LETTERE AL NUOVO CIMENTO VOL. 37, N. 16 20 Agosto 1983

Lie-Isotopic Lifting of the Special Relativity for Extended Deformable Particles.

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(ricevuto il 9 Maggio 1983)

PACS. 03.30. – Special relativity.

Summary. – We recall the variation of the speed of light with the local physical conditions of the material media in which it propagates, and identify a corresponding class of generalized metrics. The underlying group of isometries is constructed via a Lie-isotopic lifting of the envelope, algebra and group structure of Lorentz transformations. It is shown that the generalized transformations, called Lorentz-isotopic, are apparently capable of characterizing an isotopic lifting of the special relativity for extended, and therefore deformable particles. The current experimental information on the apparent approximate character of the conventional Lorentz transformations in particle physics are reviewed, and a number of direct tests suitable for the resolution of the issue are indicated.

As is known, the constancy of the speed of light, the underlying Lorentz's invariant in Minkowski space

$$x^2 = x^t \eta x = x^\mu \eta_{\mu\nu} x^\nu = x^1 x^1 + x^2 x^2 + x^3 x^3 - x^4 c_0^2 x^4 , \qquad x^4 = t \tag{1}$$

and the special relativity at large, were specifically conceived for motion in vacuum (intended as empty space), as limpidly stated in the historical contributions by LORENTZ, POINCARÉ, EINSTEIN, and others (1).

Nevertheless, it is known that the speed of light is not an absolute constant through the Universe, but possesses a rather complex functional dependence on the characteristics of light itself (*e.g.*, wave-length) and on the local quantities of the physical medium in which it propagates (*e.g.*, time, co-ordinates, velocity, index of refraction, density, etc.). Thus a more adequate invariant should express the local functional dependence of the speed of light, $c = c(t, \boldsymbol{r}, \dot{\boldsymbol{r}}, \ldots)$, as well as the general inhomogenuity and unisotropy

(*) Supported by the U. S. Department of Energy under contract number DE-AC02-80ER10651 A002.

(1) An excellent account still remains that by W. PAULI: *Relativitätstheorie* (Lipsia, 1921).

of material media, *e.g.*, it should be of the type

$$x^{\hat{2}} = x^t g x = x^\mu g_{\mu\nu}(x, \dot{x}, \ldots) x^\nu = x^1 b_1^2 x^1 + x^2 b_2^2 x^2 + x^3 b_3^2 x^3 - x^4 c^2 x^4 , \tag{2}$$

which preserves the topological structure of (1). Further generalizations, *e.g.* via a metric with a functional dependence of the type $g_{kk} = f_{kk}(b)$, $g_{44} = f_{44}(c)$, should not be excluded.

In regard to the motion of particles, the hystorical contributions [1] also stated, quite limpidly, that the applicability of invariant (1) should be restricted to pointlike particles moving in empty space [2]. In fact, it was known that, whenever particles cannot be effectively approximated as massive points, their motion does not generally occur in empty space, but rather in material media. Even though not necessarily unique, a measure of the extended character of particles moving in material media is therefore given by a departure from the perfectly homogeneous and isotropic character of empty space, *i.e.* is provided by generalized invariants (at least) of type (2).

The validity of invariant (1) for electromagnetic interactions is now established by a truly impressive amount of experimental evidence. However, authoritative doubts on the validity of the same invariant for physical conditions of particles different than those conceived by LORENTZ, POINCARÉ, and EINSTEIN, have been expressed, since the early part of this century. For instance, in regard to the interior of strongly interacting particles and their nuclear forces, Fermi clearly expressed « doubts as to whether the usual concepts of geometry hold for such small regions of space » [3].

A systematic study of Fermi's legacy was suggested in ref. [4], and then conducted at the yearly *Workshops on Lie-admissible Formulations* [5]. The approach is based on the notion of closed, variationally non-self-adjoint systems [6]. These are systems which, when seen from the outside, verify conventional, total conservation laws. Nevertheless, their internal forces are generally (nonlocal and) non-Hamiltonian due to contact interactions among extended constituents for which the notion of potential energy has no physical meaning. The admission of an internal non-Hamiltonian interaction then implies the lack of exact character of the analytic, geometric and algebraic foundations of special relativity, in favour of suitable generalizations. In particular, the model implies that, while the center-of-mass motion of a hadron in vacuum is fully conformed to invariant (1), the motion of its constituents could be governed by the more general invariant (2). Thus, under the approximation of an isotropic medium with $b_1 = b_2 = b_3 = b$, invariant (2) characterizes the maximal speed

$$v_{\max} = c(t, \boldsymbol{r}, \dot{\boldsymbol{r}}, \ldots)/\sqrt{3}\, b(t, \boldsymbol{r}, \dot{\boldsymbol{r}}, \ldots) \gtreqless c_{\text{vacuum}} = c_0 \tag{3}$$

according to the hypothesis submitted in ref. [7], with $v_{\max} < c_0$ suggested for nuclear

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(2) The early, well-written, treatises on special relativity stressed explicitly its restricted applicability to massive points (see, *e.g.*, the title of Chapt. VI of P. G. BERGMANN: *Introduction to Special Relativity* (Englewood Cliffs, N.J., 1942)). It is regrettable that this sound scientific caution has generally disappeared in more recent treatises on the subject.

