

LETTER TO THE EDITOR

On a possible non-Lorentzian energy-dependence of the  $K_S^0$  lifetime

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**Abstract.** A fit to the experimental data reported by Aronson *et al* on the  $K_S^0$  lifetime shows a good agreement with a generalized expression for time dilation, obtained by a Lie isotopic lifting of the Poincaré symmetry.

In 1982 Aronson *et al* [1] discussed the behaviour of the  $K^0-\bar{K}^0$  system at the energies available at Fermilab at the time (30–110 GeV). They found that the basic  $K^0-\bar{K}^0$  parameters (i.e. the  $K_L^0-K_S^0$  mass difference, the  $K_S^0$  lifetime, and the CP-violation parameter  $|\eta_{+-}|$ ) exhibit an anomalous dependence on energy of the type

$$x = x_0(1 + b_x^{(N)}\gamma^N) \quad (1)$$

where  $x$  is any of the above parameters,  $\gamma = E_k/m_0 = (1 - \beta^2)^{-1/2}$  is the usual relativistic factor, the  $b_x$  are suitable slope parameters and  $N$  is an integer equal to 1 or 2.

In particular, as far as the K lifetime is concerned, equation (1) is clearly at variance with the standard Einstein relation

$$\tau = \tau_0\gamma \quad (2)$$

and is compatible instead with a Blokhintsev-Redei [2] behaviour of  $\tau$ , obtained via the introduction of a fundamental length.

A possible role of non-local forces in an anomalous behaviour of hadron lifetimes has been stressed by Kim [3], in agreement with the original proposal by one of us [4]. More recently, Nielsen and Picek [5], studying mechanisms of breakdown of Lorentz invariance within unified gauge theories, showed that spontaneous symmetry breaking in the Higgs sector implies deformations of the spacetime metric in the interior of mesons. This leads to a non-Einsteinian behaviour of meson meanlife with speed that depends on the Lorentz-non-invariance parameter (different for pions and kaons).

A geometrization of the above theoretical results has been achieved by one of us [6, 7] in the context of the so-called Lie-isotopic generalization of Lie's theory [8–10]. In particular, the Lie-isotopic lifting of the Poincaré symmetry is obtained essentially by the following generalization of the conventional Minkowski metric  $g_{\mu\nu} = \text{diag}(1, 1, 1, -1)$  ( $\mu, \nu = 1, 2, 3, 4$ ):

$$g_{\mu\nu} \rightarrow \eta_{\mu\nu} = \text{diag}(b_1^2, b_2^2, b_3^2, -b_4^2) \quad (3)$$

where the  $b_\mu$  have an arbitrary, generally non-local, dependence on all local variables (like coordinates, velocities, density, temperature, etc). The space characterized by metric (3) is called an isotopic Minkowski space. We refer the reader to the quoted literature for further details. Let us only stress that, in the Lie-isotopic approach, the Lorentz (or Poincaré) symmetry is not broken, but is realized in a generalized form, while Einstein's special relativity is no longer valid (see [6-10]). Then, for instance, one gets the isotopic law of time dilation

$$\tau = \tau_0 \hat{\gamma} \quad \hat{\gamma} = (1 - \beta^2)^{-1/2} \quad \beta = \frac{b_3 v}{b_4 c_0} = \frac{b_3}{b_4} \beta \quad (4)$$

(where  $c_0$  is the usual light speed in vacuum), that is clearly a covering of the Einstein relation (2).

As recently shown by Aringazin [11], a simple dependence of only  $b_3$  on speed is enough to get a Blokhintsev-Redei behaviour of the  $K^0$ - $\bar{K}^0$  parameters, in agreement with equation (1).

The aim of the present letter is a re-analysis of the data reported in [1] on the  $K_S^0$  lifetime, in the light of the above theoretical results, with special attention to the isotopic law (4). The difference with respect to the investigations of Aronson *et al* lies in the fact that we are not trying to fit the experimental data by *any* law, but to testing the validity of a *specific* law, derived from theoretical considerations.

