmetry of a K-dimensional space $S(x, m, F)$ with coordinates x and metric m over a numeric field F,

$$G: \quad x' = a(w)x, \quad y' = a(w)y, \quad x, y \in S, \quad w \in F, \tag{102}$$

leaving invariant the interval

$$(x' - y')^\dagger m (x' - y') \equiv (x - y)^\dagger m (x - y) \tag{103}$$

with main property

$$\begin{aligned} &(x' - y')^\dagger a^\dagger(ww) m a(ww)(x - y) \equiv (x - y)^\dagger m (x - y), \\ &a^\dagger(w) m a(w) \equiv m, \quad \forall x, y \in S. \end{aligned} \tag{104}$$

Then, all infinitely possible iso-symmetries $\hat{G}$ on iso-spaces $\hat{S}(\hat{x}, \hat{M}, \hat{F})$, where $\hat{M} = \hat{m}\hat{I} = (\hat{T}_i^k m_{kj})\hat{I}$ over iso-fields $\hat{F}$ with iso-unit $\hat{I} = 1/\hat{T}$

$$\begin{aligned} &\hat{G}: \quad \hat{x}' = \hat{A}(\hat{w}) \hat{\times} \hat{x} = (\hat{a}\hat{I})\hat{T}\hat{x} = \hat{a}\hat{x}, \\ &\hat{y}' = \hat{A}(\hat{w}) \hat{\times} \hat{y} = (\hat{a}\hat{I})\hat{T}\hat{y} = \hat{a}\hat{y}, \quad \forall \hat{x}, \hat{y} \in \hat{S}, \quad \hat{w} \in \hat{F}, \end{aligned} \tag{105}$$

leave invariant the iso-interval

$$\begin{aligned} &(\hat{x}' - \hat{y}')^\dagger \hat{\times} \hat{A}^\dagger(\hat{w}) \hat{\times} \hat{M} \hat{\times} \hat{A}(\hat{w}) \hat{\times} (\hat{x} - \hat{y}) \equiv \\ &\equiv (\hat{x} - \hat{y})^\dagger \hat{\times} \hat{M} \hat{\times} (\hat{x} - \hat{y}), \end{aligned} \tag{106}$$

with main property

$$\hat{A}^\dagger(\hat{w}) \hat{\times} \hat{M} \hat{\times} \hat{A}(\hat{w}) \equiv \hat{M}, \quad \forall \hat{x}, \hat{y} \in \hat{S}, \quad \hat{w} \in \hat{F}, \tag{107}$$

and all so-constructed iso-symmetries $\hat{G}$ are isomorphic to the original symmetry G.

The verification of the above theorem by all space time iso-symmetries [57]–[67] is an instructive exercise by the interested reader. Note that all iso-symmetries are uniquely and unambiguously characterized by the original symmetry and the infinite class of possible isotopic elements $\hat{T} > 0$.

We finally note that the *iso-exponentiation* [41]

$$\hat{e}^{X_k w_k} = (e^{X_k \hat{T} w_k})\hat{I} = \hat{I}(e^{w_k \hat{T} X_k}), \tag{108}$$

allows the explicit construction of iso-transformations (105).

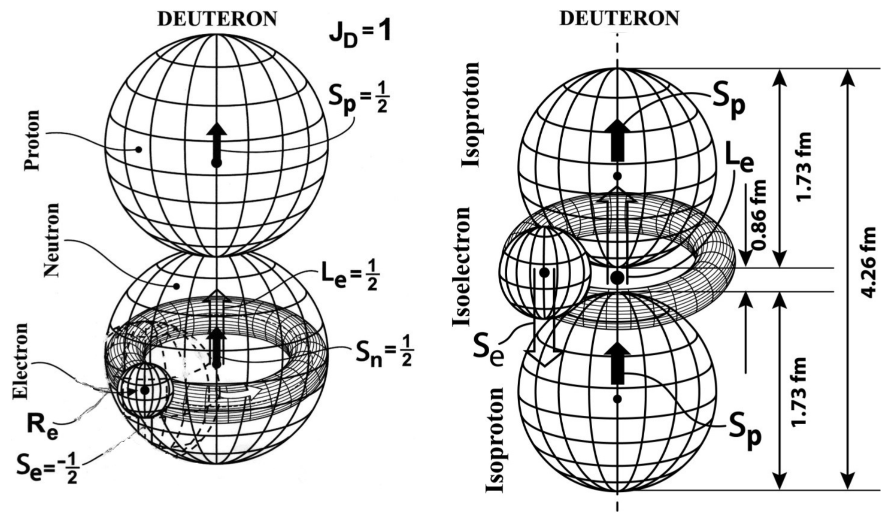

Fig. 6: In the left view, we illustrate the new axial triplet coupling of a proton and a neutron that has achieved the first known representation of the spin $S_D = 1$ of the Deuteron in its true ground state, that with null orbital contributions (Insufficiency II of Sect. 1.2) [44, 102]. In the right view, we illustrate the decoupling of the iso-electron from the neutron to achieve the first known representation of the stability of the Deuteron despite the natural instability of the neutron (Insufficiency IV of Sect. 1.2). Note that said stability is possible if and only if the proton and the electron are the actual physical constituents of the neutron.

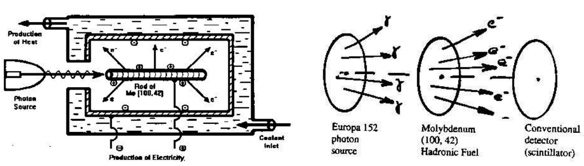

Fig. 7: In thus figure, we illustrate the stimulated nuclear transmutations (174) which are predicted to be triggered by irradiation with resonating photons γ_r with energy $E_r = E_{\hat{e}} = 1,293\,\mathrm{MeV}$ [115] which has been tentatively verified in [117]. In this figure, we reproduce the original drawing of paper [115] showing (from the left) a beam of resonating photons irradiating a cylinder of Mo(100, 42, 0) which emits electrons easily trapped by a metal casing with the production of a clean DC electric current of nuclear origin, plus clean heat triggered by said metal screen absopbing electrons with 0.782 MeV kinetic energy. In the right of this figure, we reproduce the original figure of paper [115] illustrating the simplicity as well as the low cost of experimental verifications consisting of the purchase of a small sample of the commercially available radioisotope Europium-152 (emitting photons with E_r) and of the pure isotope Mo(100, 42, 0), which samples are placed next to each other. In the event of confirmation of the emission of electrons from the Mo(100, 42, 0) sample, or of traces of Ru(100, 44, 0) in the originally pure sample of Mo(100, 42, 0), the production of clean nuclear energies via stimulated neutron decays would be confirmed.

3.4 Lorentz iso-symmetries

As it is well known, the Lorentz symmetry characterizes the propagation of point-like particles and electromagnetic waves in the homogeneous and isotropic vacuum represented by the Minkowskian space. The six generators of the connected component of the Lorentz algebra $so(3.1)$ are given by the (Hermitean) generators of rotations J_k, $k = 1, 2, 3$ and the Lorentz boosts M_k on the Hilbert space $\mathcal{H}$ over the field of complex numbers $calC$ with commutation rules

$$
\begin{aligned}
&[J_i, J_j] = -\epsilon^k_{i,j} J_k\,,\\
&[M_i, M_j] = c^2 \epsilon^k_{ij} J_k, \quad [J_i, M_j] = -\epsilon^k_{ij} M_k\,.
\end{aligned}
\tag{109}
$$

The exponentiation of the above commutation rules according to Lie's theorems yields the celebrated *Lorentz transformations* on the (3, 4)-Minkowski space $M(x, \eta, I)$ according to Theorem 3.2.1

$$
x'^3 = \gamma\left(x^3 - vt\right), \quad x'^4 = \gamma\left(t - \frac{vx^3}{c^2}\right), \tag{110}
$$

where

$$
\beta = \frac{v^2}{c^2}\,, \quad \gamma = \frac{1}{\sqrt{(1-\beta^2)}}\,, \tag{111}
$$

whose historical role has been *the invariance of the speed of light c in vacuum expressed in line element (84).*

The infinite family of *Lorentz iso-symmetries* $\widehat{\mathrm{SO}}(3.1)$, first introduced in the 1983 Nuovo Cimento paper [61] following the preparatory papers on the *rotational and spin iso-symmetries* [57]–[60] (whose knowledge is here assumed to prevent a prohibitive length), are defined on the Iso-Minkowski iso-spaces $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I})$.