(3) E. FERMI: *Nuclear Physics* (Chicago, Ill., 1949), p. 111.

(4) R. M. SANTILLI: *Hadronic J.*, **1**, 574 (1978).

(5) *Proceedings of the Second Workshop on Lie-admissible Formulations*, Parts A and B, *Hadronic J.*, **3**, (1979); *Proceedings of the Third Workshop on Lie-admissible Formulations*, Parts A, B and C, *Hadronic J.*, **4** (1980); see also the *Proceedings of the First International Conference on Nonpotential Interactions and Their Lie-admissible treatment*, Parts A, B, C and D, *Hadronic J.*, **5** (1982).

(6) R. M. SANTILLI: *Foundations of Theoretical Mechanics*, Vol. I (Berlin, 1978).

(7) R. M. SANTILLI: *Lett. Nuovo Cimento*, **33**, 145 (1982).

costituents, and $v_{\max} > c_0$ suggested for hadronic constituents. Subsequent independent studies [8] within the context of unified gauge theories in a curved space have produced the first estimate for hadronic constituents $v_{\max} = 75c_0$.

Possible internal deviations from invariant (1) in closed non-Hamiltonian systems, even though not directly detectable from the outside, can manifest themselves in a number of indirect ways. For instance, it has been suggested for a long time that possible nonlocal internal effects may imply deviations from standard predictions of the mean life of unstable hadrons in flight [9]. A systematic study of this possibility has been conducted in ref. [10] (and quoted earlier papers) for the weak decays of hadrons within the context of unified gauge theories. The main idea is that a departure from invariant (1) occurs in the Higgs sector of spontaneous symmetry breaking according to the following particular case of (2):

$$
(4)\qquad \begin{cases} x^{\hat 2} = x^\mu g_{\mu\nu} x^\nu\,, & g_{\mu\nu} = \eta_{\mu\nu} + \chi_{\mu\nu}\,, \\ \chi_{44} = \alpha\,, & \chi_{kk} = \frac{1}{3}\alpha\,, \\ \alpha = (-3.79 \pm 1.37)\cdot 10^{-3} \quad \text{for } \pi^+\,, & \alpha = (0.61 \pm 0.17)\cdot 10^{-3} \quad \text{for K}^+ \end{cases}
$$

with weighted average $\alpha = (0.54 \pm 0.17)\cdot 10^{-3}$. Subsequent independent calculations [11] give the similar value $\alpha = (3.6 \pm 5.2)\cdot 10^{-3}$, as well as predict an upper limit for experiments under way.

Deviations from invariant (1) are also at the foundation of a number of theoretical studies, such as the superluminal Lorentz transformations [12] or the Finsler treatment of local anisotropy [13], and may provide a unified interpretation of a number of experimental aspects, such as: $K^0\bar{K}^0$ regeneration experiment [14], anomalous behaviour of the magnetic moments [15], anomalous torques on electron spin [16], possible instability of the proton [17], and deviations from the discrete symmetries P [18], T [19], C [20] and PCT [21].

At any rate, the possible deformation of the charge distribution of hadrons, from the perfectly spherical shape $xx + yy + zz = 1$ into the ellipsoids $xb_1^2x + yb_2^2y + zb_3^2z = 1$, has been theoretically predicted in ref. [4] (pp. 786-797) via a Lie-admissible generalization of the enveloping algebra of SU_2, it has been quantitatively worked out in ref. [15,5], and it has been apparently confirmed via neutron interferometry to be about 1% for

