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EVIDENCE ON THE ISOMINKOWSKIAN CHARACTER OF THE HADRONIC STRUCTURE

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We review the available conceptual, theoretical and experimental evidence according to which physical media in general, with particular reference to the hyperdense media in the interior of hadrons, alter the Minkowskian spacetime with local speeds of light which are generally subluminal for media of low density (such as planetary atmospheres), and superluminal for media of high density (such as hadrons). We show that the isominkowskian geometry can provide the only known geometrization of the spacetime of physical media which is: (1) *universal*, in the sense of applying for all possible signature-preserving spacetimes; (2) *direct*, in the sense of holding within the given inertial frame of the observer for all possible spacetimes; (3) *invariant*, in a way equivalent to that of the conventional spacetime; (4) *axiom-preserving*, thus permitting the preservation of the axioms of the special relativity and their extension to arbitrary speeds; and (5) *in agreement with the available preliminary experimental data.* Specific experiments are discussed in detail for the resolution of the still open problems of the spacetime and related basic laws holding in the interior of the hyperdense hadrons.

Key words: speed of light, hyperdense media, isominkowskian geometry.

1. EVIDENCE ON LOCAL CHARACTER OF THE SPEED OF LIGHT

Strictly speaking, the speed of electromagnetic (elm) waves is not a

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"universal constant," but rather a quantity $c = c_o/n$ depending on local physical conditions representable via the index of refraction n, where c_o is the speed in vacuum. Therefore, when experimentally established, deviations from c_o are rather forceful evidence of deviations from the conventional Minkowskian spacetime of the vacuum [1a], and vice versa.

Speeds $c = c_o/n < c_o$ are known in our Newtonian environment. Lesser known is the fact that one of the first invariance studies of speeds $c < c_o$ was done by Lorentz [1b] (see the related mention in Pauli's book [1c]).

Speeds $c = c_o/n > c_o$ have been apparently measured by A. Enders and G. Nimtz [1d] in the tunneling of photons between certain guides (see review [1e] for additional references and details). Apparent speeds $c = c_o/n > c_o$ have also been identified in certain astrophysical events [1f-1h] (see also the recent data [1i]). A comprehensive review of all superluminal speeds can be found in Ref. [1j].

Note that the hopes of regaining the exact Minkowskian spacetime by reducing light to photons scattering among molecules, even though valid as a first approximation, is no longer viable because: (1) The reduction to second quantization is questionable for elm waves in our atmosphere, say, with one meter wavelength; (2) The reduction does not permit quantitative studies of superluminal speeds; and (3) The reduction eliminates the representation of the inhomogeneity and anisotropy of physical media, which have apparent, experimentally measurable effects (see below).

Recall that hadrons are not ideal spheres with isolated points in them, but rather some of the densest media measured in laboratory until now. If spacetime anomalies are established for media of relatively low density, the hypothesis that the Minkowskian spacetime can be *exact* within hadrons in its conventional realization has little scientific credibility (see below for the exact character of an axiom-preserving covering spacetime). Also, deviations are expected from the complete mutual penetration of the wavepackets of the constituents, thus resulting in the historical open legacy of the existence of nonlinear, nonlocal and nonpotential effects in the interior of hadronic.

One of the first quantitative studies of the above legacy was done by D. L. Blokhintsev [2a] in 1964, followed by L. B. Redei [2b], D. Y. Kim [2c] and others. Note that the exact validity of the Minkowskian geometry for the *center-of-mass behavior* of a hadron in a particle accelerator is beyond scientific doubts. The authors of Refs. [2a-2c] then argued that a possibility for internal anomalies due to nonlocal and other effects to manifest themselves in the outside is *via deviations from the conventional Minkowskian behavior of the mean lives of unstable hadrons with the speed* v (or energy E).

Note that the Minkowski metric can be written $\eta = \text{diag}(1, 1, 1, -c_o^2)$. Therefore, *any deviation $\hat{\eta}$ from η necessarily implies a deviation from c_o*, as one can see by altering any component of the metric and then using Lorentz transforms.

