IC/91/46

INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

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INTERNATIONAL
ATOMIC ENERGY
AGENCY

UNITED NATIONS
EDUCATIONAL,
SCIENTIFIC
AND CULTURAL
ORGANIZATION

1991 MIRAMARE - TRIESTE

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and
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THEORY OF MUTATION OF ELEMENTARY PARTICLES AND ITS APPLICATION TO RAUCH'S EXPERIMENTS ON THE SPINORIAL SYMMETRY

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MIRAMARE – TRIESTE

March 1991

Abstract

In this paper we study an open historical legacy of nuclear physics, according to which the magnetic moment of nucleons could be altered in the transition from motion in vacuum under external electromagnetic interactions (as measured until now), to motion under joint, external, electromagnetic and strong interactions, with a consequential conceivable fluctuation of the spin. The legacy is studied via the construction of the Lie-isotopic generalization of conventional field equations, i.e., generalized equations that are invariant under the Poincaré-isotopic symmetry proposed in a preceding paper. It emerges that in the transition from motion in vacuum under potential interactions, to motion within a physical medium with potential as well as contact non-Hamiltonian interactions, there is, in general, the alteration (called "mutation") of all intrinsic characteristics of particles, such as: rest energy, spin, charge, mean life, space and charge parity, electric and magnetic moments, etc. According to this view, a stable particle such as a proton when in the core of a collapsing star (or a hadronic constituent) may have physical characteristics essentially different than those of the same particle when moving in vacuum. The emerging, generalized, iso-field theory is applied to a direct and quantitative interpretation of Rauch's experimental data according to which thermal neutrons experience a deformation of their charge distributions with consequential alteration of their magnetic moments when under joint, external, electromagnetic and nuclear interactions. We then pass to the review of an intriguing generalization of Dirac's equation proposed by Dirac himself, in which the spin is mutated from $\frac{1}{2}$ to zero. We show that the generalized equation possesses an essential isotopic structure precisely of the class submitted in this work. A number of fundamental implications of the open historical legacy are pointed out. For example, we indicate the truly basic implications for the very conception of the controlled fusion for a possible mutation of the magnetic moments of nucleons precisely at the time of activation of the fusion process. Similarly, we point out the possibility, offered by the hypothesis of mutation of particles, of identifying the ultimate constituents of hadrons (or of quarks) with physical particles simply produced in the spontaneous decays, as studied in details in a subsequent paper of this series. The paper ends with the review of several experiments which have been proposed in the literature for some time, but regrettably ignored until now, for the final resolution of the problem, whether the intrinsic characteristics of particles are rigidly immutable, or they can change under suitable physical conditions.

1 Introduction

One of the basic open legacies of the Founding Father of nuclear physics is the possibility that the magnetic moments of protons and neutrons may change in the transition from electromagnetic to nuclear interactions (see, e.g., ref.s [1,2,3]).

The hypothesis emerged quite naturally when, following the knowledge of the magnetic moments of protons and neutrons, values of the total magnetic moments of nuclei began to be known in the 40's. It was soon evident that quantum mechanics, while so effective in other areas of nuclear physics, could not allow a consistent interpretation of the total magnetic moments of nuclei starting from known values of the magnetic moments of the individual nucleons. In fact, in ref. [1], p.31, one can read

> *"It is possible that the intrinsic magnetism of a nucleon is different when it is in close proximity to another nucleon."*

Similar statements can be found in ref.s [2,3].

As of today, the total magnetic moments of nuclei are still essentially unresolved, despite numerous semiphenomenological corrections (the so-called relativistic, cooperative and spin-orbits effects within the context of the Schmidt limits).

The plausibility of the above legacy is incontrovertible. Protons and neutrons are not point-like particles (as approximated by nonrelativistic and relativistic quantum mechanics), but possess an extended charge distribution with a radius of the order of the range of the strong interactions ($1F = 10^{-13}$ cm). Since perfectly rigid bodies do not exist in the physical reality, we must assume that such charge distribution can experience a deformation of its shape under sufficiently intense external fields. In turn, such deformation *necessarily* implies an alteration of the magnetic moment, as well established in classical mechanics and atomic physics.

The physical relevance of the hypothesis is equally incontrovertible. It is sufficient here to recall the now vexing efforts to achieve the controlled fusion via magnetic (and other) confinements. As well known, these efforts are fundamentally dependent on the established value of the magnetic moments of nucleons. *The historical legacy then establishes the possibility that protons and neutrons experience an alteration of their magnetic moments exactly at the time of initiation of the fusion process. This could imply the invalidation of the very conception and experimental realization of the current magnetic*

confinement apparata precisely at the initiation of the fusion process.

Despite the above well known plausibility and physical relevance, the historical legacy has been ignored in recent times. The apparent reason is that the hypothesis is incompatible with orthodox trends on more than one count. In fact, the hypothesis requires, first, the *representation of hadrons as extended objects* which is known not to be possible within the context

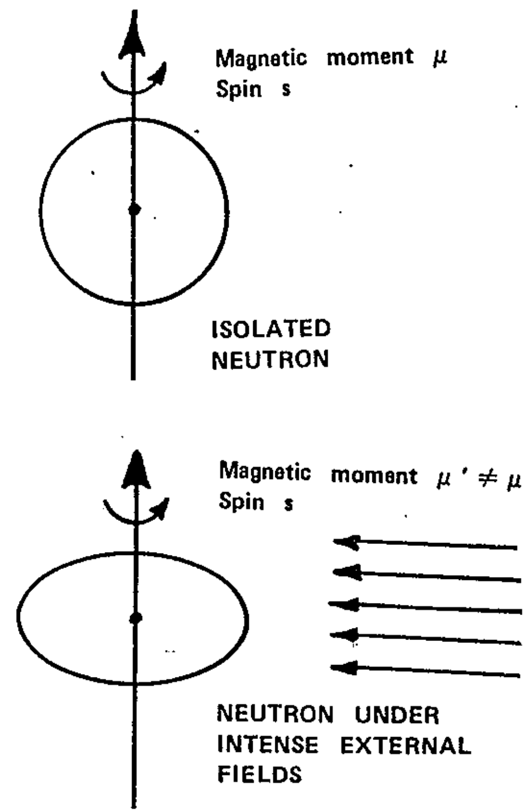

FIGURE 1: The manifest plausibility of the open historical legacy under consideration in this paper: the expected deformability of the charge distribution of hadrons under sufficiently intense external fields, and the consequential alteration of the magnetic moment. This fundamental hypothesis was formulated in the early stages of nuclear physics as soon as the value of the total magnetic moment of nuclei began to be known in the 40's. The underlying physical framework is fully established in atomic physics via experiments proving precisely the alteration of the total magnetic moment of an atom under deformation of its charge distribution caused by external fields. It is then quite plausible to expect a similar alteration at the level of elementary particles. This paper is devoted to a quantitative study of this possibility via the use of the Lie-isotopic generalization of the conventional Lie's theory and the possibility of representing the extended-deformable character of hadrons via the generalized unit of the theory.