Our main results are graphically summarized by figures 1-3. Figure 1 pictures a linear fit to the data of [1] by the standard formula (2). The fit parameter is  $a = \tau_0/m_0$ ; the reduced chi-square is  $\chi_n^2 = 0.9$ . The value found for  $\tau_0$  is  $\tau_0 = (0.9375 \pm 0.0021) \times 10^{-10}$  s, to be compared with the *Particle Data Book* [12] value  $\tau_0 = (0.8922 \pm 0.0020) \times 10^{-10}$  s. Moreover, the confidence level (CL) is 0.39, giving a probability of 61% that  $\tau_0$  is greater than the obtained value. It is therefore easily seen that, although a linear formula could fit the data, a linear fit is not able to reproduce the correct value of the  $K_S^0$  lifetime at rest as provided by low-energy (less

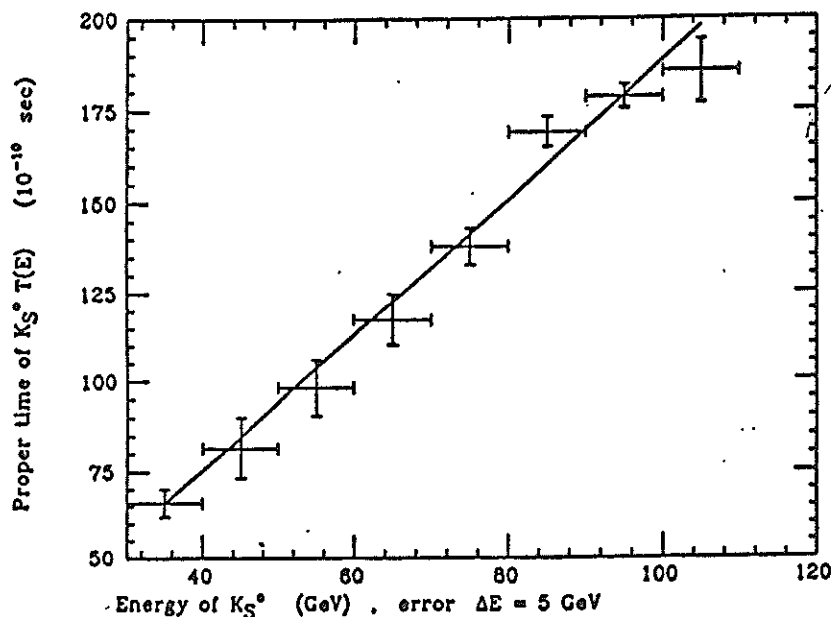


Figure 1. Linear fit of experimental data on  $K_S^0$  lifetime using Einstein's relation (2): see text for details.

