# A Possible, Lie-Admissible, Time-Asymmetric Model for Open Nuclear Reactions.

R. M. SANTILLI (*)

*The Institute for Basic Research* - 96 Prescott Street, Cambridge, Mass. 02138, U.S.A.

*Summary.* – We show that an isotopic lifting of the Hilbert space implies a time-asymmetry for open nuclear reactions, while recovering the time-reversal invariance for center-of-mass trajectories of the implementation of the system into a closed form. The conceptual, mathematical, and experimental plausibilities of the model are indicated.

Without doubt, the origin of the time asymmetry of our macroscopic world constitutes one of the most intriguing (and fundamental) open problems of contemporary physics.

At the *Newtonian level*, the situation is sufficiently (yet incompletely) understood. Consider our Earth as seen from an outside observer. Its center-of-mass trajectory is manifestly time symmetric. Nevertheless, interior, open (nonconservative) systems are manifestly time asymmetric. Particularly important for this note is the fact that the time asymmetry results to be ultimately due to the *non-Hamiltonian* character of the forces, and to the consequential, *noncanonical* nature of the time evolution, as established, say, by a satellite during re-entry. Besides conventional, closed Hamiltonian systems (*e.g.* the planetary and atomic systems), Nature clearly exhibits more general systems of closed non-Hamiltonian type, *i.e.* systems verifying conventional conservation laws of total quantities, yet the internal forces are outside the capabilities of Hamiltonian mechanics. This novel situation has stimulated the construction of the so-called Birkhoffian [1] generalization of Hamiltonian mechanics [2] for the exterior closed treatment, and of the complementary Birkhoff-admissible mechanics [3] for the interior open case.

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(1) G. D. BIRKHOFF: *Dynamical Systems*, Amer. Math. Soc. Providence, R.I. (1927).
(2) R. M. SANTILLI: *Foundations of Theoretical Mechanics*, Vol. II: *Birkhoffian Generalization of Hamiltonian Mechanics* (New York, N.Y. and Heidelberg, 1982).
(3) R. M. SANTILLI: *Lie-Admissible Approach to the Hadronic Structure*, Vol. II: *Covering of the Galilei and Einstein Relativities?* (Mass., 1982).

At the *statistical level*, fundamental advances in the non-Hamiltonian origin of irreversibility have been made by PRIGOGINE [4] and his group for both classical and quantum-statistical ensembles. Further advances have been made by FRONTEAU, TELLEZ-ARENAS, SALMON, GUIASU, GRMELA, *et al.*, this time for the non-Hamiltonian origin of irreversibility at the level of each individual constituent of a statistical ensemble, as reported at the recent Orléans International Conference [5]. The unity of thought of these statistical studies with the Newtonian profile is remarkable. In fact, the Birkhoffian mechanics is a rather natural analytic counterpart of Prigogine's statistics for closed systems, while the Birkhoff-admissible mechanics is the analytic basis of the statistics advocated by FRONTEAU *et al.*, for open systems, with the understanding that a deeper unity of mathematical structure exists [2,3,6].

At the *particle level*, the situation is fundamentally unresolved to this writing. A primary objective of this note is that of stressing the need for a systematic consideration of all plausible views on the problem, owing to its relevance. In fact, as it has been the case at the Newtonian and at the statistical level, irreversibility may imply a revision of the foundations of particle dynamics, with implications ranging from controlled fusion to solid-state physics, as well as to other branches of sciences, such as theoretical biology.

Considerable difficulties have been recently identified for the compatibility between conventional Hamiltonian/unitary time evolutions of particles and the established irreversibility of the physical world [5]. Some of these difficulties are due to the manifest problematic aspects of any quantitative attempt to achieve the established *noncanonical* time evolution of the Newtonian systems of our environment via a large collection of *unitary* time evolutions for its constituents. Other difficulties are of statistical/thermodynamical nature.

The line of study submitted for consideration in this note is the following. There is no doubt that canonical formulations have a clear applicability in Newtonian and (classical) statistical mechanics. However, they are unable to represent the totality of systems. In fact, whenever particles cannot be effectively approximated as being pointlike, we have the emergence of *contact interactions* for which the notion of potential energy has no physical basis. Under these conditions, the construction of covering mechanics has resulted to be necessary for the treatment of systems in the frame of the observer [2-5].