of quantum mechanics (without quantum field theoretical extensions). Second, the hypothesis requires the *representation of the deformation of the extended charge distribution of hadrons*, which is beyond the representational capabilities of quantum mechanics, as well as in known conflict with Einstein's Special Relativity. Finally, when truly realized in technical language, the hypothesis leads to an *alteration of the Casimir invariants of the Poincaré group* (as we shall see), thus confirming the inability of orthodox theories of providing a genuine, quantitative representation of the historical open legacy.

Still another fundamental open legacy in nuclear physics is the concept of spin under nuclear interactions (see, also, ref.s [1,2,3]). As well known, the current spin theory is that characterized by the $SU(2)$ symmetry which allows only discrete transitions in the spin value (for instance, a neutron can experience the transition from its spin $\frac{1}{2}$ to a value $\frac{3}{2}$ in which case it becomes one of its possible resonances). However, this rigid setting, when complemented with that of the $SO(3)$ symmetry for the orbital motion, has been unable to reach a final resolution of the problem of the total nuclear angular momentum, particularly for the case of the light nuclei.

This occurrence was well identified in the early (but not the more recent!) treatises in nuclear physics. In fact, in ref. [1], p. 254, one can read:

> *"...the (conventional) theory is not adequate to predict the nuclear spins and magnetic moments."*

Note the evident link of the problem of the total magnetic moment with that of the spin. In fact, any alteration of the magnetic moment of an individual nucleon while a member of a nuclear structure could imply some corresponding form of alteration/fluctuation of the spin.

It should be stressed here that we are referring to possible *internal* alterations/fluctuations of one nucleon in the interior of a nucleus while all other nucleons are considered as external, and under the evident condition that the total quantities are conserved and conventionally quantized.

The plausibility of this spin alteration/fluctuation is also incontrovertible. In fact, at the classical level, the deformation of a charge distribution generally implies a change in the intrinsic angular momentum, and some form of image could well exist at the operator level.

Note that the description of this possibility is beyond the technical capabilities of the conventiona $SO(3)$ symmetry and its $SU(2)$ covering. In fact, these symmetries are defined as isometries of spaces with the Euclidean metric which can at best idealize a perfect sphere. Any deformation of such sphere would imply the breaking of the original symmetry. At any rate, the rotational symmetry is known since its inception to apply, specifically, for *rigid bodies only* while being manifestly broken for the *deformable bodies* under consideration here.

In summary, an inspection of the current status of nuclear physics reveals that, despite truly outstanding advances, the problem of the value of the intrinsic magnetic moment and spin of one individual nucleon when a member of a nuclear structure is still fundamentally unresolved as of now.

It is then easy to see that the problem cannot be restricted to magnetic moments and spins, but must be extended to all intrinsic characteristics of a nucleon (and, therefore, of an elementary particle at large) such as: charge, space and charge parity, rest mass, etc. In fact, the alteration of a Casimir invariant of the Poincaré symmetry would inevitably imply, in general, that of all physical characteristics of a particle.

In the final analysis, we must admit that all available knowledge on the intrinsic characteristics of particles is restricted to the arena of their current measures, under *external electromagnetic interactions*, while no resolutory knowledge exists at this time for the value of the same characteristics under *external strong interactions*. The understanding, to be stressed repetitiously in this paper to minimize possible misrepresentations, is that possible deviations are conceivable only locally in the interior of hadronic matter, while total quantities must remain conventionally quantized.

In this paper we shall continue our studies of these open historical legacies. In particular, we shall study the physical conditions under which the intrinsic characteristics of extended, and therefore deformable particles, can experience an alteration of their value as currently measured until now, under long range electromagnetic interactions. As we shall see, these alterations, while not possible for conventional Hamiltonian theories, became conceivable when certain nonlocal, non-Hamiltonian short-range effects of strong interactions are taken into account.

To state our objective in different terms, let us recall that quantum mechanics and its relativistic extension (e.g., via Dirac's equation) have provided the final characterization of *one particle under external electromagnetic interactions.* In this paper we shall continue our studies on the characterization of *one hadron under external strong interactions.*

The primary purpose of this paper is to submit a generalization of the conventional field theory under the name of *iso-field theory* which admits interactions analytically more general than those of the conventional theory. After a number of draftings and re-draftings in the organization of the material, it became necessary to render this paper selfsufficient, by including a review, not only of the experimental background which is known only by a restricted number of experts, but also of the underlying theoretical and mathematical background which is virtually unknown to the physical audience at large. These reviews also became so essential for the understanding of the theory by the non-initiated reader, to discourage their presentation in appendices.

In the next section we shall review a series of experiments on the spinorial symmetry of thermal neutrons under external nuclear interactions via neutron interferometric techniques conducted by H. Rauch and others [4-14]. Even though far from being final, available experimental data [8,9] appear to confirm the historical legacy on the possible alteration of the magnetic moment of neutrons and, as such, they provide a good experimental ground for theoretical studies. In Sections 4 and 5 we shall review the research done until now on the historical legacy along the conjecture of "mutation" of (massive) particle when in conditions of total or partial immersion within dense hadronic matter [15]. As we shall see, these studies have lead to a certain generalization of the Poincaré symmetry called *Poincaré-isotopic* [41] which can indeed represent extended and deformable particles via a generalized form of the unit of the underlying algebra and related methodology (universal enveloping algebra, etc.). This Poincaré-isotopic symmetry is locally isomorphic to the conventional symmetry under certain topological restrictions on the generalized unit. Nevertheless, it implies the existence of

generalized forms of the Casimir invariants, exactly as desired.