Along these lines, R. M. Santilli [2d] submitted in 1982 the hypothesis that *contact-nonpotential interactions (thus including the strong interactions as per the above legacy) can accelerate ordinary (positive) masses at speed bigger than the speed of light in vacuun* much along the subsequent astrophysical measures [1f-1h]. The above hypothesis implies that *photons can travel inside the hyperdense hadrons at speeds bigger than that in vacuum*. V. de Sabbata and M. Gasperini [2e] conducted the first phenomenological verification within the context of the conventional gauge theories supporting the hypothesis of Ref. [2d], and actually reaching limit speeds up to $75c_o$ for superheavy hadrons.

The hypothesis of Ref. [22d] is also supported by the phenomenological calculations conducted by H. B. Nielsen and I. Picek [2f] via the spontaneous symmetry breaking in the Higgs sector of conventional gauge theories, which have resulted in the anomalous Minkowskian metrics (here written in the notation above)

$$\begin{aligned} \pi : \hat{\eta} = \text{diag}[&(1 + 1.2 \times 10^{-3}), (1 + 1.2 \times 10^{-3}), \\ &(1 + 1.2 \times 10^{-3}), -c_o^2(1 - 3.79 \times 10^{-3})], \end{aligned} \tag{1}$$

$$\begin{aligned} K : \hat{\eta} = \text{diag}[&(1 - 2.0 \times 10^{-4}), (1 - 2.0 \times 10^{-4}), \\ &(1 - 2.0 \times 10^{-4}), -c_o^2(1 + 6.00 \times 10^{-4})]. \end{aligned} \tag{2}$$

As one can see, calculations [2f] confirm speeds of photons $c = c_o/n > c_o$ for the interior of kaons, as conjectured in Ref. [2d]. Recall that: spacetime anomalies are expected to increase with the density; all hadrons have approximately the same size; and hadrons have densities increasing with mass. Therefore, results similar to (2) are expected for all hadrons *heavier* than kaons, as supported by phenomenological studies [2e].

The first direct experimental measurements on the behavior of the meanlife of K_S^o with energy, $\tau(E)$, were done by S. H. Aronson *et al.* [3a] at Fermilab and they suggested *deviations* from the Minkowskian spacetime in the energy range of 30 to 100 GeV. Subsequent direct measurements also for K_S^o were done by S. H. Aronson *et al.* [3b] also at Fermilab, suggesting instead *no deviations* of $\tau(E)$ from the Minkowskian form in the *different* energy range of 100 to 400 GeV.

More recently, a test of the decay law at short decay times was made by the OPAL group at LEP [3c]. In the latter experiment the ratio of number of events $Z^0 \to \tau^+\tau^-$ with deviations of τ from the conventional law to number of "normal" events was $(1.1 \pm 1.4 \pm 3.5)\%$.

2. ISOMINKOWSKIAN GEOMETRIZATION OF PHYSICAL MEDIA

A geometrization of the deviations from the Minkowskian spacetime was submitted by Santilli [4a] in 1983 under the name of *isominkowskian geometry* (see Ref.s [4b,4f] for the latest accounts) and resulted to be: (1) "directly universal," in the sense of admitting all infinitely possible, well behaved, signature-preserving and symmetric modifications of the Minkowski metric (universality), directly in the inertial frame of the observer (direct universality); (2) "invariant," in the sense of admitting a symmetry isomorphic to the Poincaré symmetry, the Poincaré-Santilli isosymmetry [4a-4d,4e]; and (3) "axiom-preserving", in the sense that the isominkowskian geometry and related symmetries are isomorphic to the conventional versions, a property denoted with the prefix "iso."

Moreover, the isominkowskian geometry has permitted the *exact* reconstruction of the special relativity under *arbitrary* local speeds of light [*loc. cit.*]. Refs. [4] have therefore established that, contrary to a popular belief (see, e.g., the "Lorentz asymmetry" of Ref. [2f]), the Minkowskian axioms, the Lorentz and Poincaré symmetry and the special relativity remain *exact* under all the above *spacetime anomalies*, of course, when properly formulated.