In order to have the prerequisites for unity of physical thought, we propose a similar approach for particle physics. The arena of unequivocal applicability of the conventional Hamiltonian/unitary time evolution can be identified with physical conditions under which particles can be effectively approximated as being pointlike. This includes the virtual totality of the electromagnetic interactions, as well as several aspects of weak interactions (*e.g.*, semi-leptonic decays). However, we expect the existence of physical conditions of massive, extended, particles (such as hadrons) under which pointlike approximations become excessive, and suitable corrections of the Hamiltonian/unitary time evolution must be theoretically identified and experimentally tested. For this purpose, we essentially recommend the same pattern used for the construction of quantum mechanics, but now referred to the generalized context. These guidelines restrict possible generalizations of the Hamiltonian/unitary time evolution to forms possessing *the same mathematical structure as that of their Newtonian and statistical counterparts.*

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[4]   See I. PRIGOGINE: *Nobel Lecture* (1977) and quoted works.
[5]   *Proceedings of the First International Conference on Nonpotential Interactions and Their Lie-admissible Treatment, Hadronic J.*, **5**, no. 2, 3, 4, 5 (1982).
[6]   R. M. SANTILLI: *Hadronic J.*, **5**, 264 (1982).

Recall that Hamiltonian time evolutions, whether classical or quantum-mechanical, are a realization of Lie's theory. Two progressive generalizations of Lie's theory (beyond grading sypersymmetric extensions) have been recently identified by mathematicians and theoreticians, one called of *Lie-isotopic* type, and a more general one called of *Lie-admissible* type (see bibliography-index [7] and proceedings [5]). Now, the Birkhoffian time evolution is a realization of the Lie-isotopic theory [2], while the same conclusion can be reached for Prigogine's time evolution owing to its derivation via a nonunitary transformation of conventional evolutions [6]. Similarly, the Birkhoffian-admissible time evolution is a realization of the Lie-admissible theory [3], while the same result is known to hold for the time evolution advocated for open statistical ensembles by FRONTEAU *et al.* [5,6].

These results suggest rather naturally the study of a conceivable generalization of quantum mechanics (here called « atomic mechanics ») into a covering discipline for contact interactions among extended particles called « hadronic mechanics », as originally proposed in ref. [8] and currently under study by a number of authors (see again ref. [7,5]). Hereinafter, we shall consider the branches of hadronic mechanics treating the exterior/closed/non-Hamiltonian case [9] and the interior/open/non-Hamiltonian case [10]. The former theory is essentially characterized by one, single, left and right isotopic lifting of the atomic formulations. Let $\Phi$ be the enveloping associative algebra of an atomic model with operators $A, B, \ldots$ and conventional product $AB$. An isotope $\hat{\Phi}$ of $\Phi$ is the vector space $\Phi$ equipped with the new (associative) product $A * B = ATB$, where where $T = T(r, p \ldots)$ is a fixed invertible operator. The unit $I = \hbar$ of $\Phi$, $IA = AI = A$, is now mapped into the operator unit $\hat{I} = T^{-1}$ of $\hat{\Phi}$, $\hat{I} * A = A * \hat{I} = A$. The isotope $\hat{\mathscr{H}}$ of the Hilbert space $\mathscr{H}$ of the atomic model with product $(a, a')$ is then characterized by the lifting $\widehat{(a, a')} = \hat{I}(a, Ta') = (aT, a')\hat{I}$ under suitable topological restrictions on $T$ (Hermiticity, positivity, etc.). For the algebra $\hat{\Phi}$ to act linearly on $\hat{\mathscr{H}}$, the field of scalars $C$ must be subjected to the lifting $C = \{\hat{c}|\hat{c} = \hat{I}c; c \in C\}$. All the various properties and operations on a Hilbert space can then be subjected to an isotopic generalization [9]. We recall here, for example, that an operator $U$ is isotopic-unitary when it verifies the rules $U * U^{-1} = U^{-1} * U = \hat{I}$, in which case it admits the exponential form $U = \hat{I} \exp[iw * A] = \exp[iA * w]\hat{I}$, where $A$ is an isotopic-Hermitian operator (*i.e.* a hadronic observable).

