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Recent Theoretical and Experimental Evidence on the Apparent Synthesis of the Neutron from Protons and Electrons

Ruggero Maria Santilli

Istituto per la Ricerca di Base, Castello Principe Pignatelli, I–86075 Monteroduni (IS), Molise, Italy;
E–mail ibrrms @ pinet. aip. org

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Abstract: Recent experiments have provided direct evidence on the apparent existence of the synthesis of the neutron from a proton and an electron according to the reaction $p+e \rightarrow n+\nu$, much along Rutherford's original conception of the neutron. These findings have received indirect, yet significant experimental confirmations in the Bose–Einstein correlation, the behaviour of the meanlife of unstable hadrons with speed, superconductivity, and other fields, which are such to warrant additional studies. In this paper we present a quantitative theoretical study on the apparent tendency of all massive particles to form a bound state at distances equal or smaller than their wavepackets which is enhanced at low energy. The study is centrally dependent on the isominkowskian geometrization of the mutual wave–overlapping and related isopoincaré symmetry proposed by this author back in 1983. While prohibited in the conventional Minkowski space under the conventional Poincare' symmetry, the synthesis herein considered become possible under our covering isotopic formulations thanks to new renormalizations of intrinsic characteristics of particles originating from nonlocal and nonhamitonian effects in mutual wave–overlappings. If experimentally confirmed, the studies here submitted would therefore imply that, not only the neutron can be produced via the proton–electron synthesis, but all unstable hadrons can be artificially produced via the synthesis of ligther hadrons usually selected in the decay with the lowest mode, which would then acquire the meaning of tunnel effect of the constituents. Comparison and compatibility with conventional quark models are also considered for detailed study elsewhere.

Keywords: Electrosynthesis of neutron, Isodirac equation, Isopoincaré symmetry.

1. Statement of the Problem

In recent papers, Kadeisvili [1] reviews the nonlinear–nonlocal–noncanonical isotopies of Lie's theory, and Lopez [2] reviews the axiom–preserving isotopies of quantum mechanics (QM), called hadronic mechanics (HM) (originally submitted in [3], see refs. [4] for details), and their axiomatization of Q–operator–deformations, here called Q–isodeformations.

In this note we shall apply isosymmetries / Q–isodeformations to a speculative, yet intriguing novel problem, the synthesis of elementary particles, defined as the apparent tendency of massive particles to form a bound state at short distances (<1 fm) in singlet state which is enhanced at low temperature (or very low energies).

According to these novel views, we expect that unstable elementary particles can be artificially produced via the synthesis of lighter massive particles suitably selected in their spontaneous decays. Such a synthesis occurs for each individual particle in our space–time only, without any unitary interior space and, thus, without the possibility of defining a quark. Nevertheless, compatibility with quark theories is apparently achieved by considering families of particles via the addition of unitary internal spaces. This aspect is studied elsewhere [5] via the isotopies $S\tilde{U}_Q(3) \approx SU(3)$ characterizing isoquarks, which have all conventional quantum numbers, yet more general nonlinear–nonlocal–nonhamiltonian interactions.

A central problem in the achievement of the above syntheses is the need for new renormalizations of the intrinsic characteristics of particles as characterized by QM: rest energy, spin, charge, magnetic moments, etc. In fact if particles preserve their conventionally renormalized intrinsic characteristics, no synthesis is possible (see below).

The physical origin of these novel renormalizations is seen in the nonlinear–nonlocal–nonhamiltonian interactions expected in the total mutual penetration of wavepackets–wavelengths–charge distributions of particles one inside the other, and represented with the isotopic operator $Q = Q(s,x,\dot{x},\ddot{x},\psi,\partial\psi,\partial\partial\psi,\cdots)$ [1–5]. By recalling that all interactions must produce renormalizations, and all conventional renormalizations are Lagrangian–Hamiltonian, the novelty of the renormalizations follows from their nonlagrangian–nonhamiltonian character.

Among all possible deformations, we select the isotopies $\hat{O}_Q(3.1)$ and $\hat{P}_Q(3.1)$ of the Lorentz $O(3.1)$ and Poincaré $P(3.1)$ symmetries first introduced by the author back in 1983 [6]①, which are constructed with respect to the isounit $\hat{I} = Q^{-1}$, and imply a generalization of the very notion of particle into the covering notion of isoparticle possessing precisely the generalized characteristic needed for the synthesis of the neutron.

In this note we shall denote all ordinary particles characterized by $P(3.1)$ with the familiar symbols $e^{\pm}$, $\mu^{\pm}$, $p^{\pm}$, etc. and use the symbols $\hat{e}^{\pm}$, $\hat{\mu}^{\pm}$, $\hat{p}^{\pm}$, etc. for their isotopic conditions characterized by $\hat{P}_Q(3.1)$, which represent their immersion within the hyperdense media in the interior of hadrons, called hadronic media [3].

A first understanding is that all Q–operators are selected in such a way to recover the trival unit $I = \mathrm{diag.}(1, 1, 1, 1)$ for distances > 1 fm, when motion returns to be in vacuum, in which case HM recovers QM identically and in its totality. Also, in the transition from interior motion within a hadronic medium to exterior motion in vacuum, isoparticles reacquire their conventional $P(3.1)$–invariance, plus possible secondary emissions (e.g., of γ and ν) due to the original excitations.

Quantitative representations of the following syntheses are now available [4], here expressed in self–explanatory notations (see later for secondary emissions)

$$\text{Electron pairs} = (e^-,\ e^-)_{\mathrm{QM}} \Rightarrow \text{Cooper Pair} = (\hat{e}^-,\ \hat{e}^-)_{\mathrm{HM}}, \tag{1.1a}$$

$$\text{Positronium} = (e^-,\ e^+)_{\mathrm{QM}} \Rightarrow \pi^0 = (\hat{e}^-,\ \hat{e}^+)_{\mathrm{HM}}, \tag{1.1b}$$

$$\text{Muonium} = (\mu^-,\ \mu^+)_{\mathrm{QM}} \Rightarrow \eta = (\hat{\mu}^-,\ \hat{\mu}^+)_{\mathrm{HM}}, \tag{1.1c}$$

$$\text{Pionium} = (\pi^-,\ \pi^+)_{\mathrm{QM}} \Rightarrow K^0_s = (\hat{\pi}^-,\ \hat{\pi}^+)_{\mathrm{HM}}, \tag{1.1d}$$

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①In all preceding works the isotopic element is indicated with the symbol T rather than Q.

$$\text{Hydrogen atom} = (p^+,\ e^-)_{\text{QM}} \Rightarrow n = (\hat{p}^+,\ \hat{e}^-)_{\text{HM}} \tag{1.1e}$$

Numerous other syntheses are then consequential, such as $\Lambda = (\hat{n},\ \hat{\pi}^0)_{\text{HM}}$, $\sum^{\pm} = (\hat{n},\ \hat{\pi}^{\pm})_{\text{HM}}$, etc. A primary objective of the isosymmetry $\hat{P}_Q$ (3.1) is therefore the reduction of all massive elementary particles to electrons and protons, for which purpose the construction of hadronic mechanics was suggested [3]. In this approach isoquarks emerge as being suitable HM bound states of electrons and / or protons resulting in fractional charges which are notoriously extraneous to P(3.1), but rather natural for $\hat{P}_Q$ (3.1)–invariance under generalized interactions [5].

The first and most fundamental synthesis (1.1*a*) is fully established experimentally, and it is given by the Cooper Pair (CP) in superconductivity (see, e.g., [7] and quoted references). Its interpretation via conventional quantum mechanics is manifestly problematic owing to the highly repulsive Coulomb interactions at short distances under P(3.1)–invariance. However, $\hat{P}_Q$ (3.1)–isoinvariance with a particular selection of the isounit $\hat{I}$ [8] does permit a consistent interpretation of the CP.

Once the experimental evidence of the $(e^-,\ e^+)$ synthesis is admitted, one has the inevitability of the $(e^-,\ e^+)$ synthesis (also called compressed positronium). In fact, as shown since 1978 (see [3], Sect.5), under the use of the same isounit of isotopy (1.1*a*), the charge radius of 1fm and the meanlife of $0.83 \cdot 10^{-16}$ sec, the state $(\hat{e}^-,\ \hat{e}^+)_{\text{HM}}$ represents all characteristics of the π^0, such as rest energy (134.96 MeV), spin charge, meanlife, electric and magnetic moments, etc., as well as the decay with lowest mode $\pi^0 \rightarrow e^+,\ e^-(<2.10^{-6})$ as a tunnel effect of the constituents.

Once the mechanism of the synthesis is understood at the level of electrons, its extension to the remaining mesons is straightforward. In fact, the compression of the muonic (1.1*c*) and of the pionic (1.1*d*) atoms follows the same rules as those of the electrons. The identification of the states with the η and K^0_S particles, respectively, is rendered inevitable by the uniqueness of the total characteristics, as it was inevitable for identification (1.1*b*) (see Sect. 2). The isopoincaré symmetry $\hat{P}_Q$ (3.1) then permits the interpretation of all remaining mesons as syntheses of lighter (massive) particles suitably selected in their spontaneous decays [4].