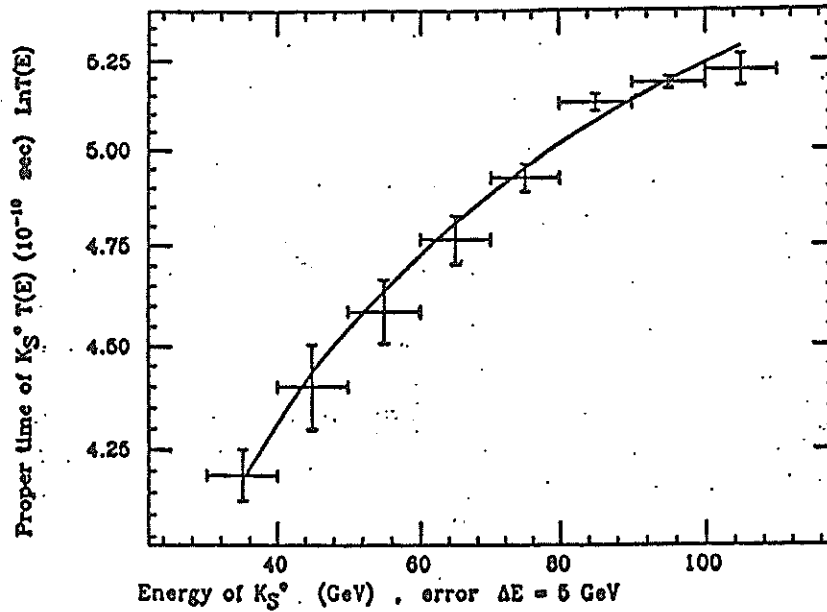


Figure 2. Logarithmic fit of experimental data on  $K_S^0$  lifetime using the power law (5): see text for details.

than 3–5 GeV) measurements. This first fit shows clearly that the Lorentzian law of time dilation is unable to account for the available data in a satisfactory way, and that a *non-linear* dependence of  $\tau$  on  $E$  is needed.

In order to further test the non-linearity of the  $\tau$ -dependence on energy, we performed a second (logarithmic) fit by the formula

$$\tau = (\tau_0/m_0)E^n \quad (5)$$

by taking  $a = \tau_0/m_0$  and  $n$  as parameters. The results of this second fit are shown in

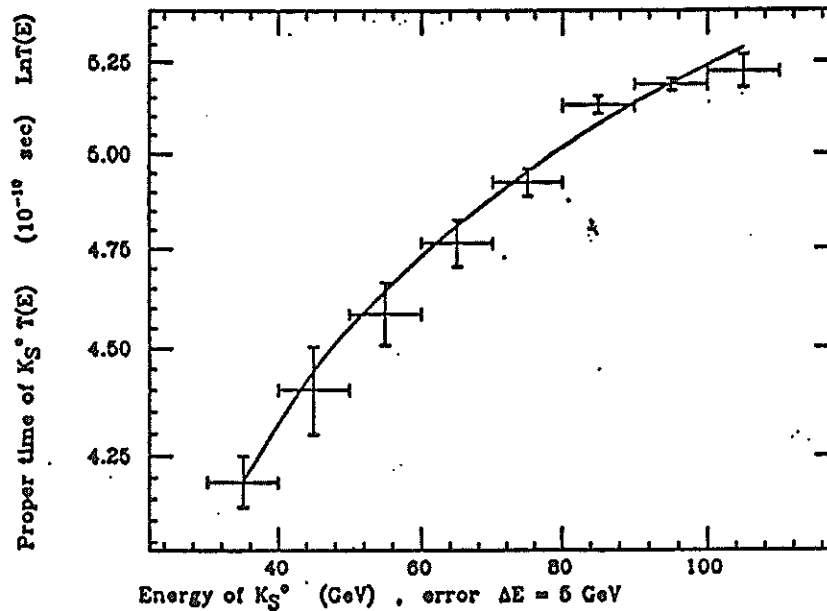


Figure 3. Logarithmic fit of experimental data on  $K_S^0$  lifetime using the isotopic formula (4): see text for details.

figure 2. We have now  $\chi_n^2 = 0.85$ , and the value obtained for the exponent of  $E$  is  $n = 1.013 \pm 0.003$ . Moreover, the upper limit on  $n$  is  $1.023 \pm 0.003$  (95% c.l.). The deviation from linearity is 1.3% at 100 GeV, and therefore an effect of 13% at 1 TeV is expected.

The last (logarithmic) fit (figure 3) was finally aimed at directly testing the isotopic law (4). We considered the ratio  $b_3^2/b_4^2$  as a single parameter  $a$ . We found for  $a$  the value  $0.898 \pm 0.021$ , with  $\chi_n^2 = 0.86$ . The deviation of  $a$  from unity is 10.2% at 100 GeV.