The theory characterizes the following Lie algebra preserving generalization of Heisenberg's equations originally proposed in ref. [8] (p. 752) as an operator image of Birkhoff's equations [1]:

$$(1) \qquad i\dot{A} = [A, B]^* = A * B - B * A = ATB - BTA \;,$$

where $B$ is called the Birkhoffian operator to emphasize the care needed for its identification with the total energy owing to novel degrees of freedom. The isotopy permits a consistent generalization of the main theorems of Lie's theory, such as Lie's first, second and third theorems, the Poincaré-Birkhoff-Witt theorem, etc. [2]. Equation (1) is therefore integrable to the finite form originally identified in ref. [8] (p. 783)

$$(2) \qquad A(t) = \hat{I} \exp[it * B] * A(0) * \exp[-iB * t]\hat{I} \;,$$

[7]  M. L. TOMBER *et al.*: *Hadronic J.*, **3**, 507 (1979); **4**, 1318, 1444 (1981).
[8]  R. M. SANTILLI: *Hadronic J.*, **1**, 574 (1978).
[9]  H. C. MYUNG and R. M. SANTILLI: *Hadronic J.*, **5**, 1277 (1982).
[10]  H. C. MYUNG and R. M. SANTILLI: *Hadronic J.*, **5**, 1377 (1982).

which is a realization of a one-dimensional, continuous, «Lie-isotopic group» [2,7,5]. The conventional Schrödinger's equation is also generalized into the isotopic eigenvalue form studied in ref. [9] following the original identification by MIGNANI [11]

$$(3) \qquad B * \psi = BT\psi = \hat{b} * \psi = b\psi .$$

Predictably, the generalized Heisenberg's and Schrödinger's equations are connected by an isotopic-unitary transformation under certain topological restrictions [9].

The isotopic branch of the hadronic mechanics is clearly set for the direct representation of closed non-Hamiltonian systems. In fact, the anticommutativity of the product of eq. (1) permits a formulation of total conservation laws much along conventional lines [2]. However, the acting forces are now a combination of potential/Hamiltonian and contact/non-Hamiltonian forces. They are represented by the Birkhoffian $B$ and by the isotopic operator $T$ according to an intriguing freedom of selection called Birkhoffian gauges [2].

The complementary Lie-admissible branch of the hadronic mechanics is essentially characterized by two, generally different isotopies, one for the action to the right and one for the action to the left [10]. For the objectives of this note we shall call «forward» («backward») the action to the right (left) and denote the isotopies with the symbol «>» («<»), with the understanding that the physical time has only one direction. We then have the isotopic liftings $\Phi^>, \mathscr{H}^>, C^>$, and $^<\Phi, ^<\mathscr{H}, ^<C$ with products $A \star B =$

$$= A \overset{\star}{T} B \text{ and units } \overset{\star}{I} = (\overset{\star}{T})^{-1}, \overset{\star}{T} = T^>, ^<T .$$

This more general theory characterizes the following Lie-admissible generalization of Heisenberg's equations:

$$(4) \qquad i\dot{A} = (A, B) = A < B - B > A = A^{<}TB - BT^{>}A$$

with integrated form

$$(5) \qquad A(t) = I^> \exp[it > B] > A(0) < \exp[-iB < t]^{<}I ,$$

proposed in ref. [8] (p. 741-746) as an operator image of the equations originally conceived by HAMILTON, those with external terms, only rewritten in our Birkhoff-admissible form for certain reasons of algebraic consistency [3]. Equation (5) is a realization of a one-dimensional, continuous, «Lie-admissible group» [3,7,5]. Note that structure (5) constitutes a group continuously connected to the identity transformation. Nevertheless, the algebra in the neighborhood of such identity is a *non-Lie*, Lie-admissible algebra with product $(A, B)$. Note also that structures (2) and (5) are modular and bimodular, respectively. The generalized Schrödinger's equations characterized by eq. (5) are those identified by MIGNANI [11], which we write in the form $B > \psi = b_1\psi$, and $\varphi b_2 = \varphi < B$.

The Lie-admissible branch of hadronic mechanics is naturally set for the direct representation of open non-Hamiltonian systems. In fact, the *lack* of anticommutativity of the product now ensures the highest possible *nonconservations* of physical characteristics, as desired, *i.e.* in a way compatible with total conservations [3].