In the transition to baryons new fundamental problems expectedly emerge whose solution required systematic studies on the isotopies $S\hat{U}_Q(2)$ of the $SU(2)$–spin symmetry [4,9] (see also the review in [1]). The origin of the syntheses here considered can then be traced back to Rutherford's [10] historical conception of the neutron as a compressed hydrogen atom. The existence of the neutron was subsequently confirmed by Chadwick [11], but Rutherford's conception of the neutron was abandoned because contrary to QM on numerous counts (impossibility to represent the total rest energy of the neutron because of the need of a "positive binding energy", inability to represent the total spin, meanlife, size and other characteristics of the neutron). As well known, these difficulties lead to the conception of the isospin $SU(2)$ which subsequently leads to the $SU(3)$ theories.

However, the above QM difficulties were inconclusive, because of the unplausibility of the underlying assumptions such as: the electron freely orbits within one of the densest medium measured in laboratory; the treatment deals with a tiny atom inside the proton; etc. As a result of such inconclusive character, studies on Rutherford's historical conception of the neutron were continued by various authors.

The most salient recent results are the following. The first representation of all characteristics of the neutron via HM, including spin via the use of the $S\hat{U}_Q(2)$ symmetry, was reached in ref. [12] of 1990. However, the problem of the total spin of the neutron (which requires a null total angular momentum for Rutherford's isoelectron $\hat{e}^-$ when compressed inside the proton) was first solved by Dirac (without his knowledge) in two of his last (and little known) papers of 1971–1972 [13,14], where he introduced a generalization of his equation, which subsequently emerged as possessing an essential isotopic structure.

The first preliminary, yet direct and impressive experimental verifications of the synthesis of protons and electrons into neutrons via the reaction at low energy $p^+ + e^- \rightarrow n + \nu$ have been achieved by an experimental team headed by don Borghi [15], reviewed in Sect.8 with a number of indirect experimental confirmations.

Under these theoretical and experimental results, the role of hadronic mechanics is then essentially reduced to the identification of their appropriate theoretical framework (see the more detailed presentation [16] and [4]).

It is hoped the reader can see the intriguing implications synthesis (1.1e). In fact, if confirmed, it will imply the possibility not only of producing unstable particles via synthesis, but also their artificial disintegration. By recalling that currently available technologies are based on mechanisms in the structure of molecules, atoms and nuclei, the studies of this paper are motivated by the possibility of resulting in a new technology, called hadronic technology [20], which is based on mechanisms, this time, in the interior of hadrons.①

2. Nonrelativistic Treatment

The radial nonrelativistic treatment of the synthesis of particles has been known since 1978 [3] (see [4,16] for recent accounts). The central hypothesis is the generalization of Planck's constant $\hbar = 1$ into the isounit $\hat{I} = Q^{-1} = \hat{I}(t, p, \dot{p}, \psi, \partial\psi, \partial\partial\psi, \cdots) > 0$ which represents nonlinear–nonlocal–nonhamiltonian interactions, although $\hat{I} \equiv h$ for mutual distances $r > 1$ fm.

The isotopy of the unit then implies corresponding compatible isotopies of the totality of the structure of QM into that of HM [4] (see the outline [2]), including: isotopy of field $F(n, +, \times) \Rightarrow \hat{F}_Q(\hat{n}, +, *)$, $\hat{I} = Q^{-1}$, $* = \times Q \times$, $Q =$ fixed (in this note $\hat{F} = \hat{R}_q$ or $\hat{C}_q$), with isonumbers $\hat{n} = n\hat{I}$, conventional sum $+$, and isotopic product $\hat{n} * \hat{m} = \hat{n} Q \hat{m}$, Q fixed, $\hat{I} * \hat{m} \equiv \hat{n} * \hat{I} \equiv \hat{n}$, $\forall \hat{n} \in \hat{R}$, and consequential generalization of all operations

①Quark theories are known to have no practical application of any nature. Under isotopic $S\hat{U}_Q(3)$ symmetries the situation is different. Isoquarks are perennially confined in a strict sense (with identically null transition probabilities for free quarks due to the total incoherence of the interior isohilbert and the exterior Hilbert space), while their isoconstituents are ordinary massive particles which can be freely produced in spontaneous or stimulated decays. Practical applications are then conceivable [4, 5, 16].

[1–5];isotopy of the conventional Euclidean space $E(r, \delta, R) \Rightarrow \hat{E}_Q(r, \hat{\delta}, \hat{R})$, $\hat{\delta} = Q\delta$; isotopy of Hilbert spaces $H: \langle\psi|\varphi\rangle \in C \Rightarrow \hat{H}_Q$: $\langle\hat{\psi}\hat{|}\hat{\varphi}\rangle = \langle\hat{\psi}|Q|\hat{\varphi}\rangle\hat{I} \in \hat{C}_Q$; isotopy of eigenvalue equations $H|\psi\rangle = E^0|\psi\rangle \Rightarrow H * |\hat{\psi}\rangle = HQ|\hat{\psi}\rangle = \hat{E} * |\hat{\psi}\rangle \equiv E|\hat{\psi}\rangle, E \neq E^0$; isotopy of enveloping operator algebras, Lie algebras, Lie groups, etc.

The Q–isodeformation operator is then selected to yields the isotopy [3] (see [8] for recent detailed studies)

$$\left[-\frac{d}{2mr^2 dr} r^2 \frac{d}{dr} \pm \frac{e^2}{r}\right]\psi = E^0\psi \Rightarrow \left[-\frac{d}{2mr^2 dr} r^2 \frac{d}{dr} \pm \frac{e^2}{r}\right] * \hat{\psi} = E\hat{\psi}$$

$$\approx \left[-\frac{Q^{0-1}}{2m} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \pm \frac{e^2}{r} - V_0 \frac{e^{-R^0 r}}{1 - e^{-R^0 r}}\right]\hat{\psi}(r) = E\hat{\psi}(r) \tag{2.1}$$

where Q^0 and R^0 are positive constants. By recalling that the Hulten potential behaves at small distances like the Coulomb one, isotopy (2.1) can then be reduced to

$$\left[-\frac{d}{2mr^2 dr} r^2 \frac{d}{dr} \pm \frac{e^2}{r}\right]\psi = E^0\psi$$

$$\Rightarrow \left[-\frac{Q^{0-1} d}{2mr^2 dr} r^2 \frac{d}{dr} - V \frac{e^{-R^0 r}}{1 - e^{-R^0 r}}\right]\hat{\psi} = E\hat{\psi} \tag{2.2}$$

where $V = V^0 - (\pm e^2 R^0)$. The radial structure equations of the synthesis submitted in ref. [3], Eq. (5.1.14), p.836, are then given by $(\hbar = 1)$

$$\left[-\frac{Q^{0-1}}{2m} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} - V \frac{e^{-R^0 r}}{1 - e^{-R^0 r}}\right]\hat{\psi}(r) = E\hat{\psi}(r) \tag{2.3a}$$

$$E^{\text{tot}} = \sum_{k=1,2} E_{k,\text{const.}} - E(\text{MeV}), \tau^{-1} = 4\pi\lambda|\hat{\psi}(0)|^2 \alpha E^{\text{kin}}(\text{sec}^{-1}), R^0(\text{cm}), \tag{2.3b}$$

where E^{tot}, τ^{-1} and R^0 are the total energy, meanlife and charge radius, respectively, of the isobound state.

When applied to synthesis (1.1b), the above model permitted the representation of the totality of the characteristics of the π^0, beginning with the suppression of the original spectrum of the positronium into one, single, admissible energy level: 134.896 MeV [3]. similar results hold for all other syntheses (1.1). A realization of the isotopic element Q verifying (2.1) has been identified by Animalu [8] in the expression $Q = Q^0 \exp\{it|E^0|\langle\psi|\hat{\psi}\rangle\}$. A comprehensive study is provided in [4]. We then have the following isopostulates of the synthesis of particles [3, 4, 16]:

2.1 Isorenormalizations. The intrinsic characteristics of the constituents of syntheses (1.1) are isorenormalized (" mutated" in the language of [3]) because of the nonlinear–nonlocal–nonhamiltonian interactions expected in the total mutual penetration of the wavepackets–wavelengths–charge distributions.

2.2 Energy Balance. Conventional QM bound states have a "negative binding energy" because $E_T < 2E_{\text{const}}$. All syntheses (1.1) instead, if QM treated, would require a "positive binding energy" because $E_T > 2E_{\text{const}}$ thus resulting in inconsistent indicial equations [3, 4, 16]. A necessary condition to resolve this problem is the renormalization of the rest energy of the constituents given in Eqs. (2.1) by $mc_0^2 \Rightarrow mc_0^2 Q^0$, under which binding energies can return to be negative. While conventional fusion processes "release" energy, syntheses (1.1) "require" energy①.

2.3 Suppression of Triplet States. Only singlet isobound states of spinning particles are stable, because triplet couplings under total mutual penetration imply highly repulsive nonlinear–nonlocal–nonhamiltonian forces due to the spinning of each particle inside and against that of the other (this occurrence was represented in [3] via the so–called "gear model").

2.4 Charge Independence. The mechanism of syntheses (1.1) is the dominance of nonlinear–nonlocal–nonhamiltonian forces at distances < 1fm which are attractive in singlet couplings and absorb the Coulomb interactions resulting in attractive total interactions irrespective of the attractive or repulsive character of the original Coulomb interactions.

2.5 Constancy of Size. Another difference between bound states in QM and HM is that the size of the former increases with mass, as established in nuclear and atomic structures, while the size of the latter remains approximately constant with the increase of the mass, as established for hadrons. This occurrence is quantitatively interpreted by HM via the unification of the seemingly disparate occurrences: 1) the range of the nonlinear–nonlocal–nonhamiltonian interactions due to total mutual penetration; 2) the range of the strong interactions and 3) the minimal hole needed to activate Hulten's potential. The addition to an isobound state of a further constituent does then increases its rest energy (and density, thus increasing the isorenormalizations), but leaves the size essentially unaffected.