The Lie-Santilli iso-algebra $\widehat{\mathrm{SO}}(3.1)$ is characterized by six iso-generators defined on the Hilbert-Myung-Santilli iso-space $\hat{\mathcal{H}}$ [90] over the iso-complex iso-field $\hat{C}$ [89] (Eq. (54), p. 44 of [58])

$$\hat{J}_k = J_k \hat{I}\,, \quad \hat{M}_k = M_k \hat{I}\,, \tag{112}$$

(where J_k, M_k are the conventional 4-dimensional matrix generators of SO(3.1)) with explicit expressions (Eq. (10), p. 550 of [61])

$$\begin{aligned} &\hat{J}_1 = n_2 n_3 J_1\,, \quad \hat{J}_2 = n_1 n_3 J_2\,, \quad \hat{J}_3 = n_1 n_2 J_3\,, \\ &\hat{M}_k = n_k M_k\,, \end{aligned} \tag{113}$$

with iso-commutation rules

$$\begin{aligned} &[\hat{J}_i \hat{,} \hat{J}_j] = \hat{J}_i \hat{\times} \hat{J}_j - \hat{J}_j \hat{\times} \hat{J}_i = -\epsilon^k_{ij} \hat{J}_k\,, \\ &[\hat{M}_i \hat{,} \hat{M}_j] = c^2 \epsilon^k_{ij} \hat{J}_k, \quad [\hat{J}_i \hat{,} \hat{M}_j] = -\epsilon^k_{ij} \hat{M}_k\,, \end{aligned} \tag{114}$$

with related Casimir iso-invariant (Eq. (13), p.551 of [61])

$$\begin{aligned} &\hat{C}_1 = \hat{J}^{\hat{2}} - \frac{1}{c} \hat{M}^{\hat{2}} = -3\hat{I}\,, \\ &\hat{C}_2 = \hat{J} \hat{\times} \hat{M} = \hat{J}_k \hat{T}^{kk} \hat{M}_k = 0\,. \end{aligned} \tag{115}$$

The realization of the Lorentz-Santilli iso-group via iso-exponents (108) (Eq. (11), p. 550 of [61]) yields the *Lorentz-Santilli iso-transformations* in the (3, 4) plane (see [42] for the general case) for motion of an extended particle with speed v along the x^3-axis *under the initial assumption that its density has unit value,* $n_4 = 1$ (Eq. (15), p. 551 of [61])

$$\begin{aligned} &x'^3 = \hat{\gamma}(x^3 - vt)\,, \\ &t' = \hat{\gamma}\left(t - \frac{v b_3^2 x^3}{c^2}\right) = \hat{\gamma}\left(t - \frac{v x^3}{n_3^2 c^2}\right), \end{aligned} \tag{116}$$

where

$$\hat{\beta} = \frac{v^2 b_3^2}{c^2}, \quad \hat{\gamma} = \frac{1}{\sqrt{(1 - \hat{\beta}^2)}}\,. \tag{117}$$

By reinstating generic values of the density $n_4 \neq 1$, and by noting that

$$\begin{aligned} &\hat{\beta} \frac{n_3}{n_4} = \frac{v_3/n_3}{c/n_4} \frac{n_3}{n_4} = \frac{v_3}{c}\,, \\ &\hat{\beta} \frac{n_4}{n_3} = \frac{v_3/n_3}{c/n_4} \frac{n_4}{n_3} = \frac{v_3}{c} \frac{n_4^2}{n_3^2}\,, \end{aligned} \tag{118}$$

iso-transforms (116) acquire the symmetrized form [42, 44, 93]

$$\begin{aligned} &x'^1 = x^1\,, \quad x'^2 = x^2\,, \\ &x'^3 = \hat{\gamma}\left(x^3 - \hat{\beta}\, \frac{n_3}{n_4}\, x^4\right) = \hat{\gamma}\left(x^3 - \hat{\beta}\, \frac{b_4}{b_3}\, x^4\right), \\ &x'^4 = \hat{\gamma}\left(x^4 - \hat{\beta}\, \frac{n_4}{n_3}\, x^3\right) = \hat{\gamma}\left(x^4 - \hat{\beta}\, \frac{b_3}{b_4}\, x^3\right), \end{aligned} \tag{119}$$

where

$$\hat{\beta} = \frac{v_3}{c} \frac{n_4}{n_3}\,, \quad \hat{\gamma} = \frac{1}{\sqrt{1 - \hat{\beta}^2}}\,, \tag{120}$$

which achieved in 1983: 1) The invariance of the iso-Minkowskian line element (100). 2) The first known invariant description of extended, thus deformable particles or extended wave packets with semi-axes n_1^2, n_2^2, n_3^2 propagating within a physical medium with density n_4. 3) The first known *invariance of the local speed of light propagating within transparent physical media,* called *iso-light*

$$C = \frac{c}{n_4} \lesseqqgtr c\,, \tag{121}$$

which, as we shall see, is generally *smaller* than c ($n_4 > 1$) for media of low density (such as Earth's atmosphere) and *bigger* than c ($n_4 < 1$) for media of high density (such as hadrons).

The following comments are in order:

3.4.1. On historical grounds, let us recall that Lorentz first attempted the invariance of the local speed of light of his times, $C = c/n_4$, but had to restrict his study to the invariance of the constant speed c, due to unsolvable technical difficulties caused by the fact that Lie's theory is notoriously a *linear* problem, while the invariance of local speeds (121) is a highly *non-linear* problem. Hence, the Lie-Santilli iso-theory was constructed with a non-linear structure precisely for the solution of the historical Lorentz problem.

3.4.2. As it is well known, Albert Einstein justly received the 1921 Nobel Prize in Physics for the *quantized absorption,* and not for the *quantized propagation* of light. For the evident intent of preserving special relativity, light propagating within physical media is generally reduced to photons scattering among the molecules of the medium, thus traveling in vacuum with speed c rather than the Lorentz speed $C = c/n_4$. While the quantized *absorption* of light is a historical reality, the quantized *propagation* of light reduced to photons is disproved by experimental evidence, e.g. because it has to occur for all visible frequencies, thus including the reduction to photons of infrared light (see Inconsistencies 1 to 7 in Sect. 8.4.4, p. 134 of [44]). As one can see, the only way known to this author to resolve these inconsistencies is that *visible light propagating within transparent physical media is a "wave" with local speed (121),* as well known since Lorentz's time.

3.4.3. It was generally believed in the 20th century that the Lorentz symmetry is broken for locally varying speed of light within physical media. In reality, *the axioms of the Lorentz symmetry are fully preserved under isotopies* in view of the evident isomorphism SO(3.1) ≈ $\widehat{\mathrm{SO}}(3.1)$.

3.4.4. It was also believed in the 20th century that *the axioms of the Lorentz symmetry do not allow speeds bigger than c,* with ensuing academic opposition for the initiation studies on interstellar travel, and other problems implying speeds

$C > c$. In 1982, Santilli [95] pointed out that *strong interactions may accelerate particles faster than the speed of light in vacuum* under the admission that strong interactions have a contact non-potential component between the extended protons and neutrons. In fact, the acceleration of point-particles via potential energy up to c notoriously requires infinite energy. By contrast, non-potential interactions can accelerate particles without any use of potential energy and, in any case, special relativity is inapplicable under non-potential interactions.

3.4.5. In 1997, Santilli [96] (see the 2016 update [97]) showed that the following simple transformation of the Minkowski coordinates

$$x^\mu \ \rightarrow \ \tilde{x}^\mu = \frac{x^\mu}{n_\mu}\,, \tag{122}$$

maps the conventional Minkowski invariant (96) with maximal speed c into iso-invariant (100) for which the local speed of light (121) is arbitrary.