The isominkowskian geometry is essentially characterized by the lifting of the Minkowskian metric $\eta \to \hat{\eta} = \hat{T} \times \eta$, where $\hat{T}(x, v, E, \mu, \tau, \omega, ...)$ is a positive-definite 4×4 matrix with an arbitrary local dependence on coordinates x, speeds v (or energies E), densities μ, temperatures τ, frequencies ω, and any other needed variables. Jointly, the basic unit of the Minkowski space, I = Diag. (1, 1, 1, 1), is lifted by an amount which is the *inverse* of the deformation of the metric, $I \to \hat{I} = 1/\hat{T}$. The dual lifting $\eta \to \hat{\eta} = \hat{T} \times \eta$ and $I \to \hat{I} = 1/\hat{T}$ then implies the preservation of all original spacetime axioms. The lifting of the basic unit then requires, for consistency, the reconstruction of the entire mathematical apparatus of the conventional geometry into a form admitting $\hat{I}$ as the new, left and right unit. A necessary condition for invariance is therefore that the isominkowskian geometry be formulated via the *isomathematics* which consists of *isonumbers and isofields, isofunctions and isotransforms, isodifferential calculus, etc.* (see Ref. [41-4k], for mathematical studies and [4b,4l] for physical profiles).

Contrary to such apparent mathematical complexity, the isominkowskian geometry and related formalism can be entirely, uniquely and unambiguously constructed via a *noncanonical* transform at the classical level, $U \times U^t = \hat{I}$ and a *nonunitary* transform at the operator level, $U \times U^\dagger = \hat{I}$. For instance, the latter map yields: the isounit $I \to U \times I \times U^\dagger = \hat{I}$, isonumbers $n \to U \times n \times U^\dagger = n \times \hat{I} = \hat{n}$;

isocoordinates $x^\mu \rightarrow U \times x^\mu \times U^\dagger = x^\mu \times (U \times U^\dagger) = \hat{x}^\mu$; isoproduct $A \times B \rightarrow U \times (A \times B) \times U^\dagger = \hat{A} \times \hat{T} \times \hat{B} = \hat{A} \hat{\times} \hat{B}, \hat{K} = U \times K \times U^\dagger, K = A, B, \hat{T} = (U \times U^\dagger)^{-1} = \hat{I}^{-1}$; isoseparation $x^2 \rightarrow U \times x^2 \times U^\dagger = (U \times x^\mu \times U^\dagger) \times (U^{\dagger -1} \times \eta_{\mu\nu} \times U^{-1}) \times (U \times x^\nu \times U^\dagger = \hat{x}^\mu \times \hat{\eta}_{\mu\nu} \times \hat{x}^\nu$; etc.

Once constructed via the above noncanonical/nonunitary transforms, the isominkowskain geometry is then invariant under additional transforms of the same class, provided that they are written in terms of isomathematics, e.g., $U = \hat{U} \times \hat{T}^{1/2}, U \times U^\dagger = \hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} U = \hat{I}$, for which the isounit $\hat{I}$ is *numerically invariant*, $\hat{I} \rightarrow \hat{U} \hat{\times} \hat{I} \hat{\times} \hat{U}^\dagger = \hat{I}$, the isoproduct is *numerically invariant*, $\hat{A} \hat{\times} \hat{B} \rightarrow \hat{U} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{U}^\dagger = \hat{A}' \hat{\times} \hat{B}'$ (i.e., $\hat{T}$ is numerically preserved); etc.(see Refs. [4b,4f,4l] for details).

The above properties illustrate that: (1) Any mixing of conventional and isomathematics (e.g., deformation of the metric $\eta \rightarrow \hat{T} \times \eta$ referred to the conventional unit I and fields, as done in deformations) implies the loss of invariance with consequential lack of physical content; (2) Any change of the speed of light requires a noncanonical/nonunitary transform of the conventional Minkowskian setting with consequential loss of invariance (in fact, the Poincaré symmetry can only provide the invariance in vacuum); and (3) The only known invariant formulation of arbitrary speeds of light is that permitted by Santilli's isomathematics, isominkowskian geometry and isopoincaré symmetry.