Let us also state that another fit, carried out by taking  $b_3^2$  and  $b_4^2$  as distinct parameters, allowed us to get the values  $b_3^2 = 0.9023 \pm 0.0004$  and  $b_4^2 = 1.003 \pm 0.0021$ , whence the ratio  $(b_3/b_4)^2 = 0.8996 \pm 0.0023$ , in good agreement with the value obtained in the third fit. Moreover—as expected on theoretical grounds—there is evidence that both these parameters are not constants, but depend on the energy. Further details will be given in a forthcoming paper.

In conclusion, our re-analysis of the experimental data of [1] on the  $K_S^0$  lifetime supports the validity (in the energy range considered) of the generalized time dilation provided by the Lie-isotopic theory. However, some points are to be stressed. First of all, our results are far from conclusive on a number of counts, the most important of which is obviously the lack of available data on hadron lifetimes. What is needed is a systematic investigation of the energy behaviour of lifetimes not only for kaons, but for all the unstable hadrons. We hope that our preliminary study will be a valid suggestion for experimentalists.

Let us recall, in this connection, that another set of data on the  $K_S^0$  lifetime exists in the energy range 100–350 GeV, due to Grossmann *et al* [13]. They are in disagreement with the data of [1], showing no evidence for an energy dependence of  $\tau$ . However, we think that the results of [13] are not at all conclusive, and that a further measurement is needed for the *whole* energy range covered by both experiments.

In any case, we think that our analysis is reliable enough at the intermediate energies of the experiment by Aronson *et al*. Indeed, on a theoretical basis [3, 4, 6–10], we expect a standard (Lorentzian) behaviour of lifetime at low energies (less than 10 GeV), whereas, at very high energies, the parameter  $a (= b_3^2/b_4^2)$  exhibits a (presently not well known) dependence on energy. Only at intermediate energies we can safely approximate  $a$  as a constant (as we have just done in our present work). A global analysis of the whole data on  $K_S^0$  lifetime (mainly aimed to just find the explicit functional dependence of  $a$  on energy) will be carried out in a forthcoming paper.

Let us also stress that this letter only aims to provide a different interpretation of the non-linearity of the  $K_S^0$ -lifetime behaviour in the range 35–110 GeV. Let us explicitly stress that our interpretation is based on a theoretical framework completely independent from the considerations which led some years ago to the renewed hypothesis of a fifth force [14].

Finally, although there is, in our opinion, a good chance that the lifetime law for hadrons has indeed the form (4) (with, in general, *different* parameters  $b_3$ ,  $b_4$  for *different* particles: cf the result of Nielsen and Picek [5] and [6–10]), we *do not* expect, *a priori*, that a similar formula must also hold for mass (at least on the same energy scale). In other words, it is easily possible (as apparently confirmed by the existing data on hadron masses) that the  $\gamma$ -factor for mass is still the usual, Einsteinian one. This seemingly contradictory statement can be motivated as

follows. As is well known, the appearance of the same factor  $\gamma$  in the dilation laws of both time and mass in special relativity is strictly related to the point-like nature of particles (a *necessary* condition to ensure the validity of special relativity) and is simply a consequence of the Lorentz transformations (i.e. of the spacetime properties), without any connection at all with a possible particle structure. On the other hand, the Lie-isotopic theory is just aimed to account—at least in first approximation—for the internal structure of hadrons (namely, for the non-local forces among their constituents) [4, 9]. From this respect, lifetime is expected to be more sensible than mass to the composite nature of particles. This point can be easily understood by a standard thermodynamical reasoning (i.e. our ignorance of the internal coordinates of the constituents), and is also implicit in the very formulation of Lie-isotopic theories, according to which the usual dynamical behaviour is recovered for centre-of-mass quantities in the standard Minkowski space [10]†.

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† Indeed, the  $\gamma$ -factor connected with lifetime takes account of the internal energy of the physical system considered, whereas that entering the mass formula is related to the total energy of the system.