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[8] V. De Sabbata and M. Gasperini: *Lett. Nuovo Cimento*, **34**, 337 (1982).
[9] See, *e.g.*, D. Y. Kim: *Hadronic J.*, **1**, 1343 (1978), and quoted papers.
[10] H. B. Nielsen and I. Picek: *Nucl. Phys. B*, **211**, 269 (1983).
[11] R. Huerta-Quintanilla and J. L. Lucio: Fermilab preprint 83/18-THY (1983).
[12] E. Recami and R. Mignani: *Lett. Nuovo Cimento*, **4**, 144 (1972). For an updated review, see G. D. Maccarone and E. Recami: preprint INFN/AE-82/12 (1982), University of Catania, Italy.
[13] G. Yu. Bogoslovsky: *Nuovo Cimento B*, **40**, 99, 116 (1977); **43**, 377 (1978).
[14] S. H. Aronson, G. J. Bock, H. Y. Cheng and E. Fishback: *Phys. Rev. Lett.*, **48**, 1306 (1982).
[15] G. Eder: *Hadronic J.*, **4**, 634, 2018 (1981); **5**, 750 (1982).
[16] P. R. Phillips: *Rev. Sci. Instr.*, **50**, 1018 (1979).
[17] See, *e.g.*, A. Zee: *Phys. Rev. D*, **25**, 1864 (1982).
[18] See, *e.g.*, M. Forte, B. R. Heckel, N. F. Ramsey, K. Green, G. L. Green, J. Byrne and J. L. Pendlebury: *Phys. Rev. Lett.*, **45**, 2088 (1980).
[19] R. J. Slobodrian, C. Rioux, R. Roy, H. E. Conzett, P. von Rossen and F. Interberger: *Phys. Rev. Lett.*, **47**, 1803 (1981); C. Rioux, R. Roy, R. J. Slobodrian and H. E. Conzett: *Nucl. Phys. A*, **394** (1983); and R. A. Hardekopf, P. W. Keaton, P. W. Lisowski and L. R. Veseer: *Phys. Rev. C*, **25**, 1090 (1982).
[20] See, *e.g.*, the B.C.D.M.S. Collaboration: J.I.N.R. Preprint EI-82-656 (1982), Dubna, U.S.S.R.
[21] See, *e.g.*, I. I. Bigi: *Z. Phys. C*, **12**, 235 (1982).

neutrons in the intense fields in the vicinity of Mu-metal nuclei (see ref. [22] and quoted earlier tests). It is evident that a deformation of the charge distribution of hadrons, that is, a deformation of the space component of invariant (1) into that of (2) with consequential, manifest, rotational asymmetry, must necessarily imply the transition to the entire invariant (2). At a deeper study, it emerges that the deformation of the charge distribution of extended particles can be considered as the ultimate physical foundation of virtually all studies reviewed here. By recalling that pointlike particles cannot be deformed and that their rotational symmetry cannot be broken (irrespective of the interactions considered), the ultimate conceptual foundations of invariant (2) in particle physics can be seen in the extended character of hadrons, with a corresponding profile for classical settings.

In this letter, we shall summarize our studies on a possible generalization of the special relativity for extended particles. A detailed presentation will appear elsewhere. The hope is that all the independent, theoretical and experimental contributions considered here (as well as others not indicated for brevity) may, one day, result to be different foundations of one, single, underlying physical theory [23].

The mathematical methods used in our analysis are those of the so-called *Lie-isotopic theory* [24-27]. The term «lifting» is often used as one way to differentiate the emerging generalizations from other approaches, *e.g.*, those characterized by maps.

The generalized relativity will be constructed along lines parallel to those of the isotopic lifting of Galilei's relativity submitted in ref. [25]. We are referring to a generalized relativity characterized by a Lie symmetry whose abstract, co-ordinate-free form *coincides* with that of the conventional relativity. The generalized relativity, therefore, essentially consists of *the most general possible realization of a known abstract symmetry* [25-27]. The terms *Galilei-isotopic relativity* have been submitted in ref. [25] for the nonrelativistic case, while the terms *Lorentz-isotopic relativity* are submitted here, for the relativistic one. The understanding is that other terms are equally conceivable such as «Poincaré-isotpic» or «Einstein-isotopic».

For the reader's convenience, we recall that the Lie-isotopic theory is based on the following main aspects:

a) isotopic lifting of the universal enveloping associative algebra $\mathscr{E}$ with conventional associative product AB and unit I, $IA = AI = A$, into the form $\hat{\mathscr{E}}$ character-

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[22] H. RAUCH: *Hadronic J.*, 5, 729 (1982).

[23] A possible unification of the different studies considered will inhevitably call for revisions aimed at mutual compatibility. For instance, invariant (2) and related Lorentz-asymmetry is conceived to be dependent on local physical conditions and, thus, it is expected to vary not only from the weak to the strong and to other interactions, but also from reaction to reaction within each type of interaction. In fact, not only the value, but even the sign of the Lorentz asymmetry parameter α of eqs. (4) is different for the π^+ and K^+ decays. As a result, to reach unification with other models along the same lines, studies aiming at a sort of «universal Lorentz asymmetry», should be revised as to admit a local dependence of the Lorentz-asymmetry, *i.e.* a local dependence of the speed of light.

[24] The notion of isotopy is rather old in abstract algebras, although generally ignored in contemporary literature. Apparently, its first application to Lie's theory (enveloping algebras, Lie algebras, and Lie groups) was made by R. M. SANTILLI: *Hadronic J.*, 4, 223 (1978), pp. 287-290 and 329-374, as an intermediary step toward the Lie-admissible generalization of Lie's theory. A review of the state of the art in 1982 is presented in ref. [25], pp. 148-183. Specific applications to the generalization of pseudo-Euclidean spaces have been presented in ref. [26,27]. Mathematical studies can be found in ref. [5].