The compatibility and unity of conceptual, theoretical, and mathematical thought among the exterior and interior branches of hadronic mechanics and the indicated

[11] R. MIGNANI: *Hadronic J.*, **5**, 2185 (1982).

statistical and Newtonian formulations, are remarkable. As an example, Prigogine's statistics is essentially characterized by nonunitary transformations of conventional, Lie, time evolutions ([4]). But these transformations produce precisely the Lie-isotopic time evolutions ($I$) (see, *e.g.*, ref. ([2]), p. 225). This establishes the direct relationship between Prigogine's statistics and the Lie-isotopic branch of hadronic mechanics, to the point the law ($I$) can be interpreted as an algebraic reformulation for particles of Prigogine's time evolution for statistical densities. Also, nonunitary transformations of Hamiltonian time-evolutions are known to be irreversible, to be no longer Hamiltonian, and to admit the Birkhoffian mechanics as a classical limit ([2-5]). This establishes the non-Hamiltonian character of the irreversibility under consideration in this letter at both the operator and the classical levels. Fully equivalent correspondences occur for the complementary Lie-isotopic approach ([5]), and they are omitted here for brevity.

Despite these (and several other ([7,5])) attractive features, it should be stressed that hadronic mechanics is still conjectural at this time, and in need of a considerable amount of theoretical and experimental inspections. With the understanding that this task cannot be accomplished in one single paper, we shall limit ourselves here to an initial confrontation of the intrinsically irreversible character of hadronic mechanics with available experimental data on time asymmetry in nuclear physics.

To avoid a major misrepresentation of our work, it should be stressed that the time-asymmetry is admitted only for *open (nonconservative) systems* under the condition that they recover time-symmetric center-of-mass trajectories for their closed implementations, exactly as it occurs in our macroscopic reality. The virtual totality of contemporary high-energy experiments is then excluded from a direct applicability to our model for a number of reasons, such as the fact that they are traditionally formulated for the closed case, the data are elaborated via the *potential* scattering theory (rather than Mignani's nonpotential generalization ([11])), etc.

Consider an open treatment of the reaction $a+A \rightarrow b+B$ constituted by a polarized beam of nucleons a in interactions with *external nuclei A of a fixed target*, which are unpolarized and of spin $s_A$, and a corresponding open treatment of the backward reaction $b+B \rightarrow a+A$. Under these conditions, contact/non-Hamiltonian effects are expected to be small. We shall therefore approximate the isotopic operators hereon with scalar quantities. Under long-range electromagnetic interactions, nucleons can be effectively approximated as being pointlike and their spin can be exactly represented via the familiar $SU_2$ form $|s\rangle = |Is\rangle = |\tfrac{1}{2}I\sigma\rangle$. When the nucleons enter the intense fields in the vicinity of nuclei A, their pointlike approximation becomes excessive. In the hope of representing the simplest possible corrections due to the extended character of nucleons, we, therefore, consider the *forward Lie-isotopic hadronic spin* as characterized by the eigenvalue equations

$$(6) \qquad (s^{>})^{2>} > |s^{>}\rangle = \tfrac{3}{4}|s^{>}\rangle, \qquad s_3^{>} > |s^{>}\rangle = \pm\tfrac{1}{2}|s^{>}\rangle, \qquad |s^{>}\rangle = |I^{>}s\rangle = |\tfrac{1}{2}I^{>}\sigma\rangle$$

with corresponding algebra, group structure, basic invariant, etc. here omitted for brevity (see Chap. 5 of ref. ([3])). One can see that *the conventional, atomic, magnitude and third component of spin are preserved under scalar isotopy*, as originally identified by EDER ([12]). However, a number of new effects are now representable, such as a deformation of the charge distribution, the anomalous behaviour of the magnetic moment, etc. ([7,5,12]). The backward Lie-isotopic spin is then characterized by the isotopic-time-reversal operator $^{<}\tau = {}^{<}I\tau$, where $\tau$ is the atomic form. It is an instructive exercise

([12]) G. EDER: *Hadronic J*, **4**, 634, 2018 (1981); **5**, 750 (1982).