To conclude, in view of the isomorphism $\widehat{\mathrm{SO}}(3.1) \approx \mathrm{SO}(3.1)$, we can state that *the abstract axioms of the Lorentz symmetry do indeed predict arbitrary speeds of light.*

3.5 Poincaré iso-symmetries

Consider the conventional Poincaré symmetry on the Minkowskian space $M(x,\eta,I)$ over a field F, as the semi-direct product of the Lorentz symmetry SO(3.1) and the translations in space time $\mathcal{T}(3.1)$,

$$\mathrm{P}(3.1) = \mathrm{SO}(3.1) \times \mathcal{T}(3.1)\,, \tag{123}$$

with generators

$$J_{\mu\nu} = \{J_k, M_k\}, \ \ P_\mu \ \ \mu,\nu = 1,2,3,4, \ \ k-1,2,3\,, \tag{124}$$

commutation rules

$$\begin{aligned} &\left[J_{\mu\nu}, J_{\alpha\beta}\right] = i(\eta_{\nu\alpha}J_{\beta\mu} - \eta_{\mu\alpha}J_{\beta\nu} - \eta_{\nu\beta}J_{\alpha\mu} + \eta_{\mu\beta}J_{\alpha\nu})\,,\\ &\left[J_{\mu\nu}, P_\alpha\right] = i(\eta_{\mu\alpha}P_\nu - \eta_{\nu\alpha}P_\mu), \quad \left[P_\mu, P_\nu\right] = 0\,, \end{aligned} \tag{125}$$

and Casimir invariants

$$\begin{aligned} &C_1 = I\,,\\ &C_2 = P^2 = P_\mu P^\mu, (\eta_{\mu\nu}P^\mu P^\nu)\,,\\ &C_3 = W^2 = W_\mu W^\mu, \ \ W_\mu = \epsilon_{\mu\alpha\beta\rho}J^{\alpha\beta}P^\rho\,. \end{aligned} \tag{126}$$

The infinite family of *Poincaré iso-symmetries,* first presented by Santilli in 1993 at the Department of Physics of Moscow State University [63, 64], also called the *Poincaré-Santilli iso-symmetries*

$$\hat{\mathrm{P}}(3.1) = \widehat{\mathrm{SO}}(3.1)\hat{\times}\hat{\mathcal{T}}(3.1)\,, \tag{127}$$

is defined on iso-Minkowski iso-spaces $\hat{M}(\hat{x},\hat{\Omega},\hat{I})$ (where $\hat{\Omega}$-$= \hat{\eta}\hat{I}$) over iso-real iso-field $\hat{\mathcal{R}}$ with iso-generators from definition (110)

$$\{\hat{J}_{\mu\nu}\} = \{\hat{J}_k, \hat{M}_k\}\,, \quad \hat{P}_\mu\,, \tag{128}$$

and iso-commutation rules [63, 64]

$$\begin{aligned} &\left[\hat{J}_{\mu\nu}\hat{,}\hat{J}_{\alpha\beta}\right] == i(\hat{\eta}_{\nu\alpha}\hat{J}_{\beta\mu} - \hat{\eta}_{\mu\alpha}\hat{J}_{\beta\nu} - \hat{\eta}_{\nu\beta}\hat{J}_{\alpha\mu} + \hat{\eta}_{\mu\beta}\hat{J}_{\alpha\nu})\,,\\ &\left[\hat{J}_{\mu\nu}\hat{,}\hat{P}_\alpha\right] = i(\hat{\eta}_{\mu\alpha}\hat{P}_\nu - \hat{\eta}_{\nu\alpha}\hat{P}_\mu)\,,\\ &\left[\hat{P}_\mu\hat{,}\hat{P}_\nu\right] = 0\,, \end{aligned} \tag{129}$$

where one should note the appearance of the *structure functions* with the functional dependence (98), i.e. $\hat{\eta}(x,p,a,E,$-$d,\pi,\tau,\psi,....)$, rather than the traditional structure constants. Consequently, the Poincaré-Santilli iso-symmetry is *irregular*, namely, it cannot be obtained from the original symmetry via non-unitary transforms, as it is the case for regular Lie-Santilli iso-algebra [101].

The use of iso-commutation rules (128) yields the *Casimir-iso-invariants* [63, 64]

$$\begin{aligned} &\hat{C}_1 = \hat{I} > 0\,,\\ &\hat{C}_2 = \hat{P}^{\hat{2}} = \hat{P}_\mu\hat{\times}\hat{P}^\mu = (\hat{\eta}^{\mu\nu}P_\mu P_\nu)\hat{I} =\\ &\quad = \left(\sum_{k=1,2,3}\frac{1}{n_k^2}\,\hat{P}_k^2 - \frac{c^2}{n_4^2}\,\hat{P}_4^2\right)\hat{I}\,,\\ &\hat{C}_3 = \hat{W}^{\hat{2}} = \hat{W}_\mu\hat{\times}\hat{W}^\mu, \ \ \hat{W} = W\hat{I}\,,\\ &\hat{W}_\mu = \hat{\epsilon}_{\mu\alpha\beta\rho}\hat{\times}J^{\alpha\beta}\hat{\times}\hat{P}^\rho\,, \end{aligned} \tag{130}$$

and they are at the foundation of classical and operator *relativistic iso-mechanics* with deep implications for structure models of particles, nuclei and stars.

Note that *all possible Poincaré-Santilli iso-symmetries are isomorphic to the conventional Poincaré symmetry.* However, the conventional Poincaré symmetry is *linear* in view of the commutativity of the linear momenta $[P_\mu, P_\nu] = 0$, while the *Poincaré-Santilli iso-symmetry is iso-linear* because the property $[\hat{P}_\mu\hat{,}\hat{P}_\nu] = 0$ holds on iso-spaces over iso-fields, but its projection into conventional spaces over conventional fields is not, in null, $[\hat{P}_\mu, \hat{P}_\nu] \neq 0$, with ensuing non-linearity of the theory. Consequently, the iso-translations $\hat{\mathcal{T}}(3.1)$ are generally nonlinear.

3.6 Dirac iso-equations

As it is well known, the Dirac equation achieved a justly historical role for the relativistic representation of the electron of the Hydrogen atom under the external field of the proton. The infinite family of *Dirac iso-equations,* first introduced in the 1995 papers [65, 66] and called the *Dirac-Santilli iso-equations* have been constructed for the relativistic representation of the iso-electron of the neutron while considering the proton as external.

The Dirac equation is generally obtained via the linearization of the second order Casimir invariant of the Poincaré symmetry (125). The Dirac-Santilli iso-equations are then best obtained via the linearization of the second order iso-Casimir invariant (130).

The carrier iso-spaces of the Dirac-Santilli iso-equations are given by the iso-product of the real-valued, *orbital* (or) iso-Minkowskian iso-spaces and of the complex-valued, *spin* (sp) iso-Euclidean iso-space

$$\hat{M}^{tot} = \hat{M}(\hat{x}, \hat{\Omega}^{or}, \hat{I}^{or}) \hat{\times} \hat{E}(\hat{z}, \hat{\Delta}^{sp}, \hat{I}^{sp}), \tag{131}$$

with orbital specifications

$$\begin{aligned} &\hat{x} = x\hat{I}^{or}, \quad \hat{\Omega}^{or} = \hat{\eta}^{or}\hat{I}^{or} = (\hat{T}^{or}\eta)\hat{I}^{or}, \\ &\hat{I}^{or} = \text{Diag.}(n_1^2, n_2^2, n_3^2, n_4^2) = 1/\hat{T}^{or} > 0, \\ &\hat{x}^{\hat{2}} = \hat{x}^{\mu}\hat{\times}^{or}\hat{\Omega}^{or}_{\mu\nu}\hat{\times}^{or}\hat{x}^{\nu} = \left(\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - \frac{x_4^2}{n_4^2}\right)^{or}, \end{aligned} \tag{132}$$

and spin specifications

$$\begin{aligned} &\hat{z} = (z_1, z_2)\hat{I}^{sp}, \quad \hat{\Delta}^{sp} = \hat{\delta}^{sp}\hat{I}^{sp} = (\hat{T}^{sp}\delta)\hat{I}^{sp}, \\ &\hat{I}^{sp} = \text{Diag.}(\lambda^{-1}, \lambda) = 1/\hat{T}^{sp} > 0, \ \text{Det.}\hat{I}^{sp} = 1, \\ &\hat{z}^{\hat{2}} = \hat{z}^{i}\hat{\times}^{sp}\hat{\Delta}^{sp}_{ij}\hat{\times}^{sp}\hat{z}^{j} = (\lambda z_1^2 + \lambda^{-1}z_2^2)^{sp}, \end{aligned} \tag{133}$$

where λ is Bohm's "hidden variable" [91].