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Lie-isotopic energy-dependence of the  $K_S^0$  lifetime

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**Abstract.** Fits to the experimental data on the  $K_S^0$  lifetime in the energy range 30-350 GeV show a good agreement with a generalized expression for time dilation, obtained by a Lie-isotopic lifting of the Poincaré symmetry. We also get preliminary results on the energy dependence of the parameters that, in the Lie-isotopic theory, determine the deformed Minkowski metric inside hadrons.

In this paper we shall study the currently available phenomenological data on the behaviour of the meanlife of unstable hadrons with energy. According to a number of theoretical studies [1-6], this behaviour is expected to be at variance with the predictions of Einstein's special relativity, the celebrated law of time dilation

$$\tau = \tau_0 \gamma \quad \gamma = (1 - \beta^2)^{-1/2} \quad \beta^2 = v^2/c_0^2 \quad (1)$$

( $c_0$  being the light speed in vacuum).

The initiation of quantitative studies in the subject is usually associated with Blokhintsev [1] and Redei [2] who, by an examination of the phenomenological data available at that time, suggested a modification of law (1) of the type

$$\tau = \tau_0 \gamma (1 + 10^{25} \gamma^2 a_0^2) \quad (2)$$

where  $a_0$  is a fundamental length.

Later Kim [3]—resuming the historical legacy opened up by the founding fathers of strong interactions, according to which the ultimate structure of hadrons is expected to be non-local—predicted a violation of law (1) of 2.6% at 150 GeV and of 14.3% at 400 GeV.

Nielsen and Picek [4] studied the problem within the context of unified gauge theories and showed that the spontaneous symmetry breaking in the Higgs sector implies deformations of the Minkowski metric of the type

$$g = \text{diag} \left[ (1 - \frac{1}{3}\alpha), (1 - \frac{1}{3}\alpha), (1 - \frac{1}{3}\alpha), -(1 + \alpha) \right] \quad (3)$$

where the 'Lorentz asymmetry parameter'  $\alpha$  has different values (and sign) for pions and kaons.

This deformation effect results in a non-Einsteinian behaviour of the meanlife with speed of the type [4]

$$\tau = \tau_0 \gamma (1 + \frac{4}{3} \alpha \gamma^2)^{-1}. \quad (4)$$

A geometrization of the above results was achieved by one of us [5,6] via a generalization of the conventional Minkowski space  $\mathcal{M}(x, g, R)$  ( $g$  being the usual metric tensor, and  $R$  the real field) of the type

$$\begin{aligned} \mathcal{M}(x, \eta, \hat{R}) : x^\mu \eta_{\mu\nu} x^\nu &= x_1^2 b_1^2 + x_2^2 b_2^2 + x_3^2 b_3^2 - x_4^2 b_4^2 \\ \eta &= Tg \quad T = \text{diag}(b_1^2, b_2^2, b_3^2, b_4^2) \\ \hat{R} &= \{\hat{c} | \hat{c} = c\hat{I}; c \in R, \hat{I} = T^{-1}\} \end{aligned} \quad (5)$$

called a *Minkowski-isotopic space*. Here, the positive-definite isotopic element  $T$  has an arbitrary, generally non-local (i.e. integro-differential) dependence on all local variables and quantities, such as coordinates, velocities, accelerations, density, temperature, etc:

$$b_\mu = b_\mu(x, \dot{x}, \ddot{x}, \dots, \rho, \dots) \quad (6)$$

and  $\hat{R}$  is an isofield, i.e. an ordinary field with conventional sum and product given by  $\hat{c}_1 \times \hat{c}_2 \equiv c_1 c_2 \hat{I}$ .

The lifting  $\mathcal{M} \rightarrow \hat{\mathcal{M}}$  requires, for consistency, the generalization of the conventional Poincaré group of transformations (and its Lie structure) into a *Poincaré isotopic group* (with a Lie-isotopic structure). The resulting theory is then a Lie-isotopic theory (as originally proposed in [7]). The physical motivation of Lie-isotopic theories lies in their capability of representing non-local, non-Hamiltonian forces. In particular, the metric deformation (5) is representative—at least in first approximation—of the contact, non-local interactions that are expected in the interior of hadrons from the total mutual penetration of the wavepackets of the constituents. For further details, we refer the reader to the review [8].