The isominkowskian geometry provides a direct geometrization of physical media at both the classical and operator levels [4l]. Since $\hat{T}$ is positive-definite, $\hat{\eta}$ can always be diagonalized in the form $\hat{\eta} = \text{Diag.}\ (1/n_1^2, 1/n_2^2, 1/n_3^2, -c_o^2/n_4^2)$, thus providing a geometrization of: the local *inhomogeneity* (e.g., via a dependence of the n's from the density); the local *anisotropy* (e.g., via a differentiation between the space and time n's); as well as *arbitrary local speeds of elm waves* (via the expression $c(x, \mu, \omega, ...) = c_o/n_4(x, \mu, \omega, ...)$ first proposed in [4a]).

The isotopic behavior of the meanlife with speed (or energy) for isotropic space with $n_1 = n_2 = n_3 = n_s(x, \mu, \omega, ...)$ (yet with general spacetime anisotropy $n_s \neq n_4$) is given by [4a-4c]

$$\hat{\tau} = \tau_o \hat{\gamma}, \ \hat{\gamma} = (1 - \hat{\beta}^2)^{-1/2}, \ \hat{\beta} = (v/n_s)/(c_o/n_4) \tag{3}$$

and includes all existing or otherwise possible laws [2] via different power series expansions in terms of different parameters with different truncations [4e]. This eliminates the ambiguity of individually testing the several different laws of Refs. [2].

Note that, when a hadron is studied from the outside, one evidently can only use the *average of the n-quantities to constants*, called "characteristic constants" of the medium considered. Note also that a possible anysotropy of the medium implies a deviation from the conventional Doppler shift studied by Mignani [5a] and others which will be studied elsewhere as a possible complement to measurements [2,3]. Note finally that the latter anomalies are eliminated by the reduction of light to photons *moving in vacuum* and scattering among molecules.

Isotopic law (3) was applied by F. Cardone, R. Mignani, and R. M. Santilli [5b] to the experimental data of Refs. [3a,3b] resulting in the single fit of both experiments,

$$1/n_1^2 = 1/n_2^2 = 1/n_3^2 = 0.909080(\pm 0.004), 1/n_4^2 = 1.003(\pm 0.002). \tag{4}$$

Therefore, even under the assumption of the correct character of measurements [3b], they do not establish the validity of the Minkowskian geometry inside hadrons because of the above isominkowskian fit. Note also that fit (4) confirms the superluminal character of the propagation of light within the hyperdense hadronic media, a property that appears to be confirmed by other studies (see the outline in [4b]). We should also mention that nonlinear and nonlocal effects at short distances have been recently studied in Refs. [5d,5e,4l].

Note that the so-called "deformations" of the Minkowskian geometry (e.g., the k-deformed Minkowski space of Refs. [6]) is not equivalent to the isominkowskian space for numerous reasons, such as: the former is defined via conventional numbers and fields while the latter is defined via isonumbers and isofields; the former implies the loss of the fundamental Poincaré symmetry, while the latter preserves it and only realizes it in a more general way; and, last but not least, the former implies the necessary abandonment of the special relativity in favor of a yet unknon relativity, while the latter preserves the axioms of the special relativity by central requirement. Irrespective of that, it does not appear to be sufficiently known that all classical and quantum deformations are afflicted by rather serious problems of physical consistency (such as the lack of invariant units of space, time, energy, etc., as inherent in all noncanonical/nonunitary transforms, with consequential lack of applicability to real measurements; loss of Hermiticity in time, as easily proved under nonunitary transforms on conventional Hilbert spaces, with consequential lack of real observables; etc.), which inconsistencies prevent any physically meaningful application (see Refs. [7] for all details).

3. THE UNRESOLVED CHARACTER OF TESTS [3a,3b]

In summary, all available conceptual, theoretical, phenoomenological and experimental evidence suggest eviations from the Minkowskian geometry inside hadrons with the sole exception of the Fermilab tests [3b].