Let us recall the explicit form of the iso-linear four-momentum on a Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ over an iso-complex iso-field $\hat{C}$

$$\hat{p}_{\mu}\hat{\times}^{or}|\hat{b}\rangle = -\hat{i}^{or}\hat{\times}^{or}\hat{\partial}^{or}_{\mu}|\hat{b}\rangle = -i\hat{I}^{or}\partial_{\mu}|\hat{b}\rangle. \tag{134}$$

By using the iso-mass of iso-particles and the iso-speed of iso-light

$$\hat{m} = m\hat{I}^{tot}, \ \ \hat{C} = C\hat{I}^{tot} = \frac{c}{n_4}\hat{I}^{tot}, \tag{135}$$

we have the iso-linearization of the second order iso-Casimir invariant (130) acting on an iso-basis $|\hat{b}\rangle$ (see Eq. (6.1), page 189, [66])

$$\begin{aligned} &(\hat{\Omega}^{\mu\nu}\hat{\times}^{tot}\hat{P}_{\mu}\hat{\times}^{tot}\hat{P}_{\nu} - \hat{m}^{\hat{2}}\hat{\times}^{tot}\hat{C}^{\hat{2}})\hat{\times}^{tot}|\hat{b}\rangle = \\ &= (\hat{\Omega}^{\mu\nu}\hat{\times}^{tot}\hat{\Gamma}_{\mu}\hat{\times}^{tot}\hat{P}_{\nu} + \hat{i}^{tot}\hat{\times}^{tot}\hat{m}\hat{\times}\hat{C})\hat{\times}^{tot} \\ &\hat{\times}^{tot}(\hat{\Omega}^{\mu\nu}\hat{\times}^{tot}\hat{\Gamma}_{\mu}\hat{\times}^{tot}\hat{P}_{\nu} - \hat{i}^{tot}\hat{\times}^{tot}\hat{m}\hat{\times}\hat{C})\hat{\times}^{tot}|\hat{b}\rangle), \end{aligned} \tag{136}$$

which holds if and only if the following conditions are verified

$$\begin{aligned} &\hat{\Gamma}_{\mu} = \hat{\gamma}_{\mu}\hat{I}^{or}, \\ &\{\hat{\gamma}_{\mu}, \hat{\gamma}_{\nu}\}^{or} = \hat{\gamma}_{\mu}\hat{\times}^{or}\hat{\gamma}_{\nu} + \hat{\gamma}_{\nu}\hat{\times}^{or}\hat{\gamma}_{\mu} = 2\hat{\eta}_{\mu\nu}, \end{aligned} \tag{137}$$

with realization given by the *Dirac-Santilli iso-gamma matrices*

$$\begin{aligned} &\hat{\gamma}_k = \frac{1}{n_k}\begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \\ &\hat{\gamma}_4 = \frac{i}{n_4}\begin{pmatrix} I_{2\times2} & 0 \\ 0 & -I_{2\times2} \end{pmatrix}, \end{aligned} \tag{138}$$

where $\hat{\sigma}_k$ are the *Pauli-Santilli iso-matrices* first proposed in Eq. (6.8.20), p. 248, [42]

$$\begin{aligned} &\hat{\sigma}_1 = \begin{pmatrix} 0 & \lambda \\ \lambda^{-1} & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -i\lambda \\ i\lambda^{-1} & 0 \end{pmatrix}, \\ &\hat{\sigma}_3 = \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & -\lambda \end{pmatrix}, \end{aligned} \tag{139}$$

with Lie-Santilli iso-commutation rules

$$\begin{aligned} &[\hat{\sigma}_i \hat{,} \hat{\sigma}_j] = \hat{\sigma}_i\hat{\times}\hat{\sigma}_j - \hat{\sigma}_j\hat{\times}\hat{\sigma}_i = \\ &= \hat{\sigma}_i\hat{T}\hat{\sigma}_j - \hat{\sigma}_j\hat{T}\hat{\sigma}_i = i2\epsilon_{ijk}\hat{\sigma}_k, \end{aligned} \tag{140}$$

with iso-eigenvalues on $\hat{\mathcal{H}}$ over $\hat{C}$

$$\begin{aligned} &\hat{S}_k = \frac{\hat{1}}{2}\hat{\times}\hat{\sigma}_k = \frac{1}{2}\hat{\sigma}_k, \\ &\hat{\sigma}_3\hat{\times}|\hat{b}\rangle = \hat{\sigma}_3\hat{T}|\hat{b}>= \pm|\hat{b}\rangle, \\ &\hat{\sigma}^{\hat{2}}\hat{\times}|\hat{b}\rangle = (\hat{\sigma}_1\hat{T}\hat{\sigma}_1 + \hat{\sigma}_2\hat{T}\hat{\sigma}_2 + \hat{\sigma}_3\hat{T}\hat{\sigma}_3)\hat{T}|\hat{b}\rangle = 3|\hat{b}\rangle, \end{aligned} \tag{141}$$

clearly showing the representation of the spin 1/2 of the considered iso-particle.

The Dirac-Santilli iso-equations can then be written

$$\begin{aligned} &(\hat{\Omega}^{\mu\nu}\hat{\times}^{or}\hat{\Gamma}_{\mu}\hat{\times}^{or}\hat{P}_{\nu} + \hat{i}\hat{\times}^{or}\hat{m}\hat{\times}\hat{C})\hat{\times}^{or}|\hat{b}\rangle = \\ &= (\hat{\eta}^{\mu\nu}\hat{\gamma}_{\mu}\hat{\times}^{or}\hat{P}_{\nu} + \hat{i}\hat{m}\hat{\times}\hat{C})\hat{\times}^{or}|\hat{b}\rangle = 0, \end{aligned} \tag{142}$$

which will be used in Sect. 3.9 for the relativistic representation of the neutron structure.

To avoid insidious, because unfounded inconsistencies in applications, the reader should keep in mind that the iso-metric $\Omega^{\mu\nu}$ for iso-momenta is the *contra-variant version* of the iso-metric $\Omega^{\mu\nu}$ for iso-coordinates.

3.7 Iso-spinorial Poincaré iso-symmetries

In view of the spin 1/2 of the electron, the space time symmetry for the relativistic treatment of the Hydrogen atom is given by the spinorial covering of the Poincaré symmetry

$$\mathcal{P}(3.1) = \mathcal{SL}(\in.C) \times \mathcal{T}(\ni.\infty), \tag{143}$$

with realization of the generators in terms of the Dirac gamma matrices

$$\begin{aligned} &\mathcal{SL}(2.C): \quad S_k = \frac{1}{2}\gamma_k \times \Gamma_4, \quad R_k = \frac{1}{2}\epsilon_i^k\gamma_i \times \gamma_j, \\ &\mathcal{T}(3.1): \quad P_{\mu}, \end{aligned} \tag{144}$$

which verify commutation rules (136).