In Section 5 and 6 we shall be finally in a position to present the so-called isotopic generalization of the conventional Klein Gordon and Dirac's equation, respectively, which is needed to reach covariance under the Poincaré-isotopic symmetry. The generalized equations show quite clearly the existence of mutations, that is, of alteration of all intrinsic characteristics of particles under the conditions indicated above, with the understanding that such mutation has no physical or mathematical ground when the particle moves in vacuum under long range electromagnetic interactions.

In Section 7 we show that the theory provides a direct representation of the currently available data on Rauch's experiments [8,9]. In particular, we show that, if taken "prima facie", these data imply a breaking of the $SU(2)$-spin symmetry for neutrons under external nuclear interactions. Despite that, we prove that the $SU(2)$-spin symmetry remains exact at the covering isotopic level, as it occurred for virtually all isotopic space-time symmetries studied until now.

In Sect. 8 we enter into what may be the most intriguing part of our analysis, which is a mere *review* of a generalization of Dirac's equation identified by P.A.M. Dirac himself in two of his last papers [60,61]. These papers have remained largely ignored in the literature perhaps because of the manifest lack of alignment of the results with contemporary trends. In fact, Dirac's generalization of his celebrated equation implies the mutation of the spin from $\frac{1}{2}$ down to zero! Most intriguingly, the generalized equation possesses an essential isotopic structure of which Dirac was unaware. These pioneering results by Dirac, not only confirm the historical legacy reviewed earlier [1-3], Rauch's experimental backing [4-14] and its Lie-isotopic structure [15-55], but open up the way to potentially fundamental advances, such as the possibility of identifying the constituents of quarks/hadrons with physical, already known particles. This latter possibility is investigated elsewhere [32].

In the final section we review a number of experiments proposed in the literature since some time, but regrettably ignored until now, which were conceived precisely for the final experimental resolution of this fundamental problem of contemporary nuclear and particle physics: whether the intrinsic characteristics of particles are rigidly immutable, or they can be altered by suitable physical conditions. Needless to say, the analysis of this paper is presented in the scientific spirit of attempting novel advances via the traditional process of trial and error. As such, it must be considered strictly an exercise of scientific curiosity, and it must remain so until the spinorial symmetry of neutrons under external nuclear interactions is experimentally resolved in a final way.

2 Rauch's Experiments on the Spinorial Symmetry of Neutrons.

A number of pioneering direct tests of the possible alteration of magnetic moments and spins of nucleons have been conducted by H. Rauch and his collaborators [4-9]. The experiments test the 4π spinor symmetry of neutrons via extremely sensitive interferometers on a low energy (thermal) neutron beam experiencing precession due to a long range magnetic field, while the beam is jointly under the action of a short range nuclear field.

It should be stressed from the outset that *Rauch's experiments [4-9] are unable to recover the exact spinorial symmetry as predicted by (nonrelativistic and relativistic) quantum mechanics.* As such, the tests constitute the experimental basis of our research.

We should equally stress from the outset that Rauch's experiments, in their currently available form, are far from being final. In fact, the resolution of the issue will require the repetition of the tests in a sufficient number of differentiated and independent forms.

Due to the importance of Rauch's experiments for this paper, it is recommendable to review briefly the experimental apparatus and the theoretical framework used in the elaboration of the results.

With reference to Figure 2, a *perfect crystal neutron interferometer* is essentially constituted by a neutron beam which is subjected to a coherent splitting into two branches via a perfect crystal. The neutron beam is generally monochromatic, unpolarized, and with high flux. The use of a perfect crystal (generally a Si crystal with extremely low impurities) allows a wide angle of separation of the two branches. In turn, this allows the application of an (electro-) magnet for spin precession (spin flip) to one of the two branches of the beam without affecting the other.

Typical experimental characteristics are the following: the beam cross section is of the order of $2 \times 1.5 mm^2$; the characteristic wavelength of the crystal is 1.83 $\mathring{A}$; electromagnet gaps are of the order of 1 cm; and the value of the magnetic induction to produce two complete spin flips, under the assumption of the conventional value of the neutron's magnetic moment, is 7,496 G. The stray fields outside the electromagnet gap are quite pronounced, and depend on the magnetization of the yoke. To overcome this

problem, the experimenters filled up the electromagnet gap in the latest tests [7-8] with Mu-metal sheets. It is this feature that renders the experiments of fundamental character inasmuch as they test the spinorial symmetry of the neutrons while in interaction with external Mu-metal nuclei of the fixed target.

Intriguingly, the occurrence is rather accidental, because the experimenters filled-up the electromagnet gap with Mu-metal to reduce the stray fields, and without having in mind a test of the spinorial symmetry under strong interactions. Yet, it is precisely the *joint* action on the neutron beam of external electromagnetic and strong nuclear interactions that render the experiments truly fundamental.

The theory needed for the data elaboration is presented in detain in a number of papers (see, e.g., ref.s [5,10,11]). The fundamental tool used in the data elaboration is the assumption of the exact character of the $SU(2)$-spin symmetry for neutrons, which can be expressed via the transformation of spinorial wavefunctions of neutrons

$$\Psi' = e^{-i\vec{\sigma}\cdot\frac{\vec{\alpha}}{2}}\Psi, \tag{2.1}$$

where the $\vec{\sigma}$'s are the familiar Pauli's spin matrices, and $\vec{\alpha}$ is the angle of rotation depending on the external magnetic field as well as the intrinsic magnetic moment of the neutron.

With reference again to Figure 2, let Ψ be the wavefunction of the original neutron beams, and let Ψ^I and Ψ^{II} be the wavefunctions of the two branches of the beam after separation. Let Ψ_0, Ψ_0^I and $\Psi_0^{II}(\Psi_H, \Psi_H^I$ and $\Psi_H^{II})$ be the wavefunctions of the exiting beams along direction 0 (H). We evidently have

$$\Psi_0 = \Psi_0^I + \Psi_0^{II}\ , \quad \Psi_H = \Psi_H^I + \Psi_H^{II}, \tag{2.2}$$

with $\Psi_0^I = \Psi_0^{II}$ under no phase shift. Suppose now we have a joint spin precession and a nuclear interaction in beam I only. Then, law (2.1) becomes

$$\Psi_0^{I'} = e^{i\chi}e^{-i\vec{\sigma}\cdot\frac{\vec{\alpha}}{2}}\Psi_0^I\ , \quad \Psi_H^{II'} = \Psi_H^{II} \tag{2.3}$$

where $\vec{\alpha}$ is the magnetic phase shift, and

$$\chi = -N\lambda b_c D \tag{2.4}$$

is the nuclear phase shift, where: N is the number of nuclei per unit volume; λ is the neutron wavelength; b_c is the bounded coherent scattering length;

and D is the thickness passed by neutrons (the Mu-metal thickness in the electromagnet gap).