[25] R. M. SANTILLI: *Foundations of Theoretical Mechanics*, Vol. II (Berlin, 1982).

[26] R. M. SANTILLI: *Lie-isotopic liftings of Lie's theory*. - I: *General considerations*, I.B.R. preprint DE-83-2 (1983), submitted for publication.

[27] R. M. SANTILLI: *Lie-isotopic liftings of Lie's theory*. - II: *Lifting of rotations*, I.B.R. preprint DE-83-2 (1983), submitted for publication.

ized by the product $A * B = A\mathscr{T}B$ and the new identity $\hat{I} = \mathscr{T}^{-1}$, $\hat{I} * A = A * \hat{I} = A$, where $\mathscr{T}$ is fixed and nonsingular;

b) isotopic lifting of the (continuous) Lie transformation group $G: x' = gx = \exp[Xu]|_{\mathscr{E}} x$, into the form $\hat{G}: x' = \hat{g} * x = \exp[Xu]|_{\hat{\mathscr{E}}} * x = \exp[X\mathscr{T}u]|_{\mathscr{E}} x$, where X and u are the generator and parameter, respectively, of the original group; and

c) isotopic lifting of the Lie algebra $G: [X_i, X_j] = X_i X_j - X_j X_i = C_{ij}^k X_k$ into the form $[X_i \hat{,} X_j] = X_i * X_j - X_j * X_i = \hat{D}_{ij}^k * X_k$, where $\hat{D} = D\hat{I}$;

with underlying methodology, such as the Lie-isotopic extension of the Poincaré-Birkhoff-Witt theorem (for the construction of a basis of $\hat{\mathscr{E}}$); of the Baker-Campbell-Hausdorff theorem (for the isotopic composition law); of Lie's first, second and third theorems (for the isotopic commutation rules); etc.

More particularly, we shall use the following property of the Lie-isotopic theory worked out in the recent paper ([26]).

Theorem 1. Let $G(m)$ be a m-parameter Lie symmetry group of the composition $z^\dagger \delta z = \bar{z}^i \delta_{ij} z^j$ of a n-dimensional Euclidean space $E(n, \delta, F)$ over the field F of real numbers R, complex numbers C, or quaternions Q. Then, the isotopic lifting $\hat{G}(m)$ of $G(m)$ characterized by a nonsingular, Hermitian, and sufficiently smooth metric g in the local variables, leaves invariant the generalized composition $\bar{z}^i g_{ij} z^j$ of the isotopic space $\hat{E}(n, g, \hat{F})$, $\hat{F} = F\hat{I}$.

The theorem essentially states that, when the isotopic element $\mathscr{T}$ (characterizing all associative products) is given by the new metric $g(t, z, \dot{z}, \ldots)$, the new unit is the Casimir invariant of order zero, $\hat{I} = g^{-1}$. The form invariance of the new metric is then consequential, as the interested reader can verify without the analysis of ref. ([26]).

In the subsequent paper ([27]), we have constructed the isotope $\hat{O}_3$ of O_3, that is, the generalization of the conventional rotation group which leaves invariant all possible deformations of the sphere, $x^1 b_1^2 x^1 + x^2 b_2^2 x^2 + x^3 b_3^2 x^3 = 1$. As expected, $\hat{O}_3$ turned out to be locally isomorphic to O_3 under the assumed metric (or to $O_{2,1}$ in other cases).

In this paper, we shall construct a generalization of the Lorentz's transformations verifying the following conditions.

A) The generalized transformations act in the *Minkowski-isotopic space*, *i.e.* in the lifting $\hat{E}(4, g, \hat{R})$ of $E(4, \delta, R)$ with points $x = (x^1, x^2, x^3, x^4 = t)$ characterized by the metric g of composition (2), according to the transformation laws

$$x' = \hat{A} * x = \hat{A} g x, \qquad x^{t\prime} = x^t * \hat{A}^t = x^t g \hat{A}^t, \tag{5}$$

under the condition that they leave invariant separation (2), *i.e.* verify the properties

$$x^{t\prime} * x' = x^t * \hat{A}^t * \hat{A} * x \equiv x^t * x = x^t g x, \tag{6a}$$

$$\hat{A}^t * \hat{A} = \hat{I}, \qquad (\det \hat{A})^2 = (\det g^{-1})^2. \tag{6b}$$

B) The transformations so defined constitute a Lie-isotopic group, *i.e.* they verify the isotopic group laws

$$\hat{A}(0) = \hat{I}, \quad \hat{A}(u) * \hat{A}(u') = \hat{A}(u + u'), \quad \hat{A}(u) * \hat{A}(-u) = \hat{I}, \tag{7}$$

C) The generalized transformations characterize a covering of the conventional Lorentz transformations, in the sense that they apply to a broader physical arena (extended particles moving in material media), while admitting the conventional theory (for pointlike particles) as a particular case.