Similarly, the *iso-spinorial coverings of the Poincaré iso-symmetries*, first presented in the 1995 paper [66] is given by

$$\hat{\mathcal{P}}(3.1) = \hat{\mathcal{SL}}(\hat{2}.\hat{C})\hat{\times}\hat{\mathcal{T}}(3.1)\,, \tag{145}$$

and admits the realization of the iso-generators in terms of the Dirac-Santilli iso-gamma iso-matrices $\hat{\Gamma}_\mu = \hat{\gamma}\hat{I}^{or}$

$$\begin{aligned} &\hat{\mathcal{SL}}(\hat{2}.\hat{C}): \quad \hat{\mathcal{S}}_k = \frac{1}{2}\hat{\Gamma}_k\hat{\times}\hat{\Gamma}_4, \quad \hat{R}_k = \frac{1}{2}\epsilon_{ijk}\hat{\Gamma}_i\hat{\times}\hat{\Gamma}_j\,, \\ &\hat{\mathcal{T}}(3.1): \ \hat{P}_\mu\,, \end{aligned} \tag{146}$$

which verify iso-commutation rules (128).

We also have the *rotational iso-sub-symmetries*

$$\hat{O}(3): \ L_k = \epsilon^i_{kj} r_j p_j, \quad [L_i,L_j] = \epsilon^k_{ij} n_k^2 L_k\,, \tag{147}$$

with iso-eigenvalues

$$\begin{aligned} &\hat{L}^{\hat{2}}\hat{\times}|\hat{b}\rangle == (\hat{L}_1\hat{\times}\hat{L}_1 + \hat{L}_2\hat{\times}\hat{L}_2 + \hat{L}_3\hat{\times}\hat{L}_3)\hat{\times}|\hat{b}\rangle = \\ &= (n_1^2n_2^2 + n_2^2n_3^2 + n_3^2n_1^2)|\hat{b}\rangle\,, \\ &\hat{L}_3\hat{\times}|\hat{b}\rangle = n_1n_2|\hat{b}\rangle\,, \end{aligned} \tag{148}$$

and the *spinorial iso-sub-symmetries*

$$\begin{aligned} &S\hat{U}(2): \ \hat{S}_k = \frac{1}{2}\epsilon^{ij}_k\hat{\gamma}_i\hat{\times}\hat{\gamma}_j\,, \\ &[\hat{S}_i,\hat{S}_j] = \frac{1}{n_k}\hat{S}_k \ \ (no\ sum)\,, \end{aligned} \tag{149}$$

with iso-eigenvalues

$$\begin{aligned} &\hat{S}^{\hat{2}} = (\hat{S}_1\hat{\times}\hat{S}_1 + \hat{S}_2\hat{\times}\hat{S}_2 + \hat{S}_3\hat{\times}\hat{S}_3)\hat{\times}|\hat{b}\rangle = \\ &= \frac{1}{4}\left(\frac{1}{n_1^2n_2^2} + \frac{1}{n_2^2n_3^2} + \frac{1}{n_3^2n_1^2}\right)|\hat{b}\rangle \\ &\hat{S}_3\hat{\times}|\hat{b}\rangle = \frac{1}{2}\,\frac{1}{n_1n_2}\||\hat{b}\rangle\,, \end{aligned} \tag{150}$$

that will be used in Sect. 3.9 for the identification of the main characteristics of the iso-electron in the neutron structure.

3.8 Special iso-relativities

Special Relativity (SR) has achieved a justly historical role for the characterization of *time reversal invariant, thus stable systems of point-like particles and electromagnetic waves propagating in the homogeneous and isotropic vacuum,* where the restriction to time reversal invariance follows from the dependence of Minkowski's invariant (96) on t^2.

SR is only approximately valid for the characterization of time reversal invariant, thus stable systems of *extended* particles (such as the proton in a nucleus) because extended particles imply features outside the representational capabilities of the *mathematics* underlying SR, such as the existence of contact, thus zero-range interactions without potential, the *mass/energy excess* of particle fusions (Sect. 3.1), the generally inhomogeneous and anisotropic character of the medium, and other problems.

In the author's view, SR is inapplicable (rather than violated) for an axiomatically consistent representation of *irreversible processes, such as nuclear fusions* for various axiomatic and physical reasons, including the possible violation of causality (e.g. the admission of solutions in which the effect precedes the cause) [102].

Special isotopic (i.e. axiom-preserving) relativity, or *Special Iso-Relativity* (SIR) for short, has been introduced by R. M. Santilli in the 1983 Nuovo Cimento papers [61, 62] for their classical and operator formulations, respectively, and then studied in various works [92]–[97] (see also reviews [25, 28, 32]) for the characterization of *time reversal invariant systems of extended, thus deformable and dense particles under conditions of mutual penetration, as occurring in stable nuclei, under the most general known, linear, local and potential interactions represented by a Hamiltonian H and the most general possible non-linear, non-local and non-potential interactions represented by the isotopic element $\hat{T}$ of (20).* In this paper, we use SIR with *constant n-characteristic quantities* for the representation of the neutron synthesis even though the neutron is *unstable* (when isolated), yet it decays into the original constituents (27), as a result of which the neutron synthesis can be assumed to be *reversible over time.*

The formulation of SIR used in this paper *is not* recommended for the treatment of nuclear fusions (because of the possible violation of causality indicated earlier) in favor of the *Lie-admissible relativity* studied in [24, 42] with an *irreversible axiomatic structure* [103, 104].

The correct classical formulation of SIR should be done on iso-Minkowskian iso-spaces $\hat{M}(\hat{x},\hat{\Omega},\hat{I})$ over iso-fields $\hat{\mathcal{R}}$, while the operator formulation should be done on Hilbert-Myung-Santilli iso-spaces $\hat{\mathcal{H}}$ over iso-complex iso-fields $\hat{\mathcal{C}}$. At the abstract level, SR and SIR coincide by conception and construction. Therefore, by continuing to follow [66] we present below the *projection* of SIR iso-axioms in the conventional Minkowski space $M(x,\eta,I)$ over the field $\mathcal{R}$. Said iso-axioms are then uniquely and unambiguously characterized by the iso-symmetries reviewed in preceding sections, and are expressed below for the k-direction, e.g. that of the third space component,

ISO-AXIOM I: The speed of light within (transparent) physical media is given by the locally varying speed:

$$C = \frac{c}{n_4} \ \lesseqgtr \ c\,. \tag{151}$$

ISO-AXIOM II: The maximal causal speed within physical *media is given by:*

$$V_{max,K} = c\,\frac{n_k}{n_4}\,. \tag{152}$$

ISO-AXIOM III: The addition of speeds within physical media follows the isotopic law:

$$V_{tot} = \frac{\frac{v1.k}{n_k} + \frac{v_{2.k}}{n_k}}{1 + \frac{v_1 v_2}{c^2}\,\frac{n_4^2}{n_k^2}}. \tag{153}$$

ISO-AXIOM IV: The iso-dilation of time, the iso-contraction of lengths, the iso-variation of mass with speed and the mass-energy iso-equivalence (iso-renormalization) within physical media follow the isotopic laws:

$$t'_k = \hat{\gamma}_k\, t\,, \tag{154}$$

$$\ell'_k = \hat{\gamma}_k^{-1}\,\ell\,, \tag{155}$$

$$m'_k = \hat{\gamma}_k\, m\,, \tag{156}$$

$$\hat{E}_k = m\,V^2_{max,k} = m_k c^2\,\frac{n_k^2}{n_4^2}\,. \tag{157}$$

ISO-AXIOM V: The frequency shift within physical media follows the isotopic law (for null aberration)

$$\omega_{exp} = \frac{\omega_{sou}}{\hat{\gamma}\,[1 - \hat{\beta}\,\mathrm{iso}\cos(\hat{\alpha})]}. \tag{158}$$

To avoid a prohibitive length, in regard to the experimental verifications of Iso-Axioms I–V in classical physics, particle physics, nuclear physics, astrophysics and other fields, we suggest the interested reader to inspect the 1995 [43] and the 2021 upgrade [44].