2.6 Suppression of Atomic Energy Spectrum. Yet another difference between QM and HM bound states is that the former generally have a spectrum of energy, while the latter admit only one, single, energy level (this occurrence was called "spectrum suppression" in [3]). Each given synthesis (1.1) therefore has no excited isostates at distances < 1 fm, because all excited states imply greater distances, thus recovering conventional QM energy levels at distances > 1 fm. In fact, the Hulten potential has a finite spectrum of energy levels, as well known. When all conditions of systems (2.3) are imposed, this finite spectrum reduces to only one level (see [3, 4, 15, 16]).

2.7 Nearly Free Constituents. The notion of potential energy has no mathematical or physical meaning for the contact nonhamiltonian interactions responsible for synthesis (1.1). The binding energies are then generally small, $E_{\text{bind}} \approx 0$ an occurrence which is reminiscent of

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①This illustrates the profound differences between the conventional "cold fusion" and the "synthesis" studied in this paper. It should be noted that the synthesis of the neutron and other unstable hadrons can be a conceivable source of sub–nuclear energy called "hadronic energy" [20] because originating in mechanisms, this time, in the interior of hadrons. By recalling that the neutron decays spontaneously and in a naturally esoenergetic way, the hadronic energy is predicted in the inverse reaction, the stimulated decay of the neutron. The synthesis of this paper is therefore the foundation of the stimulated decay of the hadronic energy [20] "asymptotic freedom" in quark theories [5].

3. Isotopies and Isodualities of Poincare Symmetry

We now outline results which are rather old in isotopies, but which do not appear to have propagated as yet to the literature on their direct applicability to q–deformations.

Consider the Minkowski space $M(x, \eta, R)$ with local coordinates $x=\{x, x^4\}$, $x^4=c_0 t$, c_0 = speed of light in vacuum, metric $\eta = \mathrm{diag.}\ (1, 1, 1, -1)$, separation $x^\mu \eta_{\mu\nu} x^\nu$ and invariant measure $ds^2 = -dx^\mu \eta_{\mu\nu} dx^\nu$. Its group of linear–local–canonical isometries is the ten–dimensional Poincare group $P(3.1)$ characterized by the (ordered sets of) parameters $w=\{\theta, v, a\}$ (Euler's angles θ_k, speed parameter v_k and translation parameters a), and generators, say, for a system of two particles with non–null masses $m_a, X=\{X_k\}=\{M_{\mu\nu}=\sum_a (x_{a\mu} p_{a\nu} - x_{a\nu} p_{a\mu})$, $P_\mu = \sum_a p_{a\mu}\}$, $\mu,\nu = 1, 2, 3, 4$, $a=1, 2$, in their known fundamental representation (see e.g., [17], p. 40).

Three realizations of the ten–dimensional isotopic covering $\hat{P}_Q(3.1)$ of $P(3.1)$ have been constructed via the Lie–isotopic theory, the classical [18], operator [4, 19] and abstract [6] ones. The latter can be readily constructed by following the space–time version of steps 1–5 for the isorotational symmetry [1], Sect.3.E.Step 1 is the identification of the fundamental isotopic element Q here interpreted as a 4×4 matrix generalization of q–number–deformations. The identification is done via, the deformation of the Minkowski metric η into the most general known metric $\hat{g}$ which is nonlinear, nonlocal–integral, and noncanonical in all variables, wavefunctions and their derivatives,

$$\hat{g} = \hat{g}(s,x,\dot{x},\ddot{x},\psi,\partial\psi,\partial\partial\psi,\cdots) = Q(s,x,\dot{x},\ddot{x},\psi,\partial\psi,\partial\partial\psi,\cdots)\eta, \tag{3.1}$$

under the condition of being of Kadeisvili Class III [1] (smooth, bounded, nowhere singular and Hermitean, but not necessarily positive or negative–definite). Under the assumed conditions, the Q–matrix can always (although not necessarily) be diagonalized into the form

$$Q = \mathrm{diag.}(\hat{g}_{11}, \hat{g}_{22}, \hat{g}_{33}, \hat{g}_{44}) = Q^*, \quad \det Q \neq 0 \tag{3.2}$$

The isosymmetry $\hat{P}_Q(3.1)$ is then constructed with respect to the isounit $\hat{I} = Q^{-1}$.

Step 2 is the lifting of the conventional field $R(n, +, \times)$ of real numbers n into the isofield $\hat{R}_Q(\hat{n}, +, *)$ of isoreal numbers $\hat{n} = n\hat{I}$, $\hat{I} = Q^{-1}$.

Step 3 is the lifting (necessary for consistency) of the space $M(x, \eta, R)$ on the field R into the isominkowski space $\hat{M}_Q(x, \hat{g}, \hat{R})$ on the isofield $\hat{R}$ with isoseparation [6]

$$(x-y)^{\hat{2}} = \left[(x^\mu - y^\mu)\hat{g}_{\mu\nu}(s,x,\dot{x},\ddot{x},\psi,\partial\psi,\partial\partial\psi,\cdots)(x^\nu - y^\nu)\right]\hat{I} \in \hat{R} \tag{3.3}$$

Step 4 identifies the basic isotransformations leaving invariant (3.3)

$$\begin{gathered} x' = \hat{\Lambda}(w) * x, \ \hat{\Lambda}^* \hat{g} \hat{\Lambda} = \hat{\Lambda} \hat{g} \hat{\Lambda}^* = \hat{I}\hat{g}\hat{I} \\ \hat{\mathrm{D}}\mathrm{et}\ \hat{\Lambda} = [\mathrm{Det}\ (\hat{\Lambda} Q)]\hat{I} = \pm \hat{I}, x' = x + A, \end{gathered} \tag{3.4}$$

where the quantity A will be identified shortly. The connected component $\hat{P}^0_Q(3.1) = S\hat{O}_Q(3.1) \times \hat{T}_Q(3.1)$ is characterized by $\hat{\mathrm{Det}}\hat{\Lambda} = +\hat{I}$ with structure [4, 6, 18, 19]

$$\hat{Q}_Q(3.1){:}\hat{\Lambda}(w) * x = \left\{ \prod_k e_{\xi_Q}^{iX_k * \hat{w}_k} \right\} Qx = \left\{ \prod_k e^{iX_k Q w_k} \right\} x, \tag{3.5a}$$

$$\hat{T}_Q(3.1){:}\left\{ e_{\xi_Q}^{iP\eta a} \right\} * x = \{ e^{iP\hat{g}a} \} x, \left\{ e_{\xi_Q}^{iP\eta a} \right\} * p \equiv 0, \tag{3.5b}$$

where w_k and X_k are conventional [17] and Q is given by (3.2). The isocommutation rules $\hat{P}^0_Q(3.1)$ are given by [loc.cit.]

$$[M_{\mu\nu}, M_{\alpha\beta}] = i(\hat{g}_{\nu\alpha} M_{\beta\mu} - \hat{g}_{\mu\alpha} M_{\beta\nu} - \hat{g}_{\nu\beta} M_{\alpha\tau} + \hat{g}_{\mu\beta} M_{\alpha\nu}), \tag{3.6a}$$

$$[M_{\mu\nu}, P_\alpha] = i(\hat{g}_{\mu\alpha} P_\nu - \hat{g}_{\nu\alpha} P_\nu), [P_\mu, P_\nu] = 0, \tag{3.6b}$$

where the product is the fundamental Q–isocommutator $[A, B] = A * B - B * A = A Q B - B Q A$ of the Lie–isotopic theory [1,3]. The isocasimirs are then given by

$$C^{(0)} = \hat{I}, C^{(1)} = P^2 = P * P = P_\mu \hat{g}^{\mu\nu} P_\nu \tag{3.7a}$$

$$C^{(2)} = \hat{W}^2 = \hat{W}_\mu \hat{g}^{\mu\nu} \hat{W}_\nu, \; \hat{W}_\mu = \varepsilon_{\mu\alpha\beta\rho} J^{\alpha\beta} * P^\rho \tag{3.7b}$$

The general isopoincaré transformations are then given by [loc.cit.]

$$x' = \hat{\Lambda} * x \text{ isolorentz transforms,} \tag{3.8a}$$

$$x' = x + A(s, x, \dot{x}, \ddot{x}, \cdots), \text{ isotranslations,} \tag{3.8b}$$

$$x' = \hat{\pi}_r * x = (-x, x^4), x' = \hat{\pi}_t * x = (x, -x^4), \text{ isoinversions,} \tag{3.8c}$$

$$A_\mu = a_\mu \left\{ \hat{g}_{\mu\mu} + a^\alpha [\hat{g}_{\mu\mu}, \hat{P}_{\mu\alpha}] / 1! + a^\alpha a^\beta [[\hat{g}_{\mu\mu}, \hat{P}_{\mu\alpha}], \hat{P}_{\mu\beta}] / 2! + \cdots\cdots \right\} \tag{3.8d}$$