As anticipated earlier, the transformations verifying conditions *A*), *B*), *C*), will be called *Lorentz-isotopic* and their group, denoted with $\hat{O}_{3,1}$, will be called the *Lorentz-isotopic group.*

To simplify the construction, we use the factorizations $\hat{A} = A\hat{I} \in \hat{O}_{3,1}$ and $\hat{N} = N\hat{I} \in \hat{\mathbf{R}}$, under which we ignore the lifting $\hat{\mathbf{R}}$ of the field $\mathbf{R}$ (for which $\hat{N} * x = Nx$), while conditions (6*b*) assume the simpler form $A^t g A = g$, $\det A = \pm 1$. The assumption of the metric $g = \operatorname{diag}(b_1^2, b_2^2, b_3^2, -c^2)$ then implies the preservation under lifting of the connectivity properties of $O_{3,1}$, resulting in the components $\hat{O}^{\uparrow}_{\pm 3,1}$ and $\hat{O}^{\downarrow}_{\pm 3,1}$. It is possible to prove that $\hat{O}^{\uparrow}_{+3,1}$ forms a (Lie-isotopic) group, while the remaining components, which are characterized by the isotopic inversions

$$
\begin{cases}
x' = \hat{P} * x = Px = (-\boldsymbol{r}, t)\,, \\
x' = \hat{T} * x = Tx = (\boldsymbol{r}, -t)\,, \\
x' = \hat{P} * \hat{T} * x = \widehat{PT} * x = PTx = (-\boldsymbol{r}, -t)\,,
\end{cases}
\tag{8}
$$

do not form a group unless combined with $\hat{O}^{\uparrow}_{+3,1}$, as in the standard case.

The construction of the Lorentz-isotopic group is, therefore, reduced to that of $\hat{O}^{\uparrow}_{+3,1}$, whose explicit form is given by

1) *The isotopic lifting of the enveloping associative algebra* $\hat{\mathscr{E}}$ *of* $O^{\uparrow}_{+3,1}$

$$
\hat{\mathscr{E}}: \quad \hat{I} = g^{-1}\,, \quad X_k\,, \quad X_i * X_j\,, \quad X_i * X_j * X_k, \ldots \qquad i \leqslant j\,, \quad i \leqslant j \leqslant k\,,
\tag{9}
$$

where the basis X_k consists of the ordered set of the conventional 4×4 generators of O_4, say, $\boldsymbol{J}$ and $\boldsymbol{M}$, as given, *e.g.*, in ref. [28], p. 42, under the redefinition

$$
\begin{cases}
X = \{\tilde{J}_k, \tilde{M}_k\}\,, \quad \tilde{J}_1 = b_2^{-1} b_3^{-1} J_1\,, \quad \tilde{J}_2 = b_1^{-1} b_3^{-1} J_2\,, \\
\tilde{J}_3 = b_1^{-1} b_2^{-1} J_3\,, \quad X^\dagger = -X\,, \quad \tilde{M}_k = b_k^{-1} M_k\,, \quad k = 1, 2, 3\,.
\end{cases}
\tag{10}
$$

2) *The isotopic lifting of the proper Lorentz group* characterized by the following expansions in $\hat{\mathscr{E}}$

$$
\hat{O}^{\uparrow}_{+3,1}: \quad \hat{A}_g(\theta, u) = \left(\prod_{k=1}^{3}{}^{*} \exp[\tilde{J}_k \theta_k]|_{\hat{\mathscr{E}}}\right) * \left(\prod_{k=1}^{3}{}^{*} \exp[\tilde{M}_k u_k]|_{\hat{\mathscr{E}}}\right) = \left(\prod_{k=1}^{3} \exp[\tilde{J}_k g \theta_k]\right)\left(\prod_{k=1}^{3} \exp[\tilde{M}_k g u_k]\right)\hat{I}\,,
\tag{11}
$$

where the θ's and u's are the conventional parameters of $O^{\uparrow}_{+3,1}$, and the last exponentials are the conventional ones.

([28]) P. ROMAN: *Theory of Elementary Particles* (Amsterdam, 1964).

3) *The isotopic lifting of the Lie algebra of the Lorentz group*

$$\hat{O}^{\uparrow}_{+3,1}\begin{cases}[\tilde{J}_i \hat{,} \tilde{J}_j] = -\varepsilon_{ijk}\tilde{J}_k\,, \\ [\tilde{M}_i \hat{,} \tilde{M}_j] = c^2\varepsilon_{ijk}\tilde{J}_k\,, \\ [\tilde{J}_i \hat{,} \tilde{M}_j] = -\varepsilon_{ijk}\tilde{M}_k\,,\end{cases} \tag{12}$$

with related isotopic Casimir invariants

$$\begin{cases}\hat{C}_1 = \tilde{J}^{\hat{2}} - \dfrac{1}{c^2}\tilde{M}^{\hat{2}} = \sum\limits_{k=1}^{3}\left(\tilde{J}_k g \tilde{J}_k - \dfrac{1}{c^2}\tilde{M}_k g \tilde{M}_k\right) = -3\hat{I}\,, \\ \hat{C}_2 = \tilde{J} * \tilde{M} = \sum\limits_{k=1}^{3}\tilde{J}_k g \tilde{M}_k = 0\,.\end{cases} \tag{13}$$