The Lie-isotopic lifting of the Poincaré group leads to the following *isotopic law of time-dilation* (for a boost along  $x_3$ )

$$\tau = \tau_0 \hat{\gamma} \quad \hat{\gamma} = (1 - \hat{\beta}^2)^{-1/2} \quad \hat{\beta}^2 = b_3^2 / b_4^2 \beta^2. \quad (7)$$

As shown by Aringazin [9], the isotopic formula (7) admits as particular cases (depending on the assumed expansion) all available non-Einsteinian laws (like (2) and (4)).

Some features of (7) must be stressed: Firstly, we expect significant deviations from (1) only at high energies (cf the estimates by Kim [3] reported above). In other words, the energy dependence of the isotopic time dilation at low energies is in good approximation Lorentzian (in agreement with the low-energy measurements in the range 3–5 GeV [11]). Secondly, the parameters  $b_\mu$ , on account of their physical meanings, are a priori *different* for different hadrons [5, 6] (cf the analysis by Nielsen and Picek [4]). Finally, still on physical grounds, one expects that there is a *different* dependence on energy for the spatial parameters  $b_i$  ( $i = 1, 2, 3$ ) and for the temporal one,  $b_4$ . This can easily be seen by considering that the parameters  $b_\mu$  take

account, on average, of the non-local effects inside hadrons [5-7, 10]. These effects are obviously more effective as far as the spatial part of the metric is concerned, while for the time part they only give rise, essentially, to a change of the light speed [5-7, 10]. In other words, we expect that the time parameter  $b_4$  is a very slow function of the energy (at limit constant) in comparison with the  $b_i$ 's (as it can easily be inferred from the discussion of [5-7, 10]).

Another basic point to be stressed is the *different* predictive power of the time-dilation laws (1) and (7). Indeed, the Einsteinian law (1), being linear in energy, is expected to provide the value of the hadron meanlife in the rest system,  $\tau_0$ . On the contrary, the isotopic law (7), after assuming the  $\tau_0$  value as given by the low-energy measurements (in the range 3-5 GeV, i.e.  $\tau_0 = (0.8922 \pm 0.0020) \times 10^{-10}$  s) [11], is expected to yield information on the functional dependence of  $b_3$  and  $b_4$  on energy.

In this paper, we shall consider the only available data on the meanlife of an unstable hadron at high energies, i.e. those on  $K_S^0$  in the energy range 30-375 GeV. They have been obtained by two different experiments; the first by Aronson *et al* [12] in the range 30-100 GeV and the second by Grossman *et al* [13] in the range 100-375 GeV. Indeed the two experiments do not disprove each other, because they were conducted on *different* (and *non-overlapping*) ranges of energy. As we shall show in the following, the disagreement between the results of these two different measurements is only apparent: when considered as a *single* set of data (after a suitable weighting by proper statistical weights) they provide us with a self-consistent and coherent conclusion within the framework of the Lie-isotopic predictions. For brevity we shall refer to the two sets of data as 'A' and 'G', respectively. Let us stress that we are not interested at all in problems connected with the possible existence of a fifth fundamental force [14]. Our only aim is to test the predictive power of the time-dilation laws (1) and (7).

In the following, we shall consider first the data, A, separately and then the two sets of data, A + G, together, in order to investigate the behaviour of the parameters  $b_3$  and  $b_4$  with energy. In the latter case, of course, we have properly accounted for a suitable weighting of the two sets of data by considering the proper statistical weights for the two experiments (analogously to what was already done in [13]; cf figure 3). In particular, we have taken into account: (i) the different statistical uncertainties; (ii) the acceptances of the two apparatuses; and (iii) the different binning.