The general isolorentz transformations are given by the isorotations reviewed in [1], and the isoboosts first constructed in [6]

$$x'^1 = x^1, x'^2 = x^2 \tag{3.9a}$$

$$\begin{aligned} x'^3 &= x^3 \cosh\left[v(\hat{g}_{33}\hat{g}_{44})^{\frac{1}{2}} \right] - x^4 \hat{g}_{44} (\hat{g}_{33}\hat{g}_{44})^{-\frac{1}{2}} \sinh\left[v(\hat{g}_{11}\hat{g}_{22})^{\frac{1}{2}} \right] \\ &= \hat{\gamma}(x^3 - \beta x^4) \end{aligned} \tag{3.9b}$$

$$\begin{aligned} x'^4 &= -x^3 \hat{g}_{33} (\hat{g}_{33}\hat{g}_{44})^{-\frac{1}{2}} \sinh\left[v(\hat{g}_{11}\hat{g}_{22})^{\frac{1}{2}} \right] + x^4 \cosh[v(\hat{g}_{33}\hat{g}_{44})]^{\frac{1}{2}} \\ &= \hat{\gamma}(x^4 - \hat{\beta} x^3) \end{aligned} \tag{3.9c}$$

where

$$\beta = v / c_0, \; \hat{\beta} = v^k g_{kk} v^k / c_0 g_{44} c_0, \tag{3.10a}$$

$$\cosh\left[v(\hat{g}_{11}\hat{g}_{22})^{\frac{1}{2}} \right] = \hat{\gamma} = (1 - \hat{\beta}^2)^{-\frac{1}{2}}, \;\; \sinh\left[v(\hat{g}_{11}\hat{g}_{22})^{\frac{1}{2}} \right] = \hat{\beta}\hat{\gamma} \tag{3.10b}$$

A few comments are in order. It is easy to prove the local isomorphisms $\hat{P}_Q(3.1) \approx P(3.1)$ for all $Q>0$ (but not so for all Q of generic Class Ⅲ). This illustrates that the Lorentz transformations are necessarily inapplicable (and not "violated") under isotopies, but the Poincare symmetry is preserved in an exact form, only realized in its most general possible (rather than simplest possible) form.

The "direct universality" of the isopoincaré symmetry should be noted, i.e., its applicability for all infinitely possible isoseparations (3.3) (universality), directly in the x–frame of the experimenter (direct universality).

Despite their apparent simplicity, isotransformations (3.9) are highly nonlinear–nonlocal–noncanonical owing to the unrestricted functional dependence of the Q–matrix (or elements $\hat{g}_{\mu\mu}$).The simplicity of the final invariance should also be noted. In fact, the invariance of all infinitely possible isoseparations (3.3) is merely given by plotting the given $\hat{g}_{\mu\mu}$ elements in Eqs. (3.9).

This brings us to a first application of Q–isodeformations which has not yet propagated in the literature. It is given by the capabilities of the Q–isotopies [18]; a) to represent the transition from the Minkowskian to the Riemannian geometry; b) to provide the universal invariance of general relativity and c) to achieve a geometric unification of the special and general relativities. All these results are achieved by merely assuming the particular nonlinear, yet local and Lagrangian realization $\hat{g} \equiv g(x) = \text{Riemann} = Q(x)\eta$, and then the construction of the $\hat{P}_Q(3.1)$ isosymmetries with respect to the gravitational isounit $\hat{I} = [Q(x)]^{-1}$.

Note that all Riemannian metrics admit the above Q–decomposition with $Q>0$ (trivially, from their locally Minkowskian character). Our isopoincaré symmetry $\hat{P}_Q(3.1)$ then ensures: 1) the invariance of all riemannian line elements①; 2) the crucial isomorphisms $\hat{P}_Q(3.1) \approx P(3.1)$; and 3) the isomorphisms among the underlying spaces $\hat{\mathcal{R}}(x,g,\hat{R}) \approx M(x,\eta,R)$, where the Riemannian space $\mathcal{R}(x, g, R)$ is reinterpreted as the isominkowskian space $\hat{M}(x,g,\hat{R}),g = Q\eta$, $\hat{I} = Q^{-1}$.The isotopic unification of the special and general relativities then follows via the embedding of curvature $Q(x)$ in the isounit $\hat{I}$ of the theory.

The isodual isopoincaré symmetry $\hat{P}^d_Q$ (3.1) is characterized by the antiautomorphic conjugation $Q \to Q^d = -Q^d(\hat{I} \to \hat{I}^d = -\hat{I})$ leading to isodual isofields $\hat{F}^d_Q(\hat{n}^d, +, *^d)$, isodual isospaces $\hat{M}^d_Q(x,\hat{g}^d,\hat{R}^d),\hat{g}^d = -\hat{g}$, $\hat{R}^d \approx -\hat{R}$, etc. They characterize negative–definite energies, motion flowing backward in time, etc., thus permitting an intriguing (and novel) characterization of antiparticles [1,4,18,19]. Note the isodual Poincare symmetry $P^d(3.1)$, whose identification requires the isotopic theory (owing to the need of a bona fide generalized unit $I^d = (-I)$.

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①As an example, the invariance of the Schwartzschild line element is very simply achieved by plotting in Eqs. (3.8) and (3.9) the values of $g_{11} = (1 - 2M/r)^{-1}$, $g_{22} = r^2$, $g_{33} = r^2 \sin^2\theta$, $g_{44} = 1 - 2M/r$. Similarly $\hat{P}_Q(3.1)$ provides the direct invariance of any arbitrarily given Reimannian metric with elements $g_{\mu\mu}$

For physical applications the isotopies are restricted to preserve the signature (+, +, +, −) of $M(x, \eta, R)$, called of Kadeisvili Class I [1], with realization

$$\hat{g} = Q\eta,\ Q = \mathrm{diag.}(b_1^2, b_2^2, b_3^2, b_4^2) \equiv \mathrm{diag.}(n_1^{-2}, n_2^{-2}, n_3^{-2}, n_4^{-2}),$$
$$b_\mu, n_\mu > 0 \tag{3.11}$$

where the b's called characteristic functions of the medium considered. The use of the quantity $Q^d = -Q$ then characterizes the isodual symmetry.

The unifying powers of the isopoincaré symmetry should be finally noted. On mathematical grounds, the singles abstract isotope $\hat{P}_Q$ (3.1) of Class III outlined above unifies all possible inhomogeneous ten–dimensional groups, such as $O(4) \times T(4), O(3.1) \times T(3.1)$, $O(2.2) \times T(2.2)$ all their isoduals and all their infinite isotopes [4, 18].

On physical grounds, the isotope $\hat{P}_Q$ (3.1) of Class I unifies: linear and nonlinear, local and nonlocal, Hamiltonian and nonhamiltonian, relativistic and gravitational, as well as exterior and interior systems, at both classical and operator levels [4, 18].

4. Isotopies and Isodualities of the Special Relativity

We shall now ignore gravitational profiles, and consider isotopic theories specifically built for interior relativistic dynamical problems with $Q = Q(s, \dot{x}, \ddot{x}, \cdots) > 0$ of Kadeisviei Class I.

The isotopies of the Poincaré symmetry $P(3.1) \Rightarrow \hat{P}_Q(3.1)$ imply necessary, corresponding liftings of the special relativity into a form called isospecial relativity, originally submitted in [6] and then studied in detail in [4,18]. The objective is a form–invariant description of extended particles and electromagnetic waves propagating within inhomogeneous and anisotropic physical media represented by isospaces $\hat{M}(x, \hat{g}, \hat{R})$. The special relativity is then identically admitted, by construction, when motion returns to the homogeneous and isotropic vacuum.

The isospecial relativity is based on the isopoincaré invariance on isospaces $\hat{M}(x, \hat{g}, \hat{R})$ of Class I, with consequential isotopies of all basic postulates of the special relativity. Those important for this note are the following isopostulates for realization (3.11) with $b_\mu = b_\mu(s, \dot{x}, x, \cdots), b_1 = b_2 = b_3 \neq b_4$ (see [6] for original works and [4] for comprehensive studies):

4.1 The invariant speed is the "maximal causal speed"

$$V_{\max} = |dr / dt|_{\max} = c_0 b_4 / b_3 \tag{4.1}$$

4.2 The addition of speeds u and v is given by the "isotopic addition law"

$$v' = (u + v) / (1 + u_k b_k^2 v_k / c_0^2 b_4^2) \tag{4.2}$$

4.3 Time intervals and lengths follow the isodilation–isocontraction laws

$$\hat{\tau} = \hat{\gamma}\tau_0,\ \hat{\Delta}L_0 = \hat{\gamma}\Delta L \tag{4.3}$$

4.4 The frequency follows the "isodoppler shift law"

$$\hat{\omega}' = \omega\hat{\gamma}(1 - \hat{\beta}\cos\alpha),\ \cos\hat{\alpha}' = (\cos\alpha - \hat{\beta}) / (1 - \hat{\beta}\cos\alpha) \tag{4.4}$$

4.5 The energy equivalence of mass follows the "isoequivalence principle"

$$\hat{E} = mc^2 = mc_0^2 b_4^2 = mc_0^2 / n_4^2 \tag{4.5}$$

where $\hat{\gamma}$ is defined in (3.10*b*).

The above generalized postulates are implicit in the preceding formulations, e.g., in isoinvariant (3.3) or in isolorentz transformations (3.9); they recover identically the conventional postulates in vacuum for which $b_\mu = 1$; and they coincide with the conventional postulates at the abstract, realization–free level, where we lose all distinctions between $\hat{I}$ and I, $x^{\hat{2}}$ and x^2, $\hat{\beta}^2$ and β^2, $\hat{\tau}$ and τ, $\hat{\omega}$ and ω, $\hat{E}$ and E, etc.

A most visible departure from the conventional postulates is the abandonment of the speed of light as the invariant speed in favor of quantity (4.1) which is intrinsic of the isominkowski geometry and represents the maximal causal speed as characterized by an effect following a cause due to particles, fields or other means. Note that in vacuum $V_{max} \equiv c_0$ by therefore recovering as a particular case the speed of light as the maximal causal speed.