In this note we therfore re-examine tests [3b] by focusing the attention on the range-energy selection rule which can be applied to re-elaborate the initial data on K_S decays. By taking into account the results as they were done, we performed Monte Carlo simulations of the main features of experiment [3b] and made our own fits for K_S^o. Our conclusions and recommendations are the following:

(1) We agree that the parameters in the full formula dN/dt for the proper time evolution are strongly correlated. This may cause a generally non-relevant regular dependence of the parameters on entities which are not present in the formula, such as number of runs, energy, etc., apart from the systematic uncertainties. Therefore, the above dependence may shadow the weak energy dependence we are interested in, as can be seen from the large values of the correlation elements.

(2) The authors of [7] solved the problem of non-correlated fit by selecting the K_S^o momenta greater than 100 GeV/c. By means of that energy cut, they obtained the data sample in which the CP violating terms contribute up to 1.6%. However, it seems unrealistic to look for the deviations from the Minkowskian decay law of the order of some percent. More realistic is to test the decay law on the level of 10^{-3}, as suggested by studies [2]. In fact, the assumption of 1.6 contribution from PC violation in the data elaboration of Ref. [3b] implies looking for the energy dependence of τ_s at the level $k \times 10^{-2}$, thus rendering meaningless to look for more realistic deviations of the order of 10^{-3} and smaller.

(3) We propose to suppress the CP violating terms significantly using selection rule for the ratio R/E, where R and E are K_S^o range and energy. In the experiment, R/E ranges from 2.3 to 36.1 cm/GeV. The R/E interval should be selected to make the contribution of the CP violating terms less than a desirable value, say $k \times 10^{-3}$. An effective (R, E) plot can then be calculated via Monte Carlo methods applied to the real decay volume.

The price we pay for more accurate data handling due to the range-energy selection rule will be *lower statistics.* In fact, under the above new assumptions, 60-70% events will be rejected, i.e., only 63K - 84K events of the total 220K will be useful. Apart from the loss of a major part of the data, 1/3 of the decay volume in the experiment turns out to be also useless. The large inefficiency of the experiment occurred because it had not been optimized for the problem. Basicly, the experimental design and data selection rules followed that of conventional K_S, K_L studies.

We illustrate the above arguments with two fits shown in Fig. 1. 220 000 K_S decays at six energy values (from 125 to 375 GeV) were generated in the decay volume with the ranges from 9.3

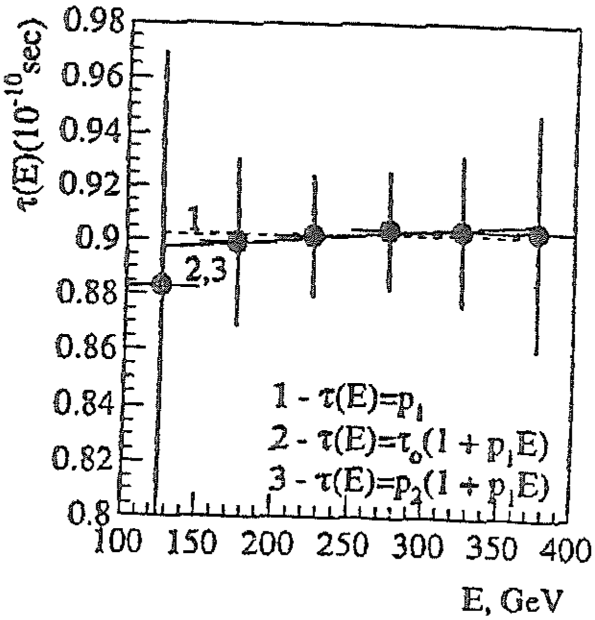

Fig. 1. Comparison of the various fitting functions (curves 1, 2, and 3) applied to the simulated lifetime $\tau(E)$ dependence of Ref. [3b] under the energy-range selection rule identified in the text.