We therefore have

$$\Psi'_0 = \Psi_0^{I'} + \Psi_0^{II'} = \left(e^{i\chi}e^{-i\vec{\sigma}\cdot\frac{\vec{\alpha}}{2}} + 1\right)\Psi_0^I \stackrel{\text{def}}{=} (U+1)\Psi_0^I . \tag{2.5}$$

The *intensity modulation* of the forward beam is then given by

$$\begin{aligned} I &= \Psi_0^{I'\dagger}\Psi_0^{I'} = \Psi_0^I(U^\dagger + 1)(U+1)\Psi_0^I \\ &= \frac{I}{2}\left(1 + \cos\chi\cos\frac{\alpha}{2} + \vec{\alpha}\cdot\vec{P} + \sin\chi\sin\frac{\alpha}{2}\right) , \end{aligned} \tag{2.6}$$

$$\alpha = |\,\vec{\alpha}\,| ,$$

where $I = \Psi_0^\dagger\Psi_0$ is the original, unmodulated intensity, and

$$\vec{P} = \frac{\Psi_0^\dagger\,\vec{\sigma}\,\Psi_0}{\Psi_0^\dagger\Psi_0} \tag{2.7}$$

is the *polarization.*

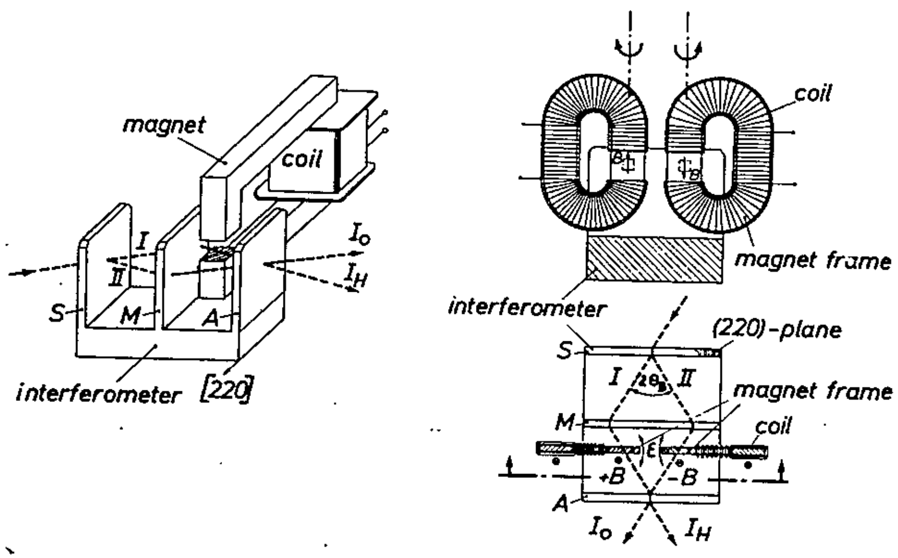

FIGURE 2: A view of the perfect crystal neutron interferometric apparatus used by Rauch and his collaborators [4-9] to test the spinorial symmetry of neutrons under external nuclear fields. The figure shows the electromagnet gap in air, although that used for the latest tests was filled up with Mu-metal sheets to reduce stray fields. This turned the experiments into a test of the $SU(2)$- spin symmetry under joint external, electromagnetic and strong/nuclear interactions. Recall that all experimental information achieved during this century on elementary particles was achieved via *external electromagnetic interactions.*The historical aspect of Rauch's experiments is that they are among the first to achieve *measure under external strong interactions,* where the external character is evidently established by the fact that the Mu-metal sheets are fixed and external with respect to the neutron beam. In turn, this external character of the strong interactions renders Rauch's experiments the first performed for the ultimate characterization of one hadron, as we shall elaborate better in the subsequent sections.

The *polarization modulation* is then given by, after some algebra,

$$\vec{P}' = \frac{1}{2I'}\left\{\left(\cos^2\frac{\alpha}{2} + \cos\frac{\alpha}{2}\cos\chi\right)\vec{P}\right. \tag{2.8}$$

$$\left. + \left[(\sin^2\frac{\alpha}{2})\hat{\alpha}\cdot\vec{P} + \sin\chi\sin\frac{\alpha}{2}\right]\hat{\alpha} + \left(\cos\chi\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\sin\frac{\alpha}{2}\right)\hat{\alpha}\wedge\vec{P}\right\} ,$$

$$\hat{\alpha} = \vec{\alpha}\,/|\,\vec{\alpha}\,| .$$

In Rauch's experiments, the neutron beams are unpolarized, $\vec{P}=\vec{0}$, in which case the intensity modulation and the polarization modulation are given by

$$I'_{\vec{P}=\vec{0}} = \frac{1}{2}(1 + \cos\chi\cos\frac{\alpha}{2})I , \tag{2.9.a}$$

$$\vec{P}'_{\vec{P}=\vec{0}} = \frac{\sin\chi\sin\frac{\alpha}{2}}{1 + \cos\chi\cos\frac{\alpha}{2}}\hat{\alpha} . \tag{2.9.b}$$

In summary, the assumption of the exact validity of the $SU(2)$-spin symmetry for neutrons under joint, external, electromagnetic and nuclear interactions has the following implications each of which can be subjected to direct tests:

A) The intensity is modulated according to law (2.9a), that is, according to a co-sinusoidal behavior depending explicitly on half of the angle of the magnetic precession;

B) The polarization is modulated according to law (2.9b), that is, according to a law which also depends explicitly on half of the angle of the magnetic precession; and

C) The phase lag between the intensity and the polarization modulation is exactly $\pi/2$. In fact, the polarization is null (maximal) when the intensity is maximal (null), and vice versa.

Rauch and collaborators focused on the property that the intensity modulation (2.9a) is absent for $\alpha = 4\pi$. Since intensities and their modulations can be measured accurately, so is the 4π symmetry of spinors.