for the researcher interested in learning the new techniques to prove that, under the assumptions identified and the methods of ref. ([9,10]), we have the following manifestly irreversible, *hadronic generalization of the atomic principle of detailed balancing*:

$$(7) \quad (I^> + \sum_j P_j^> > A_j^>)^{-1}(k_i/k_f)2(2s_\mathrm{A}+1)X_{if}^> = (^<I + \sum_j {}^<P_j < {}^<A_j)^{-1}(k_f/k_i)2(2s_\mathrm{B}+1)\,{}^<X_{fi}\,,$$

where we have preserved conventional symbols, *e.g.*, according to ref. ([13]), and only added the time character. Additional, tedious, but simple calculations then yield the result $A_j^>/^<P_j = I^>/^<I$, that is *the ratio between the analyzing power $A_j^>$ of the open, forward interactions of nucleons* a *with external nuclei* A *and the polarization $^<P_j$ of the open, backward interactions of hadrons* b *with external nuclei* B *is equal to the ratio of the corresponding forward and backward units of their respective algebras of operators.*

A few comments are in order. As desired, the time asymmetry ceases to exist in the closed treatment of reactions a+A $\rightleftarrows$ b+B, trivially, because in this case the forward and backward identities are equal. Note that atomic mechanics is a particular case of the hadronic one by construction. Thus, the time asymmetry also ceases to exist when the hadronic units recover the atomic one. Recall that an infinite number of different Hamiltonians are possible for atomic mechanics, evidently, because of the endless possibilities of potential forces. By the same token, an infinite number of different hadronic units are possible, because of the endless possibilities, this time, of nonpotential forces. Also, atomic mechanics is unable to identify uniquely the Hamiltonian without experimental information on the system. By the same token, the ratio $I^>/^<I$ must be identified via experiments; it is expected to vary from reaction to reaction; and, per each given reaction, it is expected to vary with physical conditions (*e.g.*, energy of the collisions). Finally, hadronic mechanics establishes the *the symmetries of the Hamiltonian are not necessarily the symmetries of the system*, trivially, because the Hamiltonian represents only part of the system. In fact, the time asymmetry (7) holds for time symmetric Hamiltonians.

A direct experimental information on the time asymmetry of *open* nuclear reactions is provided by the measures by SLOBODRIAN, CONZETT, et al. ([14]) which shows clear differences between polarization and analyzing power. Even though the data are for the difference $(A_j^> - {}^<P_j)$, they can be reformulated for the ratio $A_j^>/^<P_j$, thus providing a first identification of the dependence of the ratio $I^>/^<I$ on $\theta_{c.m.}$. The hadronic prediction of asymmetry variation from reaction to reaction appears to be confirmed by the two independent reactions and their inverses considered in experiments ([14]). Finally, no relevant experimental information is currently available, to my best knowledge, on the additional hadronic prediction of asymmetry variation, per each reaction, with energy, and other physical data. Unfortunately, an independent verification of the polarization measurements of one reaction by HARDEKOPF, VEESER, et al. ([15]), *does not* confirm *quantitatively* asymmetries ([14]). The experimental situation is, therefore, unsettled at this moment.

An important, additional, experimental information favoring the time asymmetry

([13]) R. J. SLOBODRIAN: *Hadronic J.*, **4**, 1258 (1981).

([14]) R. J. SLOBODRIAN, C. RIOUX, R. ROY, H. E. CONZETT, P. VON ROSSEN and F. HINTERBERGER: *Phys. Rev. Lett.*, **47**, 1803 (1981); H. E. CONZETT: in *Polarization Phenomena in Nuclear Physics*, *AIP Conf. Proc.*, **69**, Part 2, 1422 (1981) and *Hadronic J.*, **5**, 714 (1982); R. J. SLOBODRIAN: *Hadronic J.*, **5**, 679 (1982). For the most recent measures following those of ref. ([14]), see C. RIOUX R. ROY, R. J. SLOBODRIAN and H. E. CONZETT: *Nucl. Phys. A*, **394**, 428 (1983).

([15]) R. A. HARDEKOPF, P. W. KEATON, P. W. LISOWSKI and L. R. VEESER: *Phys. Rev. C*, **25**, 1090 (1982).

(again, for the *open* case only) is given by the measures by RAUCH *et al.* [16] of an apparent deformation of the charge distribution of nucleons in the external field of nuclei, which is intriguingly close to the hadronic prediction (1% deformation for neutrons in the field of Mu-metal nuclei [12]). In fact, the time-reversal operator has an essential dependence on spin. A deformation of the charge distribution of nucleons then necessarily implies a form of time asymmetry. In fact, experiments [14] identify quite clearly the origin of the asymmetry in the spin component of the nuclear forces. Studies are currently in progress to compute the amount of time-asymmetry (as well as of space asymmetry) which is characterized by Rauch's measures of rotational asymmetry. The results will then be compared with data [14,15] as well as others.