The best way to verify Isopostulates 4.1 is in the simplest possible medium, the homogeneous and isotropic water, where the speed of light is no longer c_0, but rather the familiar value $c = c_0 / n_4 < c_0$, where n_4 is the index of refraction. The insistence in keeping the speed of light as the invariant speed leads to a number of inconsistencies, such as: the violation of both the conventional and isotopic laws of addition of speeds, none of which yields the speed of light as the sums of two light speed $u = v = c = c_0 / n_4$; electrons can propagate in water at speeds bigger than the assumed invariant speed, as experimentally established by the Cherenkov light; and others. All these inconsistencies are resolved by the isospecial relativity [4, 6].

Even greater inconsistencies emerge if one insists in keeping the speed of light as the invariant speed for all media more complex than water, e.g., inhomogeneous and anisotropic atmospheres. A resolution of these inconsistencies requires the separation of the invariant speed from the speed of light, and their identity only for the particular case in vacuum.

Since isopostulates 1–5 are quantitatively different than the conventional ones, they are suitable for experimental verification. Intriguingly, all available experimental evidence appears to confirm the above isotopic postulates, not only for simple media such as water or atmospheres, but also for the more complex media, such as the hyperdense media inside hadrons (Sect.8, 9).

Isotopic theories predict the existence of a hitherto unknown universe, called isodual universe, which is characterized by the isodual isominkowski (and isoriemannian) spaces $\hat{M}_Q^d(x, \hat{g}, \hat{R})$. The isodual isospecial relativity [4, 18] is a $\hat{P}_Q^d$ (3.1)–invariant description of antiparticles in interior dynamical conditions, characterized by the image of Isopostulates1–5 on $\hat{R}_Q^d(\hat{n}^d, +, *^d)$. The isodual special relativity is a new image of the conventional relativity for antiparticles in exterior conditions characterized by the isodual Poincare invariance P^d (3.1) on $M^d(x, \eta^d, R^d)$.

5. Isorenormalization

Isominkowskian treatments with generic characteristic functions $b_\mu^2(s,\dot{x},x,\cdots)$ are valid for the local description, that is, the behaviour of a particle or an electromagnetic wave at one given point of the interior medium, such as for the value of the speed of light $c=c_0 b_4=c_0/n_4$ at one given point of an inhomogeneous and anisotropic medium. When "global" values are of interest, such as the average speed of light through the entire medium considered, $c=c_0 b_4^0 = c_0/n_4^0 = \text{const}$, the b–functions can be effectively averaged to constants, $b_\mu^0 = \text{Aver.}(b_\mu) = \text{const.} > 0$, $n_\mu^0 = \text{Aver.}(n_\mu) = \text{const.} > 0$

In this case isotransforms (3.8), called restricted isopoincaré transformations, regain linearity and locality (thus, preserving conventional inertial frames), although they remain noncanonical. Since we are interested in the "global" treatment of syntheses (1.1), in this section we shall consider the restricted isopoincaré transformations.

A primary function of the isominkowski spaces (as well as a primary mean for their experimental verification) is the geometrization of inhomogeneous and anisotropic physical media at large, and the media in the interior of hadrons, in particular. By recalling that systems are now characterized by a conventional Lagrangian or Hamiltonian plus the isotopic element Q, the desired novel isorenormalizations are expected to originate directly from the isominkowskian geometrization.

Consider the (operator) relativistic isokinematics on $\hat{M}(x,\hat{g},\hat{R})$[4] with basic expressions

$$p=(p^\mu)=(\hat{m}u^\mu)=\left(m_0\hat{\gamma}\hat{c}v^k, m_0\hat{\gamma}\hat{c}\right), \hat{m}=m_0\hat{\gamma}, c=b_0 b_4^0, \tag{5.1}$$

isoeigenvalue form

$$p_\mu * \hat{\psi} = -i\hat{I}_\mu^\nu \partial_\nu \hat{\psi} = -ib_\mu^{-2}\partial_\mu \hat{\psi}, (\text{nosum}), \tag{5.2}$$

and fundamental isoinvariant [4]

$$\begin{aligned} p^{\hat{2}} * \hat{\psi} &= \hat{\eta}^{\mu\nu} p_\mu * p_\nu * \hat{\psi} = (b_k^2 p_k * p_k - c^2 p_4 * p_4) * \hat{\psi} \\ &= (m_0^2\hat{\gamma}^2 c^2 v^k b_k^2 v^k - m_0^2\hat{\gamma}^2 c^4)\hat{\psi} \\ &= \left[-m_0^2\hat{\gamma}^2 c^4(1-\hat{\beta}^2)\right]\hat{\psi} \\ &= (-m_0^2 c^4)\hat{\psi} \end{aligned} \tag{5.3}$$

It is then easy to see that the isorenormalization needed for the syntheses is provided by Isopostulates 1–5 themselves. As an example, in going from motion in vacuum to motion within a physical medium with characteristics b^0–constants, a particle experience the following isorenormalization of the rest energy

$$E=mc_0^2 \Rightarrow E'=mc^2=mc_0^2 b_4^{02}, \tag{5.4}$$

which is precisely the relativistic version of the nonrelativistic isonormalization of Sect. 2. A similar occurrence holds for all remaining intrinsic characteristics. This is expected from the alteration of the conventional Casimir invariants into form (3.7), as well as by the isotopies of Dirac's equation (see next section).

The predictions of the isospecial relativity for particles can be independently tested via the predictions for electromagnetic waves which, under the necessary condition of propagating within inhomogeneous and anisotropic media, are expressed by the iso–plane–wave on $\hat{M}_Q(x,\hat{g},\hat{R})$[4]

$$\hat{\Psi}(x) = N\ e^{i(K^k b_k^{02} x^k - E b_4^{02} t)} \tag{5.5}$$

The novel predictions suitable for experimental tests (sect. 9) are the isodoppler's law (4.4) with isoredshift for low density media (i.e., the prediction that electromagnetic waves lose energy within such media), and isoblueshift for high density media (i.e., the complementary prediction that electromagnetic waves gain energy from such media), while there is no modification of the Doppler's law in water because of its homogeneity and isotropy (i.e., electromagnetic waves preserve their energy in water).

A number of considerations here not reported for brevity (see [4, 16, 19]) lead to the important conclusion that all hadrons beginning from kaons are of the so–called isominkowskian media of type 9 for which $\hat{\beta} < \beta, \hat{\gamma} > \gamma$, Aver. $(b_k^0) < b_4^0$[19], p. 103. The understanding is that different hadrons have different numerical values of the characteristics b^0 quantities, e.g., because they have different densities.

6. Isotopies and Isodualities of Dirac's Equation

We are now sufficiently equipped to review a fundamental application of the isospecial relativity, the isotopies of Dirac's equation, here called isodirac equation, for the characterization of isoparticles $\hat{e}^{\pm}, \hat{p}^{\pm}$, etc.

The isolinearization of 2–nd order invariant (5.3) can be done by introducing the 12–dimensional isospace $\left\{\hat{M}_Q^{\text{Orb.}}(x,\hat{g},\hat{R}) \times \hat{S}_Q^{\text{Intr}}(2)\right\} \times \left\{\hat{M}_Q^{d,\text{Orb}}(x,\hat{g}^d,\hat{R}^d) \times S_Q^{d,\text{Intr.}}(2)\right\}$ for the characterization of the orbital and intrinsic angular momenta for particles and antiparticles, respectively. The following expressions in self–explanatory notation (see [4] for details) then characterize the isogamma matrices $\hat{\gamma}$.

$$\begin{aligned} &\left(\hat{g}^{\mu\nu} p_\mu *^{\text{orb}} p_\nu + \hat{m}^2\right) *^{\text{orb}} \hat{\Psi}(x) \\ &\equiv \left(\hat{g}^{\mu\nu}\hat{\gamma}_\mu *^{\text{tot}} p_\nu + i\hat{m}\right) *^{\text{tot}} \left(\hat{g}^{\alpha\beta}\hat{\gamma}_\alpha *^{\text{tot}} p_\beta - i\hat{m}\right) *^{\text{tot}} \hat{\Psi}(x) \end{aligned} \tag{6.1a}$$

$$\{\hat{\gamma}_\mu \,\hat{,}\, \hat{\gamma}_\nu\}^{\text{tot}} = \hat{\eta}_\mu Q^{\text{tot}} \hat{\gamma}_\nu + \hat{\gamma}_\nu Q^{\text{tot}} \hat{\gamma}_\mu = 2\hat{g}_{\mu\nu} \hat{I}^{\text{orb}}, \tag{6.1b}$$

$$\hat{\gamma}_\mu = \tilde{\gamma}_\mu \hat{I}^{\text{orb}}, \{\tilde{\gamma}_\mu \,\hat{,}\, \tilde{\gamma}_\nu\}^{\text{intr}} = \tilde{\gamma}_\mu Q^{\text{intr}} \tilde{\gamma}_\nu + \tilde{\gamma}_\nu Q^{\text{intr}} \tilde{\gamma}_\mu = 2\hat{g}_{\mu\nu} I. \tag{6.1c}$$