The results of tests [4-9] are the following. First, the experiments were conducted with an air gap, yielding the median angle with related errors [4]

$$\alpha = 704^\circ \pm 38^\circ, \tag{2.10}$$

$$\alpha_{\mathrm{Max}} = 742^\circ\ , \quad \alpha_{\mathrm{Min}} = 660^\circ .$$

The uncertainty was primarily related to that of the magnetic field along the neutron path.

The tests were subsequently repeated also with an air gap, resulting in values [5]

$$\alpha = 697.5^\circ \pm 16^\circ, \tag{2.11}$$

$$\alpha_{\mathrm{Max}} = 713.6^\circ\ , \quad \alpha_{\mathrm{Min}} = 681.5^\circ .$$

Finally, the experimenters filled-up the electromagnet gap with Mu-metal sheets to reduce the stray fields and related uncertainty of the magnetic field along the neutron path. This produced the following data [6]

$$\alpha = 716.8^\circ \pm 3.8^\circ, \tag{2.12}$$

$$\alpha_{\mathrm{Max}} = 720.6^\circ\ , \quad \alpha_{\mathrm{Min}} = 713.0^\circ .$$

which were subsequently refined in the following values which are the best available at this moment [8,9]

$$\alpha = 715.87^\circ \pm 3.8^\circ, \tag{2.13}$$

$$\alpha_{\mathrm{Max}} = 719.67^\circ\ , \quad \alpha_{\mathrm{Min}} = 712.07^\circ .$$

Additional measures were performed for the intensity and polarization modulation as reproduced in Figure 3.

The essential results of the experiments are the following.

1. *The experiments do not confirm the exact nature of the (conventional) $SU(2)$-spin symmetry in their currently available form.* In fact, the angle 720° needed to confirm the symmetry is not contained in the best available, minimal and maximal values (2.13).

2. *None of the median angles coincides with the value 720° needed to establish the exact $SU(2)$-spin symmetry.* In different words, to truly establish the exact $SU(2)$-spin symmetry, it is not sufficient that the value 720° is comprised within the experimental errors, but must be sufficiently well approximated by the *median* value of the measured angle. In all experimental data [4-9] the median angle is always significantly lower than the value 720° by an order of: 2.2% for value (2.10): 3.1% for values (2.11); and 0.6% for values (2.13). Notice also that *the median angle is consistently smaller than 720° for all experiments conducted until now.* This aspect too shall have its relevance later on in the theoretical interpretation of the results.

3. *The measurements of the intensity modulation for angle (2.13) do not confirm the exact $SU(2)$-spin symmetry because of a number of additional reasons*, such as the shape of the curves of Figure 3 (which is sensibly different than a co-sinusoid), and others

Additional, independent experiments have been conducted via a wave-front division interferometer [12], the so-called NMR-system[13], and other means[14]. Nevertheless, these experiments were conducted in the 70's and have statistical errors similar to those of the early versions of Rauch's experiments. As such, they do not have a significant impact in the best available data, Eq. (2.13).

In conclusion, in their currently available form, Rauch's experiments confirm the plausibility of sufficiently small alterations/fluctuations of the magnetic moment and spin of neutrons under external electromagnetic and nuclear interactions, without establishing it. The final experimental resolution of the issue, whether in favor or against established laws, will require a significant number of additional tests.

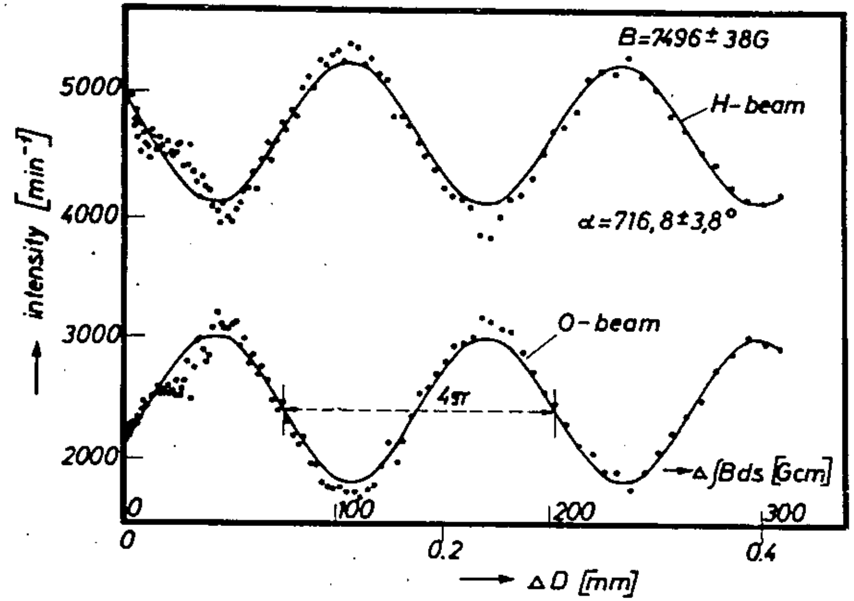

FIGURE 3: The plotting for the intensity modulation of experiments [6] which yields value (2.12). This value was subsequently refined in ref. [8] as a result of the subsequent availability of more precise measurements related to the apparatus and to universal constants. Values (2.13) were then confirmed in ref. [9]. Note that even for value (2.12), which contains 720° within the maximal and minimal value of the experimental error, the median angle is clearly lower than 720°. Value (2.12) alone, and prior to refinement (2.13), therefore shows an apparent violation of the $SU(2)$-spin symmetry, and the same situation occurs for all tests conducted until now. It should be anticipated here from the analysis of the subsequent sections that this apparent breaking occurs only when the $SU(2)$ symmetry is realized via the simplest conceivable Lie product $AB - BA$. On the contrary, the $SU(2)$ symmetry will be shown to remain exact for all values (2.10)-(2.13) when realized with a more general Lie product of the isotopic type $AgB - BgA$, where g represents precisely the deformed shape of the neutrons.

This paper will achieve a primary objective if it succeeds in stimulating the resumption of the experiments (the last one by Rauch actually performed in laboratory being that of 1978 [7]), and in the final experimental resolution of such fundamental topic of contemporary physics.