It is evident that, by construction, *the groups $\hat{O}_{3,1}$ and $O_{3,1}$ coincide at the abstract, co-ordinate-free level, by therefore being locally isomorphic* (for the metric $g = \mathrm{diag}\,(b_1^2, b_2^2, b_3^3, -c^2)$—see below for other cases). In fact, the structure constants of $\hat{O}^{\uparrow}_{+3,1}$ coincide with those of $O^{\uparrow}_{+3,1}$ for the case considered here (co-ordinates $(\boldsymbol{r}, t)$ and metric $\mathrm{diag}\,(+1, +1, +1, -c_0^2)$) The extension of the local isomorphism to the full groups $\hat{O}_{3,1}$ and $O_{3,1}$ is then trivial.

Despite that, the explicit form of the transformations of $\hat{O}^{\uparrow}_{+3,1}$ and $O^{\uparrow}_{+3,1}$ are significantly different. For instance, a Lorentz-isotopic transformation in the plane (3, 4) is given by

$$x' = \begin{pmatrix} z' \\ t' \end{pmatrix} = \hat{A} * x = \begin{pmatrix} \cosh(uc) & -(c/b_3)\sinh(uc) \\ -(b_3/c)\sinh(uc) & \cosh(uc) \end{pmatrix}\begin{pmatrix} z \\ t \end{pmatrix}, \tag{14}$$
$$b_3 = b_3(t, z, \ldots)\,, \qquad c = c(t, z, \ldots)$$

and can be written

$$\begin{cases} z' = \hat{\gamma}(z - vt)\,, & \cosh(uc) = \hat{\gamma} = (1 - vb_3^2 v/c^2)^{-\frac{1}{2}}\,, \\ t' = \hat{\gamma}(-vb_3^2 z/c^2 + t)\,, & \sinh(uc) = \dfrac{vb_3}{c}\hat{\gamma}\,. \end{cases} \tag{15}$$

Numerous other examples of Lorentz-isotopic transformations can be explicitly computed with the methods presented here, *e.g.*, via expansions (11), where the only unknown is the assumed generalized metric. Explicit examples of isotopic rotations have been computed in ref. [27].

It is an instructive exercise for the interested reader to prove that the Lorentz-isotopic transformations verify all conditions *A*), *B*), *C*), by therefore constituting a *covering* of the conventional transformations. Their most salient difference is that the former are generally *nonlinear*, as evident from the dependence on the local co-ordinates of the metric g entering the expansions (11) and related explicit forms of type (15). As a matter of fact, this intrinsic nonlinearity, expressed in a formally linear theory (at the isotopic level), renders the covering transformations particularly intriguing for a number of problems, such as the characterization of hadrons and their interactions or decays.

It should be indicated at this point that the $\hat{O}_{3,1}$ symmetry holds for all possible metrics g under the restrictions that they are nonsingular, Hermitean (and, thus, diagonalizable), and verify sufficient continuity properties. Thus, the $\hat{O}_{3,1}$ symmetry holds for a class of invariants substantially broader than (2). However, in relaxing condition C), we lose the covering nature of the generalized transformations. Also, in relaxing the positive definite character of the element g_{kk} and $-g_{44}$, the local isomorphism between $\hat{O}_{3,1}$ and $O_{3,1}$ is generally lost. The occurrence will be investigated in detail elsewhere.

Note that $\hat{O}_{3,1}$ contains as a particular case the generalized Lorentz transformations introduced in ref. [13] for the invariant

$$x^{\hat{2}} = x^{\mu} g_{\mu\nu} x^{\nu} = x^{\mu}[(\nu^{\alpha}\eta_{\alpha\beta}x^{\beta}/x^{\alpha}\eta_{\alpha\beta}x^{\beta})^{r/2}\eta_{\mu\nu}]x\nu \,, \tag{16}$$

where $\eta_{\mu\nu}$ is the conventional Minkowskian metric, ν^{α} is a vector along the direction of anisotropy, and r is a scale parameter. The underlying transformations are then given by (11), *e.g.*, by (15) with $vb_3^2 z = vz$ and $\hat{\gamma} = [(1 - v/c)/(1 + v/c)]^{r/2}(1 - v^2/c^2)^{-\frac{1}{2}}$, and they exhibit an appreciable difference with the conventional ones only for speeds very close to that of light. However, the relationship with $\hat{O}_{3,1}$ demands specific investigations.