The above formulation is excessively general for our needs in this note. We shall therefore assume the particularization

$$\hat{I}^{\text{orb}} \equiv \hat{I}, Q^{\text{orb}} \equiv Q, I^{\text{spin}} = I = \text{diag.}(1,1), \hat{I}^d = -\text{diag.}(1,1), \tag{6.2a}$$

$$(\hat{\gamma}_\mu \,\hat{,}\, \hat{\gamma}_\nu) = \hat{\gamma}_\mu Q \hat{\gamma}_\nu + \hat{\gamma}_\nu Q \hat{\gamma}_\mu = 2\hat{g}_{\mu\nu} I, \hat{I} = Q^{-1}, \tag{6.2b}$$

more space ?

$$\hat{\gamma}^k = b^k \hat{I} \begin{bmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{bmatrix}, \hat{\gamma}^4 = ib^4 \hat{I} \begin{bmatrix} I_s & 0 \\ 0 & -I_s \end{bmatrix}, I_s = \mathrm{diag.}(1,1), I_s^d = -I_s, \tag{6.2c}$$

where the $\gamma-$ and $\sigma-$matrices are the conventional ones, and $b_\mu = b_\mu(s, \dot{x}, x, \cdots)$. One can see the emergence of the isodual isospaces $S^d(2)$ characterized by $I^d = -\mathrm{diag.}\ (1,1)$ in γ^4 beginning with the conventional Dirac's equation, which then persist under isotopies to $\hat{S}^d(2)$. The desired isodirac equation can then be written:

$$\left(\hat{\gamma}_\mu * p^\mu + i\hat{m}\right) * \hat{\psi}(x) = \left(\hat{g}^{\mu\nu}\hat{\gamma}_\mu Q P_\nu + i\hat{m}\right) Q \hat{\psi} = 0, \hat{m} = m\hat{I} \tag{6.3}$$

with a simple extension to include electromagnetic potentials which, being external, are not altered by the isotopies.

The orbital and intrinsic angular momenta of particles with the lowest admissible hadronic weight are then characterized by

$$\hat{O}_Q(3): \hat{L}_k = \varepsilon_{kij} r_i p_j, [\hat{L}_i \hat{,} \hat{L}_j] = \varepsilon_{ijk} b_k^{-2} \hat{L}_k, \tag{6.4a}$$

$$\hat{L}^2 * \hat{\psi} = (b_1^{-2} b_2^{-2} + b_2^{-2} b_3^{-2} + b_3^{-2} b_1^{-2})\hat{\psi}, \hat{L}_3 * \hat{\psi} = \pm b_1^{-1} b_2^{-1} \hat{\psi}, \tag{6.4b}$$

$$S\hat{U}(2): \hat{S}_k = \frac{1}{2}\varepsilon_{kij}\hat{\gamma}_i * \hat{\gamma}_j, [\hat{S}_i \hat{,} \hat{S}_j] = \varepsilon_{ijk} b_k^2 \hat{S}_k, \tag{6.4c}$$

$$\hat{S}^2 * \hat{\psi} = (1/4)(b_1^2 b_2^2 + b_2^2 b_3^2 + b_3^2 b_1^2)\hat{\psi}, \hat{S}_3 * \hat{\psi} = \pm\frac{1}{2} b_1 b_2 \hat{\psi}, \tag{6.4d}$$

which confirm the existence of the desired nontrivial isorenormalizations of angular momentum and spin.

A simple isotopy of the conventional derivation yields the magnetic and electric isodipole moments (assumed for simplicity along the third axis)

$$\hat{\mu} = \frac{b_3}{b_4}\mu, \ \hat{m} = \frac{b_3}{b_4} m \tag{6.5}$$

first derived in [3], eqs. (4.20. 16), p. 803.

The isosymmetry of (6.3) is the isotope $S\hat{L}(2.\hat{C})$ of the spinorial symmetry $SL(2.C)$ with generators $\hat{S}_k = \frac{1}{2}\varepsilon_{kij}\hat{\gamma}_i * \hat{\gamma}_j$, $\hat{\Gamma}_k = \frac{1}{2}\hat{\gamma}_k * \hat{\gamma}_4$ and isocommutation rules (3.6a). By adding isotranslations, Eqs. (6.3) therefore characterize the spinorial covering $\hat{\mathbf{P}}_Q(3.1)$ of the isopoincare symmetry $\hat{\mathbf{P}}_Q(3.1)$ of Sect. 3. The proof that isodirac's equation transforms isocovariantly under $\hat{P}_Q(3.1)$ is instructive. Equally instructive is the proof of the isoselfduality of Dirac's equation and of its isotopic extension which justifies the assumed 12–dimensional isospace①. The true general symmetry of the conventional Dirac equation is therefore $P(3\times 1)$

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①This property requires the knowledge of the behaviour of all quantities under isoduality, e. g. the isodual complex numbers $\cdot c^d = -\bar{c}$, the isodual states [4] $|>^d = -(1>)^\dagger = -<1$, etc. It is then easy to prove that isoduality is equivalent to charge conjugation. This property then permits the novel representation of antiparticles via isodual spaces. Note that the notion of isoduality is embedded in the structure of the conventional gamma matrices because of the appearances of the isodual matrices $\sigma_k^d = -\sigma_k$ and isodual unit $I_s^d = -I_s$. We here merely consider its isotopies [4, 19].

$\times P^d(3.1)$. Additional studies on the isotopics of Dirac's equation have been done by Jannussis, Nishioka and others [21].

7. Isotopic Characterization of the Synthesis of the Neutron

We now specialize Eqs. (6.3) for the characterization of synthesis (1.1e). Recall that Dirac's equation describes the ordinary electron e^- under the external field of the proton p^+. Eqs. (6.3) are therefore ideally set to describe Rutherford's electron $\hat{e}^-$ when immersed within the hadronic medium inside the proton considered as external.

As a first step, we can therefore study the synthesis $n = (p^+, \hat{e}^-)_{HM}$ where the proton is unperturbed owing to his much greater mass, and the isoelectron $\hat{e}^-$ is represented by (6.3) where the b_μ–functions are averaged to constants b_μ^0, thus averaging all interactions, whether Lagrangians or not①. These assumptions then permit the following remarkably simple isorenormalizations.

7.1 Isorenormalization of Rest Energy. The energies involved in synthesis (1.1*e*) are: $E_n^0 = 939.57$ MeV; $E_p^0 = 938.28$ and $E_e^0 = 0.5$ MeV. As we shall see in the next section, the binding energy is very small and can be assumed to be null in first approximation. This requires the isorenormalization via (4.5)

$$E_e^0 = m^0 c_0^2 = 0.5\text{MeV} \Rightarrow \hat{E}_e^0 = \dot{m}^0 c_0^2 b_4^{02} \approx 1.3\text{MeV},\ b_4^0 \approx 1.62 \qquad (7.1)$$

first predicted in [15], p. 527②

7.2 Isorenormalization of Total Angular Momentum. The isoelectron $\hat{e}^-$ must have a null total angular momentum to permit a consistent synthesis (1.1e). This result was first reached by Dirac [13,14] via his generalization of his own equation, which results to have an essential isotopic structure with a nondiagonal Q–matrix (denoted β in [13]). The total angular momentum is then subjected to the isorenormalization $L + 1/2 \Rightarrow (n + n')/2$, $n, n' = 0, 1, 2, \cdots$, thus being null for the ground state (see [4,8] for detailed review and isotopic reinterpretation).

The isotopic $S\hat{U}(2)$–spin theory [9] permits a rigorous confirmation of this result because the only allowed addition of angular momentum and spin for a particle immersed within the hyperdense medium inside the proton is that for which: 1) the spin–spin coupling is a singlet; 2) the orbital angular momentum is along the spin of the heavier particle; and 3) at the limit of compression of the electron to the center of the proton, its orbital and intrinsic angular momenta must evidently be equal but opposite, thus resulting in a null total value (see also the

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① The understanding of this point requires the knowledge that conventional electromagnetic interactions can be represented via the generalized Lie tensor (the b–functions), with the Lagrangian representing only the kinetic energy [22], p.98–101. Despite its misleading appearance, Eqs. (6.3) represent, as written (i.e., without conventional interactions), an electron under the most general known linear and nonlinear, local and nonlocal, as well as Lagrangian and nonlagrangian interactions. Their averaging into constant b_μ^0 is possible because of the stability of the syntheses considered.

② There is a clear misprint in [15], Eqs. (2.45) yielding 16.5 rather than 1.65. As we shall see in Sect.8, the experimental value is precisely 1.65 also accounting for the binding energy.

review in [1], Sect.3.E).

Isodirac's equation (6.3) permits a quantitative interpretation of $\lim_{r\to 0} \hat{L}_{\hat{e}} \equiv -S_{\hat{e}}$ via $\hat{L}_3 \equiv -\hat{S}_3$ *and* $\hat{L}^2 \equiv \hat{S}^2$ and from Eqs. (6.4) (for $L=1$, $L=0$ being unallowed for $\hat{L}_3 = -\hat{S}_3 \neq 0$)

$$b_1^{-1}b_2^{-1} = \frac{1}{2}b_1 b_2, b_1^{-1}b_2^{-1} + b_2^{-2}b_3^{-2} + b_3^{-2}b_1^{-2}$$
$$= (1/4)(b_1^2 b_2^2 + b_2^2 b_3^2 + b_3^2 b_1^2) \quad . \tag{7.2}$$

with numerical solution for the simple case of spherical symmetry

$$b_1^2 = b_2^2 = b_3^2 = \sqrt{2} \approx 1.415 \tag{7.3}$$

which confirms a fundamental prediction of the isospecial relativity, that the nucleon is an isominkowskian medium of Type 9 ($\hat{\beta} < \beta$, $\hat{\gamma} > \gamma$, Aver.$(b_k^0) < b_4^0$).