3 The Conceptual Foundations

In 1978 [15] we submitted the hypothesis of *mutation of elementary particles*, that is, the conjecture that the intrinsic characteristics of massive particles are altered ("mutated") in the transition from motion in vacuum under external electromagnetic interactions (as measured until now), to motion within the medium in the interior of nuclei, hadrons and stars (the "hadronic medium" composed by overlapping wave-packets of hadrons). A necessary condition of the hypothesis is therefore the (partial or total) immersion of the wavepackets of the particles considered within hadronic matter. The hypothesis was nothing but an extrapolation of the historical open legacies of Section 1, from the possible mutation of the magnetic moments and spins to those of all remaining characteristics of particles.

The hypothesis of mutation was submitted for the purpose of studying the most general possible realization of the strong interactions. Recall that, for motion in vacuum under external electromagnetic interactions, we have local, action-at-a-distance, potential interactions under which only the *kinematical* characteristics of particles can change (energy, linear momentum, and angular momentum). This is the case, for instance, for an electron when passing from one atomic orbit to another. For motion within hadronic matter, we have in addition the contact interactions due to the wave-overlapping which, at the classic level are:

1. *Non-local*, in the sense that the interactions occur throughout the volume of mutual wave-overlapping and cannot be reduced to a finite number of isolated points [16,17];

2. *Non-Hamiltonian*, in the sense that they violate the integrability conditions for the existence of a Hamiltonian in the frame of the observer (the so-called *conditions of variational self-adjointness* [18]); and

3. of *zero-range* or *instantaneous character*, in the sense that they occur at the instant of "contact" [19].

Similar characteristics are then expected to persist, for consistency, at the operator level.

Note that, besides conventional, local-potential terms, sufficiently small contact, non-Hamiltonian and instantaneous terms are conceivable for all strong interactions. In fact, all hadrons have a wavepacket and a charge distribution of the order of the range of the strong interactions (about $1F = 10^{-13}$ cm). A necessary condition to activate strong interactions is therefore that hadrons enter into a state of mutual penetration of their wavepackets and charge distributions. This overlapping is minimal in the *nuclear structure*, being of the order of about 1% of the volume of nucleons (as easily established when certain nuclear volumes are compared to those of individual nucleons). As a result, we expect minimal mutations in nuclear physics. In the transition to the *hadronic structure* we expect proportionally larger mutations, because we wave 100% mutual overlapping of the wavepackets of the hadronic constituents (as well known from quantum mechanics, the minimal size of the wavepacket of a massive particle is precisely of the order of the dimension of all hadrons). In the transition to the *star structure* we expect proportionately even larger mutations because, in addition to 100% mutual immersion of the wavepackets, we have their compression, as typically the case for particles in the core of a collapsing star. (See ref. [15] for additional considerations on this hierarchy of physical conditions).

Note that contact non-Hamiltonian and instantaneous interactions are also expected in the absence of strong interactions, e.g., for leptons when penetrating within dense hadronic matter. This is the reason why the hypothesis of mutation was formulated for any massive particle moving "within hadronic matter", rather than solely for hadrons under strong interactions.

As an example, when a proton is in the core of a collapsing star, its spin could conceivably mutate because the conditions of total immersion and compression within hyperdense hadronic matter simply do not allow the particle to spin as freely as when it constitutes the nucleus of the hydrogen atom [15]. A proportionately smaller mutation is then expected when the same proton is a member of a nucleus, exactly along the historical legacy of Section 1. A similar mutation is conjectured for the ordinary electrons when penetrating within a hadron. In fact, the name *eletons* is submitted in ref. [15], p. 613, to characterize electrons when immersed within hadronic matter, in order to differentiate them from the same particles when moving in vacuum.

As we shall see, the mutation of intrinsic characteristics of massive particles is theoretically realizable primarily because of the *non-Hamiltonian* character of the interactions considered. In fact, such character requires a

generalization of the time evolution law via a suitable generalization of the Lie product. This, in turn, implies an alteration of the space-time symmetries which results precisely in a mutation of the intrinsic characteristics. However, as deeper studies have shown, the contact and instantaneous character of the interactions considered are sufficient, individually, to imply a mutation of the intrinsic characteristics.

The first concept we would like to stress to avoid misrepresentations of this work is that *no mutation is conceivable for conventional realizations of strong interactions (e.g., as in QCD), trivially, because of their strictly local-potential nature.* In order to actually reach a mutation, the very analytic structure of the strong interactions must be generalized, as we shall see.

Another concept which is important to avoid misrepresentations, is that *mutations are conceivable only as internal effects in the interior of nuclei, hadrons or stars, and they are not detectable from the outside, because the total characteristics of an isolated bound state must remain conserved and conventionally quantized.* As an example, suppose that Rauch's experiments (Sect. 2) will eventually establish a (small) mutation of the spin of a thermal neutron when in interactions with a Mu-metal nucleus. This occurrence is conceivable if and only if the Mu-metal nucleus experiences a joint spin mutation such that the total angular momentum of the system composed by the thermal neutron and the Mu- metal nucleus is conventionally quantized without mutation.

The compatibility of internal mutations with conventional total characteristics is studied in ref. [15], p. 622, via the notion of *closed non-Hamiltonian systems.*. In essence, conventional total conservation laws for isolated systems can occur, not only for potential internal forces, but also under contact, non- Hamiltonian internal effects. One merely has internal exchanges, not only of the energy, but also of all possible physical quantities, in such a way that total quantities are conserved and conventionally quantized. A classical example of closed non- Hamiltonian systems is provided, say, by Jupiter when isolated from the rest of the Solar system. In fact, its center-of-mass motion verifies all conventional space-time symmetries. Nevertheless, the interior dynamics is essentially non-Hamiltonian with local nonconservation of the angular momentum as well as of other physical quantities. The structure of nuclei, hadrons and stars is then conjectured in ref. [15] as being an operator image of the structure of Jupiter.

The above ideas of 1978 were submitted to a systematic study by a number of authors in subsequent years. The first elaboration of the hypothesis of spin mutation is presented in ref. [20] which also identifies some of the implications suitable for experimental tests (e.g., test Pauli's exclusion principle under *external* nuclear interactions). Unfortunately, the authors of ref. [20] were unaware of Rauch's experiments [4-9] at the time of writing their paper (1979). As a consequence, the spin mutation is not interpreted via Rauch's data in paper [20]. This situation is corrected in paper [21], which presents a first, quantitative treatment of Rauch's experiments via the hypothesis of spin mutation of the preceding papers [20,15]. Independent investigations along similar lines were presented in ref.s [22,23], which provide another quantitative treatment of Rauch's data via a form of spin mutation called "fluctuation".