The following comments are now in order:

3.8.1. Note that the maximal causal speed in SIR is no longer given by the speed of light, and it is given instead by value (152), because *physical media are generally opaque to light,* thus requiring the broader geometric notion $v_{max,k}$ derivable from the expression in $(3,4)$-space coordinates

$$\frac{dx_k^2}{n_k^2} - dt^2\,\frac{c^2}{n_4^2} = 0\,. \tag{159}$$

3.8.2. By recalling that we are dealing with inhomogeneous and anisotropic physical media, the reader should be aware that the numeric values of Iso-Axioms (151)–(158) generally vary with the variation of the k-direction.

3.8.3. The sole known geometric representation of the excess mass/excess rest energy of the neutron synthesis, as well as of particle fusions at large (Sect. 3.1), will be done in the next section with Iso-Axiom (157).

3.8.4. When the isotopic element $\hat{T}$, and therefore, the n-characteristic quantities, solely depend on space time coordinates $\hat{T} = \hat{T}(x)$, $n_\mu = n_\mu(x)$, iso-Minkowskian intervals (00) coincide with Riemannian intervals [67], and characterize the *Exterior General Iso-Relativity* (EGIR) for the formulation of Einstein's field equations under the universal Poincaré-Santilli *iso-symmetry* $\hat{\mathcal{P}}(3.1)$ [64] (rather than the known covariance), including the representation of the Schwartzschild metric with the isotopic element (for brevity, see Sect. 8.5, p. 155 on of [44])

$$\hat{T}_{kk} = \frac{\delta_{kk}}{\left(1 - \frac{2M}{r}\right)},\quad \hat{T}_{44} = 1 - \frac{2M}{r}\,, \tag{160}$$

with the apparent resolution of the century-old problematic aspects of general relativity [105].

3.8.5. When the isotopic element $\hat{T}$ has the general functional dependence (101), Iso-Axioms I-V characterize the *Interior General Iso-Relativity* (IGIR) which is intended to study the *origin* (rather than the sole description) of the gravitational field, that expectedly occurs in the nuclear structure (see Santilli's paper [106] from his stay at MIT in 1974–1977), thus including the structure of the neutron (see Sect. 8.6, p. 161 of [44]).

3.9 Relativistic representation of the neutron synthesis

Recall that, under the invariance of the spinorial covering of the Poincaré symmetry $\mathcal{P}(3.1) = SL(2.C) \times \mathcal{T}(3.1)$ (which is needed for the spin $S = 1/2$ of the electron), the Dirac equation has provided an exact and time invariant relativistic representation of the *point-like* electron under the *external* field of the proton in the structure of the Hydrogen atom.

The Dirac-Santilli iso-equation (142) has been constructed to attempt the exact and time invariant representation of the *extended* wave packet of the iso-electron within the *extended* proton in the structure of the neutron according to Fig. 3, thus requiring its characterization via the isotopies of the spinorial covering of the Poincaré symmetry $\hat{\mathcal{P}}(3.1) = \widehat{SL}(2.\hat{C})\hat{\times}\hat{\mathcal{T}}(3.1)$, first introduced in the 1995 paper [66] jointly with the Dirac-Santilli iso-equation and the first relativistic representation of the neutron synthesis.

For consistency, the neutron structure model (29) requires that the hadronic angular momentum of the iso-electron $\hat{L}_3$ be equal to the proton spin $\hat{S}_3$, thus requiring that

$$\hat{L}_3 = \hat{S}_3\,,\quad \hat{L}^{\hat{2}} = \hat{S}^{\hat{2}}\,. \tag{161}$$

From (148) and (150) of the iso-spinorial iso-symmetry $\hat{\mathcal{P}}(3.1)$, we therefore obtain the following two conditions on the characteristic quantities for the basic isotopic element (97) expressed in the symbols $b_\mu = 1/n_\mu$ of [66]

$$\begin{gathered} b_1^{-1} b_2^{-1} = \frac{1}{2}\, b_1 b_2\,, \\ b_1^{-2} b_2^{-2} + b_2^{-2} b_3^{-2} + b_3^{-2} b_1^{-2} = \\ \frac{1}{4}\left(b_1^2 b_2^{-2} + b_2^2 b_3^{-2} + b_3^2 b_1^{-2}\right), \end{gathered} \tag{162}$$

with numeric value confirming the expected spheroidal shape of the neutron (Eqs. (7.2), (7.3), p. 192 of [66])

$$\begin{aligned} b_1^2 = b_2^2 = \frac{1}{n_1^2} = \frac{1}{n_2^2} = \sqrt{2} = 1.415\,, \\ b_1 = b_2 = \frac{1}{n_1} = \frac{1}{n_2} = 1.189\,. \end{aligned} \tag{163}$$

Consequently, the above relativistic representation is in remarkable axiomatic and numerical agreement with the corresponding non-relativistic value (Sect. 2.3.2) via Bohm's hidden variable (78),

$$\lambda = \sqrt{b_1 b_2} = b = \sqrt{\frac{1}{n_1}\frac{1}{n_2}} = \frac{1}{n}\sqrt{2} = 1.189\,, \tag{164}$$

by therefore establishing the *compatibility between the non-relativistic and the relativistic structure models (29) of the neutron.*

The value of the third semi-axis $1/b_3^2 = n_3^2$ of the iso-electron can be found by assuming the preservation of the volume V of the original sphere with semi-axes $n_k^2 = 1,\ k = 1, 2, 3$ plus values (162) for the first two semi-axes

$$\begin{aligned} V = \frac{4}{3}\pi(n_1^2)^2 n_3^2 = 4.192\Big(\frac{1}{1.415}\Big)^2 n_3^2 = 4.19\,, \\ n_3^2 = \frac{4.19}{2.087} = 2.007\,, \end{aligned} \tag{165}$$

resulting in the values

$$n_1^2 = n_2^2 = 0.707\,, \quad n_3^2 = 2.007\,, \tag{166}$$

suggesting that the spheroidal shape of the iso-electron is *prolate* (because $n_3^2 > n_1^2 = n_2^2$).

The representation of the excess energy $\Delta E = 0.782$ MeV in the neutron synthesis from the proton and the electron (7), is done via Iso-Axiom (157), requiring a numeric value of $n_4^2 = 1/b_4^2$ which in this case, represents the density of the *proton,* since the charge of the iso-electron has no dimension.

From Iso-Axiom (157) we obtain the iso-renormalized rest energy of the neutron

$$\begin{aligned} \tilde{E}_n = m_e C^2 = \\ = m_e c^2\,\frac{b_3^2}{b_4^2} = m_e c^2\,\frac{n_3^2}{n_4^2} = 939.565\,\text{MeV}\,, \end{aligned} \tag{167}$$

from which

$$\frac{b_4^2}{b_3^2} = \frac{n_3^2}{n_4^2} = \frac{1.293}{0.511} = 2.530\,. \tag{168}$$

From values (166) we then obtain the numeric value of the density n_4^2 which is needed for the iso-renormalization of the mass/rest energy of the iso-electron here presented apparently for the first time

$$n_4^2 = \frac{1}{b_4^2} = \frac{n_3^2}{2.530} = 0.793\,, \quad n_4 = \frac{1}{b_4} = 0.891\,, \tag{169}$$

which is compatible with the density $n_4^2 = \frac{1}{b_4^2} = 0.429$ of the fireball of the proton-antiproton annihilation of the Bose-Einstein correlation [107,108], see Eq. (10.27), p.127 of [107] (see also [108]).

Intriguingly, taken in *prima facie,* the above data suggest that *the proton is about* 50% *denser than the proton-antiproton fireball of the Bose-Einstein correlation.*

4 Applications of the neutron synthesis

In this section, we briefly indicate some of the applications of the synthesis/fusion of the proton and the electron into the neutron with related references.