7.3 Isorenormalization of Magnetic Moments. Yet another prediction of the isospecial relativity is that, when ordinary electrons are immersed in the hyperdense medium inside protons, they experience a deformation–isorenormalization of both their orbital and intrinsic magnetic moments, first generically studied in ref. [3], p. 803, and then studied, nonrelativistically, in ref. [12].

The isodirac's equation permits a quantitative, simple and direct treatment of this aspect, too, via Eqs. (6.5) which yield for synthesis (1.1*e*) (for $L=1$ from (7.2), see Fig.1, p. 525, of [12] for orientations)

$$\mu_n = -1.9|e|/2m_p c_0 = \mu_p + \mu_{\hat{e}}^{\text{orb}}| + \mu_{\hat{e}}^{\text{intr}}|, \quad \mu_n = +2.7|e|/2m_p c_0, \tag{7.4a}$$

$$\mu_{\hat{e}}^{\text{tot}} = -4.6|e|/2m_p c_0 = -2.4 \cdot 10^{-3}|e|/2m_e c_0, \tag{7.4b}$$

$$\mu_{\hat{e}}^{\text{intr}} = \left(b_3^0 / b_4^0\right)\mu_e^{\text{intr}} = (1.41/1.65)\mu_e^{\text{intr}} = 0.8545\mu_e^{\text{intr}}, \tag{7.4c}$$

$$\mu_{\hat{e}}^{\text{orb}} = (-0.8545 + 0.0024)\mu_e^{\text{intr}} = -0.8521\mu_e^{\text{orb}} \tag{7.4d}$$

The latter numerical values should not be considered as final because of the need to study the general model $n = (\hat{p}^+ \uparrow \hat{e}^- \downarrow)_{\text{HM}}$ with isorenormalization of the electromagnetic properties of both the proton and the electron (including the charge which is not isorenormalized in this first treatment). Nevertheless, the latter study is expected to yield adjustments of numerical values (7.4).

We can therefore conclude by saying that the isospecial relativity does indeed provide a quantitative representation of the synthesis of protons and electrons into neutrons (plus neutrinos), with all needed, specific, numerical predictions of the quantities involved in a form ready for experimental tests. The extension of the results to other syntheses of particles is here left for brevity to the interested reader.

8. Experimental Verifications

Even though preliminary and in need of independent re–runs, a number of direct and indirect experimental verifications are today available supporting syntheses (1.1).

8.1 Direct Verifications. The first direct experimental verification of the isotopic origin of electron paring in superconductivity has been provided by Animalu [8] with rather impressive phenomenological agreement with data. The best verification of syntheses (1.1*b*)—(1.1*d*) are given by the uniqueness of represented energy levels via model (2.7) (see Sect. 9 for specific tests).

The first direct experimental verification of the synthesis (1.1*e*) was done by don Borghi et al. [15]. The experiments essentially consist in forming a gas of protons and electrons inside a metallic chamber (called clystron) via the electrolytic separation of the hydrogen. Since the protons and electrons are charged, they cannot escape the metallic chamber. Nevertheless, numerous transmutations of nuclei occurred for matter put in the outside of said chamber. The measures can then be solely interpreted, in the absence of any other neutron source, by the synthesis of the protons and electrons into neutrons which, being neutral, can escape the chamber and cause the measured transmutations.

8.2 Verifications via the Bose–Einstein Correlation. The most important indirect verification of cold fusion (1.1*e*) has been recently achieved via theoretical [19] and phenomenological [23] studies on the Bose–Einstein correlation. These results are important because they confirm, not only the fundamental isominkowskian laws underlying synthesis (1.1*e*), but also their numerical values.

In essence, studies conducted via the full use of nonlinear–nonlocal–nonhamiltonian isominkowskian geometrization of the $p-\bar{p}$ fireball result in the twopoint Boson isocorrelation function on $\hat{M}_Q(x,\hat{g},\hat{R})$ [19], Eq. (10.8), p. 122,

$$\hat{C}_{(2)} = 1 + \frac{K^2}{3}\sum_{\mu}\hat{g}_{\mu\mu}(e^{-q_t^2/b_\mu^{02}}), \quad \hat{g} = \text{Diag.}\left(b_1^{02}, b_2^{02}, b_3^{02}, -b_4^{02}\right) \tag{8.1}$$

where q_t is the momentum transfer and $K = b_1^{02} + b_2^{02} + b_3^{02}$ is normalized to 3, under the sole approximation, also assumed in conventional treatments, that the longitudinal and fourth components of the momentum transfer are very small.

Phenomenological studies conducted in [23] via the UA1 data confirm model (8.1) in its entirety, and identify the numerical values.

$$b_1^0 = 0.267 \pm 0.054, \qquad b_2^0 = 0.437 \pm 0.035,$$
$$b_3^0 = 1.661, \qquad b_4^0 = 1.653 \pm 0.015 \tag{8.2}$$

These measures have the following implications: a) They first confirm the non–linear–nonlocal–nonhamiltonian origin of the correlation, which is the expected origin of the synthesis (1.1e); b) They confirm the isominkowskian geometrization of Type 9 ($\hat{\beta} < \beta, \hat{\gamma} > \gamma$, Aver. $(b_k^0) < b_4^0$) for the $p-\bar{p}$ fireball which, having the same density of the proton, is directly applicable to synthesis (1.1e); c) They provide a numerical confirmation of rest energy isorenormalization (7.1) predicted in [12], beyond the best expectation by this author. Also, experimental value $b_4^0 = 1.653$ yields the isorenormalized rest energy $\hat{E}_z = 1.36$ MeV, thus implying the existence of the binding energy $E = -0.072$MeV, which is small, also as predicted.

8.3 Additional Experimental Verifications. Phenomenological calculations of deviations from the Minkowskian geometry inside pions and kaons were conducted in [24] via standard gauge models in the Higgs sector, resulting in the deformed metric $\hat{\eta} = \mathrm{diag.}((1-\alpha/3), (1-\alpha/3), (1-\alpha/3), -(1-\alpha))$ which is precisely of the isominkowskian type (3.11) with numerical values

$$\text{PIONS } \pi^{\pm}: \ b_1^{02} = b_2^{02} = b_3^{02} \simeq 1 + 1.2 \cdot 10^{-3}, \quad b_4^{02} \simeq 1 - 3.79 \cdot 10^{-3} \tag{8.3a}$$

$$\text{KAONS } K^{\pm}: b_1^{02} = b_2^{02} = b_3^{02} \simeq 1 - 2 \cdot 10^{-4}, \quad b_4^{02} \simeq 1 + 6.1 \cdot 10^{-4} \tag{8.3b}$$

Pions $\pi^{\pm}$ are then isominkowskian media of Type 4, while the heavier kaons $K^{\pm}$ are of Type 9 [4] . This confirms measures (8.2) because all hadrons heavier than $K^{\pm}$ are expected to be isominkowskian media of Type 9.

Independent phenomenological plots [25] on the behaviour of the meanlife of the K_S^0 (which, according to current experiments, is anomalous from 30 to 100 GeV and conventional from 100 to 350 GeV) via isominkowskian geometrization [6] , yield the following characteristic values of the K_S^0.

$$b_1^{02} = b_2^{02} = b_3^{02} \approx 0.909080 \pm 0.0004, \quad b_4^{02} \approx 1.002 \pm 0.007, \tag{8.4}$$

which are of the same order of magnitude of values (8.3b). Measures (8.4) therefore provide an independent confirmation that the interior of kaons is indeed an isominkowskian medium of Type 9, and an additional independent confirmation of the isogeometrization needed for synthesis (1.1e). Plots [25] also computed the values

$$\Delta b_k^{02} \simeq 0.007, \quad \Delta b_4^{02} \simeq 0.001. \tag{8.5}$$

This confirms the prediction of the isospecial relativity in the range 30–400 GeV that the b_4 quantity, being a geometrization of the density, is constant for the particle considered (although varying from hadron to hadron with the density), while the dependence in the velocities or, equivalents, in the eugigy rests with the b_k–quantities.

energy

9. Proposed Tests

Physics is a science with an absolute standard of value: the experiments. Experiments themselves have their own standard of value, the more fundamental the law to be tested, the more relevant the experiment. In particular, experiments such as don Borghi's verifications on the synthesis $n = (\hat{p}^+, \hat{e}^-)_{HM}$, can only be dismissed via other experiments, and simply cannot be dismissed in a credible way via theoretical considerations or personal views.

We therefore suggest the independent verification or dismissal of experiment [15] on the fundamental synthesis $p^+ + e^- \rightarrow n + \nu$.This can be first done in a number of ways, such as:

Experiment I: Repeat the experiment as originally done [15];

Experiment II: Hit a beryllium mass saturated with hydrogen and at low temperature with a cathodic ray of the threshold energy of 0.80 MeV①. The synthesis would be confirmed if a measurable flux of neutrons is produced. This latter experiment is particularly significant for the verification of the threshold energy of 0.80 MeV. In turn, such energy is fundamental for the inverse reaction, the stimulated decay of the neutron (see below). In fact, at sufficiently low temperatures, the protons of the hydrogen nuclei can be considered as being at rest in the beryllium chrystal. The threshold energy is then entirely carried by the cathodic ray. The measure of the neutron flux for cathodic rays of various energies below and above 0.80 MeV would then provide this crucial experimental information.