A preliminary analysis of the data A has already been given by us [15]. In particular, we have already checked that, as far as those data are concerned, a linear law of the type (1) fails to fit the experimental points, and must be replaced by a power law. This had already been realized by Aronson *et al* [12], who indeed proposed a generalized time-dilation law we shall not consider here, because it is a special case of the isotopic formula (7) [9].

However, for the sake of completeness and for comparison, we report here the results of the linear fit to data A (figure 1(a)). The fitted law is  $\tau = \tau_0 E/m_0$ , and  $\tau_0$  is taken as a parameter. The reduced  $\chi$ -square is  $\chi_n^2 = 0.9$ . The value found for  $\tau_0$  is  $\tau_0 = (0.9375 \pm 0.0021) \times 10^{-10}$  s. The confidence level (CL) is 0.39, giving a probability of 61% that  $\tau_0$  is greater than the obtained value.

The set of data G was analysed separately in [13]. The result of the fit by Grossman *et al*, only confined to their data set, gives of course a good agreement with the low-energy value of  $\tau$ . Indeed, we are just going to prove that it is the *whole* set of data A+G (when considered together) which seems to support the validity of the Lie-isotopic time-dilation law (7).



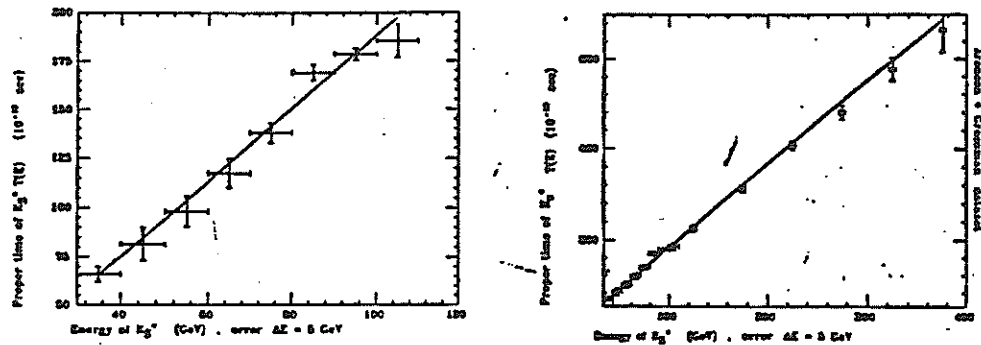


Figure 1. Linear fit of experimental data on  $K_S^0$  lifetime using Einstein's relation (1) for: (a) the set of data A; (b) the set of data A+G. See the text.

The result of the linear fit to the two sets of data A+G is shown in figure 1(b). The value obtained for the meanlife of  $K_S^0$  is  $\tau_0 = (0.91444 \pm 0.00193) \times 10^{-10}$  s. The reduced  $\chi$ -square is 1.23. The CL is 19%, with therefore a probability of 81% that the value of  $\tau_0$  is greater than that given by the fit. Let us further check the compatibility of the fitted value with that measured at low energies (see [11]) by considering the absolute values of the related errors,  $\sigma_{\tau_0}^{(f)} = 0.00193$  and  $\sigma_{\tau_0} = 0.0020$ . We have

$$\Delta\tau_0 = \tau_0^{(f)} - \tau_0 = 0.022 = 11.1\sigma_{\tau_0} = 11.5\sigma_{\tau_0}^{(f)}. \quad (8)$$

We can therefore conclude that the Einsteinian law (1) fails to predict the right experimental value of the meanlife of  $K_S^0$  at rest.