4.1 Detection of smuggled fissile material

Recall that fissile material, such as Uranium-233, Uranium-235 and Plutonium-239, are *metals* that, as such, cannot be distinguished from ordinary metals via all scanning equipment currently available at airports and ports. Thanks to the studies reported in this paper, the U.S. publicly traded company *Thunder Energies Corporation,* (now the private *Hadronic Technologies Co*) did develop, produce and sell a scanner permitting a clear detection of fissile material via the irradiation of baggages with the *Directional Neutron Source* (DNS) of Fig. 5 which produces on demand from a commercially available Hydrogen gas *a beam of thermal neutrons ($E < 100$ eV) in the desired direction and intensity,* resulting in a shower of easily detectable radiation from the disintegration of a few fissile nuclei [68]–[80].

It should be noted that various neutron sources are commercially available but they all produce *high energy neutrons* that, as such, are not recommendable for use in public places because of the risk of triggering a chain reaction which is absent for irradiation of fissile material with a controlled small beam of thermal neutrons.

4.2 Representation of nuclear stability

It appears that hadronic mechanics has permitted a quantitative solution of the problem of nuclear instability despite the neutron natural instability (Insufficiency IV of Sect. 1.2) via the *decoupling of the permanently stable electron from the neutron when members of a nuclear structure* (Fig. 6), which was first presented in Appendix C.1 and Fig. 13, p. 152 of [102]. Note that the indicated decoupling introduces a new, very strong, Coulomb *attraction* in the Deuteron structure between the iso-electron and the proton pair. Note also that *the indicated nuclear stability is possible if and only if the proton and the electron are the actual physical constituents of the neutron.*

The resolution of Insufficiency V of Sect. 1.2 (on the nuclear stability despite the very big, repulsive, protonic, Coulomb force) requires separate future studies on the *structure* of the elementary iso-charge.

4.3 Representation of the gravitational stability of the Sun

The Sun releases into light the energy of [109]

$$\Delta E^{Sun}_{out} = 2.3 \times 10^{38}\ \mathrm{MeV/s}, \tag{170}$$

which corresponds to about 4.3×10^6 t/s. Since, in a Gregorian year, there are 10^7 seconds, the loss of mass by the Sun per year ΔM^{Sun}_{year} due to light emission is given by

$$\Delta M^{Sun}_{year} = 10^{23}\ \text{metric tons per year}. \tag{171}$$

The above loss of mass by the Sun is of such a magnitude to cause a change of planetary orbits that should be detectable by contemporary, sufficiently sensitive instruments in astrophysical laboratories contrary to centuries of measurements on the stability of planetary orbits, i.e. the stability of the Sun's gravitational field.

For these and other reasons, Santilli [110] proposed in 2007 the hypothesis that *the missing energy in the neutron synthesis is provided by the ether conceived as a universal substratum with extremely big energy density,* and that the energy of 0.782 MeV is transferred from the ether to the neutron by a massless, chargeless and spinless *longitudinal impulse* called *etherino* (denoted with the letter *a* from the Latin *aether*) in the *left hand side* of the neutron synthesis

$$\hat{e}^- + a + \hat{p}^+ \ \rightarrow\ n\,. \tag{172}$$

In fact, a medium size star such as our Sun synthesizes about 10^{40} neutrinos per second [13], that requires the total energy of about

$$\Delta E^{star}_{n} = 7.8 \times 10^{39}\ \text{MeV per second}. \tag{173}$$

The etherino hypothesis [110] was formulated on grounds that the energy needed for the neutron synthesis by the Sun (173), is essentially equal to the Sun's loss of energy into light (170). Consequently, the assumption that the missing energy for the neutron synthesis is provided by the ether as a universal substratum permits a quantitative representation of the stability of the Sun's gravitational field.

In any case, the missing energy of 0.782 MeV cannot be provided by the relative kinetic energy bytween the proton and the electron, because at that value, the $e - p$ cross section is essentially null, thus prohibiting any synthesis. Similarly, said missing energy cannot be provided by the Sun because the total missing energy (173) is so big that the Sun would cool down and never produce light.

Note that the indicated representation of the gravitational stability of the Sun implies a *return to the continuous creation of matter in the universe* [112], with intriguing implications, e.g. for a realistic representation of the energy released in supernova explosions. Note finally that experiments on the *predictions* of the neutrino hypothesis [113] may be numerically representable via the corresponding *predictions* of the etherino hypothesis.

Note also that the indicated gravitational stability of the Sun requires the acceptance of the ether as a universal substratum for the structure and propagation of truly "elementary" particles and electromagnetic waves without any real conflict with special relativity due to our evident inability to reach a reference frame at rest with the ether (for the absence of the "ethereal wind" under the indicated conditions, see the 1956 paper [114] and Chapter 3 of [32]).

4.4 Stimulated decay of the neutron

The hypothesis that the neutron is a hadronic bound state of a proton and an electron implies the *possible stimulated decay of one or more neutrons when members of selected nuclear structures* via irradiation with resonating photons γ_r with energy equal to the total energy of the iso-electron $E_r = E_{\hat{e}} = 1,293$ MeV. Intriguing, said stimulated decay implies the production of nuclear energy without the emission of harmful radiation and without the release of radioactive waste, e.g. as occurring in the stimulated decay [115]

$$\begin{aligned} &\gamma_r + \mathrm{Mo}(100,42,0) \rightarrow \mathrm{Tc}(100,43,1) + \beta^-\,, \\ &\mathrm{Tc}(100,43,1) \rightarrow \mathrm{Ru}(100,44,0) + \beta^-\,, \end{aligned} \tag{174}$$

which has been tentatively verified by the experimental team [117] (see also [83]). Regrettably, no physics laboratory contacted by the author has shown interest to date in dismissing or confirming Tsagas' results via the repetition of the very simple and inexpensive measurements of transmutation (174) (see Fig. 7 for details).

4.5 The pseudo-proton hypothesis

The synthesis of the neutron via Rutherford's "compression" of an electron within the dense proton, implies the synthesis, in statistical smaller amounts of *negatively charged, strongly interacting particles* preliminarily confirmed by tests [118], such as: the *protoid* $\tilde{p}^-_1$ with spin 0, mass essentially that of the neutron and mean-life predicted to be of about 7 s, and the *pseudo-proton* $\tilde{p}^-_2$ with spin 1/2, mass equal to that of the neutron and mean-life of the order of 5 s, both representable with the synthesis/fusion $\tilde{p}^- = (\hat{e}^-, n)_{hm}$.

Note that, being negatively charged and strongly interacting, *protoids and pseudo-protons are attracted by nuclei* with new nuclear transmutations here expressed for N protoids

$$N\tilde{p}^-_1 + N(Z,A,J) \rightarrow \tilde{N}(Z-N, A+N, J)\,, \tag{175}$$

having an evident significance for possible new forms of nuclear energies and recycling of nuclear waste.

4.6 Recycling of nuclear waste

Due to known public opposition, it appears that the sole possible recycling of radioactive nuclear waste should be done by the nuclear power plants themselves via their stimulated decay. Unfortunately, the latter recycling is prohibited by quantum mechanics with ensuing mainstream academic opposition against its study. Interested readers may be interested to know that hadronic mechanics predicts a number of mechanisms for the recycling of radioactive nuclear waste via their stimulated decay triggered by irradiation with thermal neutrons (Sect. 4.1 and Fig. 5), pseudo-protons (Sect. 4.5) as well as other means, and ensuing production of new nuclear energy (see, for brevity, Sect. 8.2.10-II, p. 111 of [44] and [115, 116].

4.7 Resolution of the Coulomb barrier for nuclear fusion

In the author's view, the most important environmental implication of the synthesis/fusion of the proton and the electron into the neutron is the consequential synthesis/fusion, under the extremely strong Coulomb attraction (6), of at least a pair negatively charged electrons generally coupled in singlet and a positively charged Deuteron into a new negatively charged nucleus $\tilde{D}(-1, 2, 1)$, called *pseudo-Deuteron* with sufficiently long mean life (of the order of $\tau = 1$ s) to be attracted by a natural, positively charged Deuteron, resulting in a new nuclear fusion, called *HyperFusion,* without the historical Coulomb barrier that has prevented the achievement to date of new clean nuclear energies (see [102] for brevity).