However, Rauch's measures too are preliminary at this moment. It is, therefore, evident that a significant number of additional, different experiments are needed for a *quantitative* resolution of the problem of the origin of macroscopic irreversibility. The understanding is that the plausibility of a sufficiently small time asymmetry is out of question, as anticipated by DIRAC, since 1949 (in his celebrated paper in *Rev. Mod. Phys.*).

In fact, open nuclear reactions have been hystorically treated via *non-Hermitian Hamiltonians* with consequential *nonunitary time evolutions*. Law (5) merely provides their reformulation according to the rules

(8)
$$\begin{cases} A(t) = \exp[itH]A(0)\exp[-itH^{\dagger}] \equiv \exp[itBT^{>}]A(0)\exp[-i^{<}TB] \equiv \\ \qquad = I^{>}\exp[iB>t]>A(0)<\exp[-it<B]^{<I} , \\ \\ H = BT^{>}; \quad H^{\dagger} = {}^{<}TB; \quad H \neq H^{\dagger}; \quad B = B^{\dagger}; \quad (^{<}T)^{\dagger} = T^{>} \end{cases}$$

with infinitesimal versions

(9)
$$i\,dA/dt = AH^{\dagger} - HA \equiv A^{<}TB - BT^{>}A \equiv A < B - B > A ,$$

where the (Hermitian) Birkhoffian operator $B$ can represent the nonconserved energy.

Reformulations (8) and (9) are warranted by the fact that conventional nonunitary time evolutions do not admit a consistent algebra in their infinitesimal behaviour, trivially, because the « product » would be *trilinear* (being dependent on $A$, $H$, *and* $H^{\dagger}$), while our Lie-admissible reformulation admits a consistent *bilinear* product. In turn, this permits advances that do not appear to be readily achievable via time evolutions with inconsistent algebraic structures, as one can see for the case of time asymmetry (7).

The point is that *the general irreversibility of nonunitary time evolutions for open nuclear reactions is an incontrovertible physical truth*. We can, therefore, argue on the *amount* of the irreversibility for one given open nuclear reaction, or on its most effective theoretical treatment, but the existence of the irreversibility is out of question.

The same results can be reached in a number of alternative ways. For instance, when nucleons are approximated as being pointlike, they do not admit deformations, and their conventional (unitary or antiunitary) space-time symmetries *cannot* be broken. The conventional atomic principle of detailed balancing then follows, with its exact time reversal symmetry.

[16] H. RAUCH, A. ZEILINGER, G. BADUREK, A. WILFING, W. BAUSPIESS and U. BONSE: *Phys. Lett. A*, **54**, 425 (1976); G. BADUREK, H. RAUCH, A. ZEILINGER, W. BAUSPIESS and U. BONSE: *Phys. Rev. D*, **14**, 1177 (1976); H. RAUCH, A. WILFING, W. BAUSPIESS and U. BONSE: *Z. Phys. B*, **29**, 281 (1978); and S. HAMMERSCHMIED, H. RAUCH, H. CLERC and U. KISCHKO: *Z. Phys. A*, **302**, 323 (1981). For the latest measures indicating a rotational asymmetry of nucleons in the intense fields of nuclei see H. RAUCH: *Hadronic, J.* **5**, 729 (1982).

However, when nucleons are represented as they actually are in the physical reality, extended charge distributions, they admit (small) deformations under sufficient impacts and interactions with other nucleons. In this case the sphere $xx + yy + zz = 1$ is deformed into the ellipsoids $xa^{-2}x + yb^{-2}y + zc^{-2}z = 1$ with consequential, manifest, rotational asymmetry. A corresponding time-asymmetry then follows, as indicated earlier. Again, we can argue on the *amount* of the deformation of nucleons under given external fields, or on the time asymmetry characterized by a given rotational-asymmetry. But the existence of the time asymmetry is, again, out of question under the conditions considered.

After all, the belief that hadrons have a perfectly rigid charge distribution has no scientific ground.