The consistency of closed non-Hamiltonian systems at the classical level is studied in the original proposal [15]. A comprehensive analytic investigation is presented in monograph [19], while the consistency at the statistical level is studied in paper [24].

In regard to the operator level, the construction of the so-called *hadronic generalization of quantum mechanics* (or *hadronic mechanics* for short) was suggested in ref. [15] as one way to achieve a quantitative treatment of the hypothesis of mutation and related aspects, such as the notion of eletons, and that of operator-type closed non-Hamiltonian systems. This proposal too has been subjected to systematic studies by a number of authors (see, e.g., Proceedings [25,26,27]). We mention in particular, the Hilbert space formulation of the Lie-isotopic branch of hadronic mechanics [28] and that of the more general Lie- admissible branch [29]. A general formulation of the Lie-isotopic branch of hadronic mechanics can be found in paper [30]. In particular, this paper proves the compatibility of conventional laws for the center-of-mass of closed non-Hamiltonian systems (e.g., Heisenberg's uncertainty) with generalized laws for the individual constituents. Similar occurrences are expected for all other physical laws, such as Pauli's exclusion principle [31].

As a final introductory comment, the reader should be aware that *the hypothesis of mutation of elementary particles appears to allow the identification of the hadronic (or quark) constituents with physical, already known particles, only existing in a mutated state due to the conditions of deep mutual overlapping.* This was, after all, the objective for which the hypothesis of mutation was submitted in the first place [15]. In fact, several known arguments indicate that quarks are not elementary, but composite states of ultimate, elementary, constituents. The use of conventional quantum mechanics and related conventional intrinsic characteristics, then requires the invention of a new generation of undetected particles as the elementary constituents of hadrons. The use instead of the hypothesis of mutation and underlying methodology (hadronic mechanics) appears to permit the identification of quark constituents with already known particles, such as the ordinary electrons and positrons, but again existing in a mutated state (eletons) as discussed in the forthcoming paper [32].

Rather than being in conflict with established theories of strong interactions, the hypothesis of mutation of the intrinsic characteristics of particles and related methodology appears to offer some genuine new possibilities of resolving some of their still open problematic aspects, provided, of course, that the hypothesis is developed up to its full mathematical and experimental maturity.

4 The Mathematical Foundations

As well known, the contemporary notion of particle with immutable intrinsic characteristics is a manifestation of the fundamental space-time symmetries of contemporary physics: the *rotational symmetry*, the *Galilei symmetry* for the nonrelativistic case, and the *Lorentz (Poincaré) symmetry* for the relativistic behavior.

The hypothesis of mutation of particles was submitted in ref. [15] following the so-called *Lie-admissible generalization of the Galilei symmetry* submitted in the preceding memoir [33]. For the case of the time evolution, the generalization is essentially based in the transition from the conventional Heisenberg's equations

$$\begin{aligned} i\dot{A} &= [A,H] \stackrel{\text{def}}{=} AH - HA\,, \quad \hbar = 1\,, \quad H = H^\dagger\,, && (4.1.a)\\ A(t) &= e^{+iHt}A(0)e^{-itH} && (4.1.b)\\ &\equiv I\, e^{+iHt}A(0)\, e^{-itH}\, I\,, \\ I &= \mathrm{Diag}(1,1,\ldots,1)\,, && (4.1.c) \end{aligned}$$

to their covering, Lie-admissible form (ref. [15], p. 746)

$$\begin{aligned} i\dot{A} &= (A,H) \stackrel{\text{def}}{=} ARH - HSA \stackrel{\text{def}}{=} A \triangleleft H - H \triangleright A\,, && (4.2.a)\\ A(t) &= e^{iHSt}A(0)e^{-itRH} && (4.2.b)\\ &\equiv I^{\triangleright} e^{iH\triangleright t} \triangleright A(0) \triangleleft e^{-it\triangleleft H\triangleleft} I\,, \\ I^{\triangleright} &= S^{-1}\,, \quad {}^{\triangleleft}I = R^{-1}\,, \end{aligned}$$

with corresponding generalized form of Schrödinger's equations [28,34]

$$
\begin{aligned}
i\frac{\partial}{\partial t}\Psi &= HS\Psi \equiv H \triangleright \Psi \,, & (4.3.a)\\
-i\Psi\frac{\overleftarrow{\partial}}{\partial t} &= \Psi R \overleftarrow{H} \equiv \Psi \triangleleft \overleftarrow{H} \,, & (4.3.b)\\
H &= H^{\dagger}, \quad R^{\dagger} = S \,.
\end{aligned}
$$

The brackets (A,B) of Eq. (4.2) characterize a (nonassociative) *Lie-admissible algebra* U because the attached algebra U^-, which is the same vector space as U equipped with the brackets

$$U^- : [A.B]_U = (A,B) - (B,A) = ATB - BTA \,, \qquad (4.4)$$

$$T = R + S \,,$$

satisfies the Lie algebra axioms, as the reader is encouraged to verify [15].

Note that the attached algebra U^- is not a conventional Lie algebra, but instead an algebra called in ref. [33] *Lie-isotopic* (for certain historical reasons reported in the quoted memoir). These are algebras which coincide, as vector spaces, with ordinary Lie algebras, but are equipped with generalized brackets which preserve, by central assumption, the Lie algebra axioms.