5 Reduction of matter to protons and electrons

During his graduate studies at the University of Torino, Italy, in the mid 1960's, after learning that stars initiate their lives as aggregates of Hydrogen atoms, R. M. Santilli accepted the historical hypothesis [1,2] that matter is composed of the permanently stable protons and electrons, and could not accept the various opposing arguments [10] on grounds that Heisenberg's uncertainty principle has been experimentally verified solely for *point-like particles (i.e. as the electron) in vacuum under electromagnetic/Hamiltonian interactions*. In line with the 1935 legacy by A. Einstein, B. Podolsky and N. Rosen that *quantum mechanics is not a complete theory* [3], Santilli argued that the same principle should not be applied to the *extended protons and neutrons under strong nuclear interactions* without due scrutiny.

Subsequently, Santilli learned from experimental measurements [16, 18] that nuclear volumes are generally *smaller* than the sum of the volumes of the constituents, thus implying that the hyper-dense protons and neutrons are in conditions of partial mutual penetration in a nuclear structure. In turn, this implies the expectation that strong nuclear forces have a contact-zero range, non-linear, non-local and non-potential, thus non-Hamiltonian component under which *Heisenberg's uncertainty principle cannot be consistently formulated,* let alone tested.

In the late 1970's, when he was at Harvard University under DOE support, Santilli proposed the foundation of the EPR completion of quantum into *hadronic mechanics* for the invariant representation of extended nucleons under Hamiltonian and non-Hamiltonian interactions [23, 24]. He then initiated in 1981 studies [4] on the *completion of Heisenberg's uncertainties for strong interactions* via generalized uncertainties of the type (Eq. (2.18), p. 654 of [4])

$$\Delta r \times \Delta p \approx \frac{1}{2}\hbar F(r, p, \psi, ...), \quad F > 0\,, \tag{176}$$

and conducted systematic mathematical, theoretical, experimental and industrial studies (reported in the preceding sections) on the synthesis/fusion of a proton and an electron into the neutron.

Santilli became aware in the early 1990's that mathematical and physical theories can be completed into a form representing the astrophysical evidence that the neutron, and therefore all matter in the universe, is a collection of suitable bound states of the permanently stable protons and electrons. Final studies in the field are reported in this section on the explicit form of the uncertainty principle which is applicable under the most general possible, Hamiltonian and non-Hamiltonian strong nuclear forces.

In 1964, J. S. Bell published the theorem below under the assumption of quantum mechanics according to its Copenhagen interpretation, thus including Heisenberg's uncertainty principle, the representation of the spin 1/2 of particles via the SU(2)-invariant Pauli matrices, and other assumptions:

THEOREM 5.1 [119]: A system of two point-like particles verifying the SU(2) *Lie symmetry does not admit a classical counterpart.*

The theorem was proved by showing that a certain expression D^{Bell} (whose numeric value depends on the relative conditions of the two particles) is always *smaller* than the corresponding classical value D^{Clas},

$$D^{Bell} < D^{Clas}\,, \tag{177}$$

for all possible values of D^{Bell}.

The importance of the Theorem 5.1 for the identification of the ultimate constituents of matter is that of *strengthening* the general acceptance of Heisenberg's uncertainty principle for all possible conditions existing in the universe, thus leading to the unverified *assumption* that electrons cannot be members of a nuclear structure (Sect. 1.2).

Following the achievement of maturity of the iso-mathematical and iso-mechanical branches of hadronic mechanics [50, 89], and following the formulation of the $\widehat{\mathrm{SU}}(2)$-iso-invariant Pauli-Santilli iso-matrices reviewed in Sect. 3.6, Santilli proved in 1998 the following:

THEOREM 5.2 [5]: A system of two extended particles verifying the $\widehat{\mathrm{SU}}(2)$ Lie-Santilli iso-symmetry does admit a classical counterpart.

The theorem was first proved on grounds that contact, zero-range, non-potential interactions are outside the class of unitary equivalence of quantum mechanics, while being fully representable via a non-unitary transformation of quantum mechanical models (21). Consequently, there always exists a non-unitary transformation $UU^\dagger = \hat{I}$ of Bell's quantity D^{Bell} such to verify the equality

$$D^{hm} = U(D^{Bell})U^\dagger \equiv D^{Clas} . \tag{178}$$

Additionally, Santilli conducted a step-by-step isotopic lifting of Bell's proof of Theorem 5.1 via the Pauli-Santilli iso-matrices (139) resulting in the equality (Eq. (5.8), p. 189 of [5])

$$D^{hm} = \frac{1}{2}\,(\lambda_1\lambda_2^{-1} + \lambda_1^{-1}\lambda_2)\, D^{Bell} \equiv D^{class} , \tag{179}$$

which is always verified by particular values of Bohm's hidden variables [91] λ_1 and λ_2. [5] also provided specific examples of identity (178) in terms of the iso-Minkowski isospaces over iso-fields.

Finally, by combining the results of [4] and [5], in 2019 Santilli proved the following:

THEOREM 5.3 [6] : The iso-standard iso-deviations for iso-coordinates Δr and iso-momenta Δp, as well as their product, progressively approach Einstein's determinism for extended particles in the interior of hadrons, nuclei and stars, and achieve the full determinism at the limit of Schwartzschild's singularity (ss).

The theorem was proved by showing that the invariance under the Lie-Santilli iso-symmetry $\widehat{\mathrm{SU}}(2)$ implies the following property known as *iso-deterministic principle* derived via iso-commutation rules (50) and iso-normalization (41) (see for details Lemma 3.7, p. 34 of review [47] and its Corollary 3.7.1 on the ensuing removal of divergencies here ignored for brevity)

$$\begin{aligned} \Delta r\,\Delta p &\approx \frac{1}{2}\,\langle\hat{\psi}(\hat{r})|\,\hat{\times}\,[\hat{r},\hat{p}]\,\hat{\times}\,|\hat{\psi}(\hat{r})\rangle = \\ &= \frac{1}{2}\,\langle\hat{\psi}(\hat{r})|\,\hat{T}\,[\hat{r},\hat{p}]\,\hat{T}\,|\hat{\psi}(\hat{r})\rangle = \frac{1}{2}\,\hbar\hat{T} \ll 1 , \\ \{\Delta r\,\Delta p\}_{ss} &= 0 . \end{aligned} \tag{180}$$

Theorem 5.3 then holds in view of the fact that the isotopic element has always values smaller than $\hat{T} \ll 1$, from the fitting of all experimental data dealing with hadronic media [43], and the value of the isotopic element is null for gravitational collapse (160) $\hat{T}_{ss} = 0$.

It is easy to see that Theorems 5.2 and 5.3 resolve Objection 1.1 and 1.2 against electrons being members of a nuclear structure. In fact, under iso-principle (180), electrons would have the *sub-luminal speed*

$$v \geq \frac{\hbar}{\Delta r \times m_e} = 5.79\,\hat{T}\,10^{10}\,\mathrm{m/s}\,, \quad \hat{T} \ll 1 . \tag{181}$$

Similarly, the linear momentum uncertainty would have the value

$$\Delta p = 1.05\,\hat{T}\,10^{20}\,\mathrm{kg\,m/s}\,, \quad \hat{T} \ll 1 , \tag{182}$$

as a result of which the energy of the electrons can be the expected value $E_{\hat{e}} = 1.293$ MeV, thus being much less than the 18.5 MeV predicted via Heisenberg's uncertainty principle (4). The understanding is that the final numerical values of the isotopic element for the neutron require additional studies as well as experimental measurements. Objection 1.3 has been resolved in Section 2.3.3 by showing that excessive value (5) of the magnetic moment of the electron for nuclear standards is counterbalanced by the magnetic moment of the constrained angular momentum within the proton structure.

In conclusion, rather than adapting experimental evidence to a preferred theory, it appears that mathematical and physical methods can indeed be completed to verify the evidence that the permanently stable proton and electron are the constituents of the neutron, with ensuing reduction of all matter in the universe to protons and electron in conditions of increasing complexity.

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