Similarly, isotopic transformations (11) can be enlarged to include the case $x'^{\hat{2}} = -x^{\hat{2}}$, by therefore permitting a generalization of the superluminous transformations of ref. [12] to the case of *variable* speeds of light. Other cases will be indicated elsewhere.

We now touch the problem of the *Lorentz-isotopic relativity*, that is, the relativity characterized by transformations (11). Our remarks will be as elementary as possible and restricted to the plane (3, 4) with $g_{33} = b^2$ and $g_{44} = -c^2$. The understanding is that at least two space dimensions are needed for an effective characterization of extended particles.

1) *Maximal speed of massive, ordinary particles.* The reader should keep in mind that the Lorentz-isotopic relativity is specifically conceived for the case when the speed of light, not only is different than that in vacuum, but possesses a dependence on local quantities of the type (2). Geometrically, this implies a deformation of the light-cone according to fig. 1. The following particular cases are then relevant.

Case 1A) $v_{\max} = c_0$, $c < c_0$. This is the case, for instance, of the Čerenkov light in water which travels at the speed $c = c_0/n < c_0$, while ordinary electrons can travel at $v_{\max} = c_0 > c$. The case is readily represented by the Lorentz-isotopic relativity with $b = 1/n$ and $c = c_0/n$.

Case 1B) $v_{\max} < c_0$, $c < c_0$. This case is predicted by the theory as a modification of case 1A) when particles cannot be considered as pointlike. Additional studies are, however, needed to reach quantitative predictions for each given medium and each given extended particle.

Case 1C) $v_{\max} < c_0$, $c > c_0$. This is the case submitted in ref. [7] for nuclear constituents, where the speed c is referred not necessarily to light itself, but to any causal signal propagating within hadronic matter.

Case 1D) $v_{\max} > c_0$, $c > c_0$. This is the case submitted in ref. [7] for the hadronic constituents, where the interpretation of c is the same as that of case 1C).

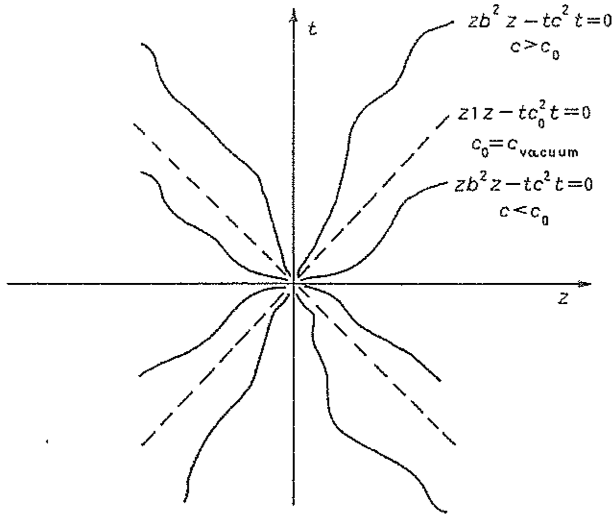

Fig. 1. – A schematic view of the deformation of the cone of light in vacuum predicted by the Lorentz-isotopic relativity submitted in this paper. The most dominant physical concept results to be the extended character of the particles considered (such as hadrons), with consequential possibility of the deformation of their spherical charge distribution into ellipsoids for sufficiently intense, short-range, external forces. In turn, these deformations imply deviations, first, from the rotational symmetry, and then from the Lorentz symmetry. Deformations of the light-cone are then unavoidable under the conditions considered. In the final analysis, they merely represent the dependence of the speed of light on the local characteristics of the material medium in which it propagates. Only two cases are considered in the figure. The first with $c < c_0$ depicts the Čerenkov light (which is fully represented by the Lorentz-isotopic relativity); while the second with $c > c_0$ depicts the hypothesis submitted in ref. (7), according to which causal signals can propagate within hadronic matter with speeds higher than that of light in vacuum owing to expected contact forces among extended particles which, besides not admitting a Hamiltonian, are instantaneous by conception. The deformations of the light-cone here submitted appear to be confirmed by a number of experimental data, although in a preliminary way, such as: the measures of rotational-asymmetry of ref. (22), the data of Lorentz asymmetry of ref. (19), and others.

2) *Isotopic composition of speeds.* The use of successive transformations (15) readily yields the composition of speed according to the isotopic rule

$$v_{\text{tot}} = (v_1 + v_2)/(1 + v_1 b^2 v_2/c^2)\,.$$

The maximal possible speed of particles is then compatible with (3). This is in full agreement with experimental evidence for the case of the Čerenkov light (case 1*A*)), and appears plausible for the remaining cases.