Let us now consider the Lie-isotopic law (7). In this case, we assume the value of  $\tau_0$  is known at low energies, and want to find the functions  $b_3(E)$  and  $b_4(E)$ . We consider three parameters, i.e.  $b_3$ ,  $b_4$ , and their difference,  $\Delta = b_4^2 - b_3^2$ . This last parameter is introduced because, as stressed above, we expect a different dependence of the two parameters on energy. The results of the fits are shown in figures 2(a) and (b) (data A and A+G, respectively). Let us summarize the fit results:

(i) data A (figure 2(a)):

$$\begin{aligned} \chi^2/n &= 0.86 & \Delta(b_3^2, b_4^2) &= (1.4 \pm 0.003) \times 10^{-7} \\ b_3^2 &= 0.9023 \pm 0.0004 & b_4^2 &= 1.0003 \pm 0.002 \end{aligned} \quad (9)$$

(ii) data A+G (figure 2(b)):

$$\begin{aligned} \chi^2/n &= 0.71 & \Delta(b_3^2, b_4^2) &= (3.926 \pm 0.002) \times 10^{-7} \\ b_3^2 &= 0.909080 \pm 0.00004 & b_4^2 &= 1.002 \pm 0.007. \end{aligned} \quad (10)$$

Denoting by  $\sigma_{A, B, b_3, 4}$  the absolute values of the errors on  $b_3^2$ ,  $b_4^2$  for the two sets of data (the lower index B referring to the set of data A+G), we have

$$\Delta b_3^2 = 0.007 = 17\sigma_{A, b_3} = 170\sigma_{B, b_3} \quad (11)$$

$$\Delta b_4^2 = 0.001 = 0.50\sigma_{A, b_4} = 0.14\sigma_{B, b_4}. \quad (12)$$

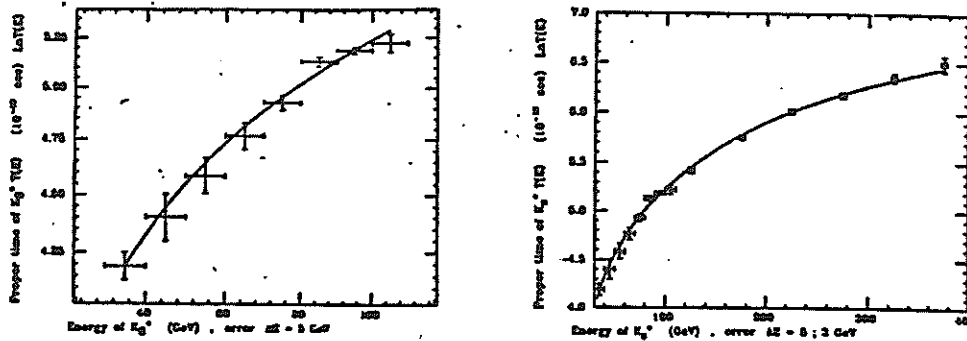


Figure 2. Logarithmic fit of experimental data on  $K_S^0$  lifetime using the isotopic formula (7) for: (a) the set of data A; (b) the set of data A+G. See the text.

Therefore, our analysis supports a prediction of the Lie-isotopic theory, namely,  $b_3$  changes significantly with energy, whereas  $b_4$  is almost constant in the energy range considered (30–375 GeV).

A detailed investigation of the explicit functional form of  $b_3(E)$  and  $b_4(E)$  will be given, by a completely new method, in a forthcoming paper.

In conclusion, we want to stress the main results of the present study: the apparent failure of the standard time-dilation law in giving the right value of  $\tau_0$  for  $K_S^0$  (when the whole energy range 30–375 GeV is considered), and the first experimental check of the energy dependence of the parameters  $b_3$ ,  $b_4$  predicted by the Lie-isotopic theory.

Of course, our analysis is far from conclusive for at least two reasons. Firstly, the experimental data on the  $K_S^0$  lifetime in the energy range considered have been obtained by two different measurements carried out in two disjointed energy intervals. In this respect, a unique measurement in the same energy range would be in order. Secondly, as stressed above, the parameters entering into the Lie-isotopic law are expected to be different for different hadrons. Therefore, it would be worth performing further high-energy measurements of the meanlife of other unstable hadrons (charged, if possible) to further check the validity of the law (7) and find the parameters  $b_3$ ,  $b_4$  for each hadron separately. This last result would enable us to get useful information on the inner structure of hadrons.

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