m to 25.3 m. The energy dependence of the lifetime was assumed in the form $\tau(E) = \tau_S(1 + \epsilon E)$ with $\tau_S = 0.8927$, the world average of the mean lifetime, and $\epsilon = 4{\cdot}10^{-5}$. After applying the range-energy selection rule, a sample of 64K events was chosen for which the contribution of the CP violating terms was less then 0.008. Namely we deal with the following distribution for the proper lifetime:

$$\frac{dN}{dx} = N[\exp(-x) \ + \ \mathrm{CPV}], \tag{5}$$

where N is a normalization constant, $x = t/\tau(E)$ and CP violating terms are equal to

$$\begin{aligned} \mathrm{CPV} =& \mid \eta_{+-} \mid^2 \exp(-xy) \\ &+ \ 2D \mid \eta_{+-} \mid \ \cos(\Delta m \ t - \phi_{+-})\exp(-x(1+y)/2), \end{aligned} \tag{6}$$

where y stands for $\tau_S(E)/\tau_L$.

The values of other parameters are taken as the world average values from Ref. [4]. These are $\mid \eta_{+-} \mid = 2.284 \cdot 10^{-3}$, the magnitude of the CP-nonconservation parameter in $K_L^o \to \pi^+\pi^-$ decay, $\phi_{+-} = 43.7^o$, and $\Delta m = 0.5333 \cdot 10^{10} \hbar sec^{-1}$ is the mass difference of $K_L^o - -K_S^o$. The dilution factor D is defined as the ratio $(N - \bar{N})/(N + \bar{N})$ where N $(\bar{N})$ is the number of K^o $(\bar{K^o})$ produced by the proton beam on the target. We accepted the value D=0.75.

In Fig. 1 the sequence of the mean proper lifetimes is plotted versus E, K_S^o laboratory energies. The dependence was obtained by simulations of K_S^o decays in the experimental volume under the conditions described above. The figure presents also two one-parameter fits: (a) the energy-dependent formula of the type $\tau(E) = 0.8927(1 + p_1 E)$ with the obtained value $p_1 = (4 \pm 5) \cdot 10^{-5}$ and $\chi^2/\text{ndf} = 0.38/5$ (solid line); (b) fit by a constant function $\tau(E) = c$, with c=0.90$\pm$0.01 and $\chi^2/\text{ndf} = 0.7/5$ (dashed line).

For comparison, we performed also the two-parameter fit to the formula of Ref.[3b], $\tau(E) = p_2(1 + p_1 E)$. In this case, the value of the crucial parameter p_1 is equal to $(4 \pm 23) \cdot 10^{-5}$ with $\chi^2/\text{ndf} = 0.38/4$.

There is a difference in interpretation of parameters in the two fitting formulae with the energy dependence. The parameter p_2 in the fit from the cited paper was interpreted as the zero-energy mean value of the proper lifetime. We think that it is difficult to extrapolate the fitting formulae from the energy interval 100-400 GeV to zero. Instead, we try to find the energy dependence in the limited energy interval by fit starting from a definite point. This difference in interpretation is important because, in general, various approaches in fitting procedures may lead to crucially different numerical results.

Thus, in the selected amount of the events, both fits dig up well the mean value of the hidden parameter ϵ determining the energy dependence in the simulated K_S^o decays, however the error bars differ strongly. Though both results for fitting values of p_1 are still insignificant statistically even in the selected sample of events, the 100% error bar in our fit being rather promising. It opens the door for new manipulations with the selection procedure aiming to improve the result. So we encourage the re-elaboration of the original data of [7] under the modified selection rules to obtain possible hopeful estimations of $\tau(E)$ instead of previous hopeless ones.

We finally note that no firm spacetime anomalies can be established via the above re-elaboration for PC violating contributions smaller than 1.6% because said anomalies are visually within the errorbars (Fig. 1) due to insufficient statistuics and other reasons. Corresponding deviations cannot be considered for PC violating contribution larger than 1.6% because the latter are experimentally known to be excluded for the energy range of measures [3b].

Despite that, the analysis of this note establishes the insufficiencies of tests [3a,3b] and the need for final, more accurate measurements as the only way to resolve the now vexing fundamental problem of the spacetime geometry and physical laws holding in the interior of the hyperdense hadrons. After all, as indicatd earlier, the isominkowskian fit [5b] of experiments [3a,3b] establishes the existence of spacetime anomalies with superluminal speeds in the interior of hadrons even in the event that measurements [3b] result to be correct.

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