3) *Isotopic proper time, dilation, and contraction.* The remaining aspects of the Lorentz-isotopic relativity can be developed via suitable generalizations of the conventional setting. For instance, the proper time of the theory is the $\hat{O}_{3,1}$-scalar

$$\mathrm{d}\tau^2 = \mathrm{d}t^2 - \mathrm{d}r b^2 \mathrm{d}r/c^2 = \mathrm{d}t^2 \hat{\gamma}^{-2}\,.$$

The time dilation is then given by $\Delta t = \Delta t_0 \hat{\gamma}$, while the Lorentz contraction is given by $\Delta l = \Delta l_0 \hat{\gamma}^{-1}$. Thus, the Lorentz-isotopic relativity implies deviations from the standard predictions for the case of extended particles moving within material media, whether a macroscopic charge moving within a liquid, or a hadron moving whitin hadronic matter. In the final analysis, this is precisely the case of the deviations from the mean life of unstable hadrons considered earlier [9,10].

As concluding remarks, we can say that, on classical grounds, the Lorentz-isotopic relativity represents in full the physical conditions of the Čerenkov light (for which no further test is needed) and extrapolates them to the case of extended particles (for which additional theoretical studies are needed to achieve quantitative predictions suitable for experiments).

In regard to particle physics, the Lorentz-isotopic relativity has been conceived to represent extended particles, that is particles that can be deformed under sufficiently intense external fields, resulting first in the rotational asymmetry $xb_1^2x + yb_2^2y + zb_3^2z = 1$, and then in Lorentz-asymmetric invariants of type (2). Deviations from the standard mean life and other predictions of the special relativity are then consequential.

The available direct experimental infomation reviewed earlier is encouragingly in favour of the need for a generalization of the special relativity, hopefully of the unifying type submitted here, although the information is far from a final form.

The needed fundamental tests are evidently those on the *continuous* part of the conventional Lorentz symmetry (besides those on the discrete part calling for a separate analysis), *e.g.*:

i) Finalization of the apparent Lorentz-boost asymmetry in the mean life of unstable hadrons (and *not* leptons) in flight, with particular reference to pions and kaons, according to the experiments reviewed, *e.g.*, in ref. [10]. In order to be effective for the selection of the suitable generalization of the law $\Delta t = \Delta t_0(1 - v^2/c^2)^{-\frac{1}{2}}$ and of the underlying metric, the experiments should finalize possible deviations per each individual particle (because expected to be different for different particles), and at different energies (because important for the selection of the generalized metric whether, *e.g.*, with a Minkowskian or a Finslerian topology);

ii) Finalization of the apparent 1% deformation/rotational-asymmetry of low-energy neutrons within the intense fields in the vicinity of nuclei, as reported in ref. [22] via interferometric measures on the periodicity of the neutron wave functions for two spin flips. The tests should then be repeated according to a number of suitable variations, *e.g.*, for $2n$ spin flips, $n = 1, 2, 3, \ldots$ (apparently, current technology in neutron interferometry can permit up to 50 spin flips); or via linear increases of the width of the material penetrated by the neutron beam (whose nuclear fields are responsible for the apparent deformation/rotational asymmetry); etc.

iii) Finalization of possible, sufficiently small deviations from Pauli's exclusion principle in open nuclear reactions, as theoretically predicted in ref. [4], elaborated by a number of authors in ref. [5] and not excluded in the tests of neutron-tritium scattering of ref. [22] and quoted recent experiments.

It is evident that the ultimate roots of tests i), ii), iii) (and several others that are conceivable along the same lines) are given by the possible deformation of the charge distribution of hadrons (and, possibly, of their constituents) under sufficiently intense, short-range fields. In fact, such deformation (tests ii)) implies a manifest, generally small, rotational-asymmetry. In turn, the transition from the space part of invariant (1) to that of (2) implies the necessary transition to the entire invariant (2). A deviation from the Lorentz boosts is then consequential (tests i)). On the other side, a rotational asymmetry may well imply a corresponding, sufficiently small deviation from the exact fermionic character of nucleons. A corresponding deviation from Pauli's exclusion principle is then also consequential (tests iii)). (Apparently, the rotational asymmetry seems also to imply deviations from the discrete symmetries because, while the generators $\hat{P}$, $\hat{T}$, etc. isotopically commute with $\hat{O}^{\dagger}_{3,1}$ in the frame in which invariant (2) is diagonal, such commutativity demands specific studies in arbitrary frames for which the metric is not diagonal.)

But, above all, the most encouraging aspect is that all tests i), ii), iii) and others, are well within current technical capabilities (besides being of quite moderate cost when compared to high-energy experiments). The physics community has, therefore, reached in full the capability to resolve experimentally the apparent approximate character of the conventional Lorentz transformations in particle physics.

On theoretical grounds the basic issue is so simple to appear trivial. The special relativity was conceived for pointlike particles moving in empty space. When extended particles moving in material media are considered, deviations from the special relativity are expected to be consequential. The selection of the appropriate generalization will of course be the result of a long scientific process of trial and error. But the insufficiency for extended particles of the relativity of pointlike particles should be out of question.

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20 Agosto 1983
Lettere al Nuovo Cimento
Serie 2, Vol. 37, pag. 545-555