A mature experimental study of the synthesis $p^{+}+e^{-}\rightarrow n+v$ also calls for a re–inspection of existing means for the production of neutrons. As an example, the currently available neutron sources based on beryllium, at a deeper inspection, may well result to be a manifestation of the synthesis of the neutron as proposed in this paper.

Needless to say, the above lines should not be solely restricted to the neutrons. For instance, the synthesis of the neutral pion $e^{+}+e^{-}\rightarrow \pi^{\circ}$ is equally significant and therefore deserving consideration. Numerous additional syntheses predicted by our isotopic generalization of the special relativity can then be studied.

Moreover,we suggest the conduction of experiments on the inverse process which is at the foundation of the hadronic energy [20], the possible stimulated decay of the neutron $\gamma+n\rightarrow p^{+}+e^{-}+\bar{v}$ which is predicted by the isospecial relativity to occur for gammas with excitation frequencies of 3.129×10^{20}Hz (corresponding to the rest energy 1.294 MeV of Rutherford's electron inside the proton) less adjustments for the binding energy. An alternative excitation frequency also deserving experimental test is the value 1.236×10^{20} Hz (corresponding to the rest energy 0.511 MeV of the ordinary electron outside the proton) which is conceivable via conventional weak processes. The following experiments are then recommended;

Experiment III: Hit with gammas of 1.294Mev (and, independently of 0.5111 MeV) a pellet of ${}_{30}Zn^{70}$ as pure as possible (note that gammas of 1.294 or 0.511 MeV are available from commercially sold radioisopes and ${}_{30}Zn^{70}$, being a stable, light, natural element, is also commercially available). The stimulated decay of one of the peripheral neutrons in the Zn–nuclei would then imply the reaction $\gamma+{}_{30}Zn^{70}\rightarrow {}_{31}Ga^{70}+\beta^{-}$. In turn, ${}_{31}Ga^{70}$ is known to be unstable and to have the spontaneous decay in 21min. ${}_{31}Ga^{70}\rightarrow {}_{32}Ge^{70}+\beta^{-}$ where ${}_{32}Ge^{70}$ is also a (rather rare) stable, light, natural element and beta minus has energy ranging from 0.44 to 1.66Mev. The measure via ordinary mass spectrometers of traces of the rare ${}_{32}Ge^{70}$ in the pellet originally of ordinary ${}_{30}Zn^{70}$ after sufficient bombardment would evidently establish beyond scientific doubts the first stimulated transmutation of matter on record. Note that such transmutation is expected to exist only at the sharp excitation frequency with ignorable proba-

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① This experiment has been independently suggested to the author by Y. Oganessian of the JINR, Dubna, Russia, and M. Farouk of the Atomic Energy Research Institute in Riyadh, Saudi Arabia (private communications).

bility to occur in a natural environment.

Experiment IV: Hit a pellet of $_{42}Mo^{100}$ (which is also a stable, light, natural element commercially available in sufficient purity) with gammas of 1.294 MeV (or 0.511 MeV) excitation frequency. The isospecial relativity predicts the stimulated transmutation $\gamma + {}_{42}Mo^{100} \rightarrow {}_{43}Tc^{100} + \beta^-$. Then, $_{43}Tc^{100}$ is also unstable with the spontaneous decay in 18.9 sec $_{43}Tc^{100} \rightarrow {}_{44}Ru^{100} + \beta^-$, where $_{44}Ru^{100}$ is again another (rather rare) stable, light, natural element and the beta minus has energy ranging from 2.2 to 3.8 MeV. Again, measures via ordinary mass spectrometers of traces of the rare $_{44}Ru^{100}$ in the pellet originally of $_{42}Mo^{100}$ after sufficient bombardment with gammas of said excitation frequency would establish the stimulated transmutation here considered.

It should be indicated that the lack of validity of the excitation frequency 3.129×10^{20}Hz does not necessarily mean that the stimulated decay of the neutron does not exist. In fact, that particular frequency has been reached via the isoequivalence principle for the electron inside the proton $E^0 = m_0 \; c_0^2 \; b_4^{02}$ by using the numerical value of b_4^0 obtained from the geometrization of the density of the $p - \bar{p}$ annihilation in the UA1 experiments at CERN [23]. The correct value for b_4^0 should be that obtained via independent experiments, specifically, on the geometrization of the medium inside a proton which is expected to be numerically different than that for the p $\bar{p}$ annihilation at the UA1 energy (evidently because the densities are different). The above indicated excitation frequency is therefore the best estimate possible at this writing which is plausible because the density of the $p - \bar{p}$ annihilation is of the same order of magnitude of the density of the proton. The understanding is that revisions are conceivable when a more precise value of the characteristic quantity b_4^0 of the proton is available. The main point is that the hadronic energy is based on a mechanism which occurs spontaneously in nature: the decay of the neutron. The identification of mechanisms for its stimulated occurrence is then only a matter of time. In particular it is conceivable that such a mechanism exists even if the synthesis of the neutron is disproved experimentally.

Note that all the above transmutations are of the type $(A,Z) \rightarrow (A,Z+1)$ and, therefore, necessarily release energy (evidently because the atomic weight of (A,Z) is bigger than that of $(A, Z+1)$). Also, one original gamma would produce two nuclear electrons of energy in the MeV range (about 10^4—10^5 time the energy of the electrons hitting television or computer screen). The hadronic energy indicated earlier then essentially consists in the capture of these energetic electrons via a metallic screen with dual production of electricity and heat (see [20] for details, including a description of patents pending). Note that the original nuclei of $_{30}Zn^{70}$ or of $_{42}Mo^{100}$ would merely act as receptacles, and this illustrates the subnuclear–hadronic character of the energy. Note finally that all the above proposed experiments are rather simple, particularly when compared to high energy or interferometric experiments, their cost is comparatively minimal with a comparatively much higher potential return, and they can be done in any nuclear laboratory.

Intriguingly, if confirmed, the primary characteristics of such hadronic energy are: 1) Complete lack of the harmful radiations emitted by the nuclear energy. The process implies the transition from a stable, light, natural element to another of the same characteristics but of lower atomic weight, while the only particles produced are the electrons, which are captured to utilize their electricity and kinetic energy, and the harmless neutrinos; 2) No restriction on weight due to super–heavy shields as needed for the nuclear energy. This renders conceivable portable, light–weight realizations; and 3) Lack of the critical mass needed for the nuclear energy. This renders conceivable realizations of the hadronic energy in large or small scale, including miniaturizations.

Experiment V: Measure the total cross section of the reaction $\gamma+n\to p^{+}+e^{-}+\nu$ as a function of the energy of the gamma. The isospecial relativity predicts that such cross section peaks at the excitation energy of 1.294 MeV (again, less adjustments due to the binding energy), with secondary peaks at multiples and submultiples of said excitation energy. Note that this experiment could confirm or deny the fundamental law of the chemical synthesis of particles, the isoequivalence law (4.5) from which all other isotopic laws can be derived via mere compatibility arguments. In particular, the plausibility of law (4.5) should be kept in mind because it merely expresses the expectation that the speed of light c in the hyperdense medium inside a proton is not equal to the speed c_0 in vacuum [26]. For additional, classical and particle experiments, see [4,18].

As stressed in this paper, the above possibilities are not permitted by relativistic quantum mechanics and require its structural generalization for novel physical conditions within the hyperdense media inside hadrons beyond those of its original conception and experimental verification. As well known, the conception of the atomic–nuclear energy at the turn of this century required the necessary relativistic generalization of pre–existing theories. It is hoped this paper has conveyed the evidence that the conception and quantitative treatment of fundamentally novel subnuclear energies now requires a structural generalizations of the pre–existing relativistic theories.

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About the Author

Ruggero Maria Santilli was born and educated in Italy where he received his Ph. D. in theoretical physics in 1967 from the University of Torino. In 1967 he moved with his family to the USA where he held academic positions in various institutions including the Center for Theoretical

Physics of the University of Miami in Florida, the Department of Physics of Boston University, the Center for Theoretical Physics of the Massachusetts Institute of Technology, the Lyman Laboratory of Physics and the Department of Mathematics of Harvard University. He is currently President and Professor of Theoretical Physics at the Institute for Basic Research, which operated in Cambridge from 1983 to 1991 and then moved to Florida and president of the Istituto per la Ricerca di Base, Monteroduni, Italy. Santilli has visited numerous academic institutions in various Countries. He is currently a Honorary Professor of Physics at the Academy of Sciences of the Ukraine, Kiev, and a Visiting Scientist at the Joint Institute for Nuclear Research in Dubna, Russia. Besides being a referee for various journals, Santilli is the founder and editor in chief of the Hadronic Journal (sixteen years of regular publication), the Hadronic Journal Supplement (nine years of regular publication) and Algebras, Groups and Geometries (eleven years of regular publication). Santilli has been the organizer of the five International Workshops on Lie-admissible Formulations (held at Harvard), the co-organizer of five International Workshops on Hadronic Mechanics (held in the USA, Italy and Greece) and of the First International Conference on Nonpotential Interactions and their Lie-Admissible Treatment (held at the Universite d'Orleans, France). He is the author of over one hundred and fifty articles published in numerous physics and mathematics journals; he has written nine research monographs published by Springer-Verlag (in the prestigious series of "Texts and Monographs in Physics"), the Academy of Sciences of Ukraine and other publishers; he has been the editor of over twenty conference proceedings; he is the originator of new branches in mathematics and physics, some of which are studied in this paper; he has received research support from the U. S. Air Force, NASA and the Department of Energy; and he has been the recipient of various honors, including the Gold Medals for Scientific Merits from the Molise Province in Italy and the City of Orleans, France.

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