It should be pointed out from the outset for subsequent needs that *the nonassociativity of Eq.s (4.2) is referred, specifically, to the brackets of the time evolution and not to the underlying envelope which remains associative.* To see this point, consider the universal enveloping associative algebra $\mathcal{A}$ underlying Heiselberg's equations

$$\mathcal{A} : AB \,, \quad IA = AI = A \,, \quad \forall A \in \mathcal{A} \,. \qquad (4.5)$$

The covering equations (4.2) were constructed by generalizing, first, the algebra $\mathcal{A}$ into the so-called *associative-isotopic* form [33] underlying the intermediary, Lie- isotopic structure(4.4)

$$
\begin{aligned}
\hat{\mathcal{A}} \;:\;& A * B \stackrel{\text{def}}{=} ATB \,,\; T = \text{fixed and invertible} \,,\\
& \hat{I} * A = A * \hat{I} = A \,,\; \forall A \in \hat{\mathcal{A}} \,, \quad \hat{I} = T^{-1} \,, \qquad (4.6)
\end{aligned}
$$

and then by differentiating the envelopes for the action to the right and to the left into the corresponding forms

$$
\begin{aligned}
\mathcal{A}^{\triangleright} \;&:\; A \triangleright B \stackrel{\text{def}}{=} ASB \,, \quad I^{\triangleright} \triangleright A = A \triangleright I^{\triangleright} = A \,, \quad \forall A \in \mathcal{A}^{\triangleright} \,,\; I^{\triangleright} = S^{-1} \,,\\
{}^{\triangleleft}\mathcal{A} \;&:\; A \triangleleft B \stackrel{\text{def}}{=} ARB \,, \quad {}^{\triangleleft}I \triangleleft A = A \triangleleft {}^{\triangleleft}I = A \,,\; \forall A \in {}^{\triangleleft}\mathcal{A}, {}^{\triangleleft}I = R^{-1} \,, (4.7)
\end{aligned}
$$

which are, individually, associative-isotopic, i.e., they individually verify the associative law.

In the subsequent paper [35] the structure of Eq.s (4.2) was identified in more details as being that of a *Lie-admissible bimodule.* In essence, the structure of Heisenberg's equations is that of a trivial Lie module, in the sense that the modular action is the trivial associative multiplication $A\psi$ for $A \in \mathcal{A}$ and ψ and element of a Hilbert space $\mathcal{H}$, while the right and left modules are trivially related. In the transition to the covering equations (4.2) we have, first, a generalization of the right modular action of associative-isotopic nature $A \triangleright \psi$, and then a nontrivial differentiation between the right and left modules.

A corresponding generalization of the notion of representation was also indicated. In the transition from the one-dimensional unitary group of time evolution (4.1b) to its covering (4.2b) we have the transition from conventional, modular, *representations of Lie algebras* for Eq. (4.1a), or *groups* for Eq. (4.1b) to *bi-representations of Lie-admissible algebras* for Eq. (4.2a) or *groups* for Eq. (4.2b) [35]. These ideas were then investigated in more details in monographs [36,37].

The central mathematical concept underlying this paper can therefore be expressed as follows:

> *POSTULATE 4.1 [15]: An eleton (or quark, or hadronic constituent when all other constituents are considered as external) is a bi-representation of a Lie-admissible algebra with inequivalent actions to the right and to the left (forward and backward in time).*

The mutation of the intrinsic characteristics of particles was originally conceived as being due precisely to the above reviewed generalization of the enveloping associative algebra of quantum mechanics, as subsequent studies [20-23] proved to be correct.

An understanding of the above mathematical structures is essential to avoid misrepresentations of the mathematical content of this work.

We now pass to an outline of the essential physical notions underlying Postulate 4.1. *The Lie-admissible generalization of Heisenberg's equations was submitted in ref. [15] to represent one (massive) particle under the most general possible, external interactions, that is, under the most general possible nonconservative conditions caused by any combination of local-potential and non-local non-Hamiltonian external forces.* An understanding of this physical context is also essential to avoid misrepresentations of the physical profile of this paper.

First, the representation of conventional, local-potential interactions is trivially achieved by Eq. (4.2) via the Hamiltonian H which represents any desired conventional model.

But, besides the Hamiltonian H, Eq.s (4.2) possess two additional operators, the isotopic operators S and R for the forward and backward motion in time. Their first implication is that of characterizing *the nonconservation of the energy H of the particle considered* according to the self-explanatory algorithm

$$i\dot{H} = (H, H) = H(\triangleleft - \triangleright)H = H(S - R)H \neq 0 . \tag{4.8}$$

Thus, the maximization of the interactions experienced by the particle considered was achieved via the maximization of its nonconservative conditions. Mutation of the intrinsic characteristics was consequential, as we shall see. In different terms, Eq.s (4.2) characterizes the *time-rate-of-variation of physical quantities.* Conservation is recovered as a particular case when forward and backward time evolutions coincide ($R = S$).

The *non-Hamiltonian* character of the interactions considered is also readily seen by the fact that the operator S and R *multiply* the Hamiltonian H in Eq.s (4.2) (rather than being additive to it, as in conventional models).

Finally, the *non-local* character is represented by the explicit forms of the units $I^{\triangleright}$ and ${}^{\triangleleft}I$ which are generally of integro-differential nature, as illustrated in ref. [32].

To complete these introductory comments it is important to identify the connection between Eq.s (4.2) and the conventional treatment of *dissipative effects in nuclear physics* (see, e.g., ref.s [1,2,3]). The latter are traditionally realized by generalizing the (conserved, Hermitean) Hamiltonian H into nonconserved, non-Hermitean forms of the type

$$H \longrightarrow H_T = H + H_D \quad , \quad H_D^\dagger \neq H_D , \tag{4.9}$$

where, in general, H_D is of the imaginary form $iV(r, \dot{r}, \ldots)$.

Heisenberg's equations (4.1) are then generalized into the non-unitary form

$$i\dot{A} = AH_T^\dagger - H_T A . \tag{4.10}$$

The following comments are in order. First and above all, *representation (4.10) of nonconservative systems (4.9) does not possess a consistent algebraic structure.* This is due to the fact that the emerging generalized brackets

$$A \circ B \stackrel{\text{def}}{=} AB^\dagger - BA , \quad B^\dagger \neq B , \tag{4.11}$$

violate the conditions to characterize an algebra (right and left distributive laws and the scalar law) and, as such, they do not characterize an algebra as commonly understood in contemporary mathematics.

The implications of this occurrence are rather deep. *Conventional formulation (4.10) of dissipation cannot provide a characterization of the mutation of physical quantities, such as the spin, because the original, perfectly defined $SU(2)$-spin algebra cannot be replaced with a consistent algebraic structure.* As a consequence, fundamental physical questions such as "What is the spin for systems (4.9)?" cannot be consistently defined, let alone answered.

But there exist other problematic aspects for structure (4.10). As well known, Heisenberg's time evolution (4.1b) has a well defined topological group structure, because of its well define Lie-algebra content in the neighborhood of the identity, Eq. (4.1a). No known, well defined group structure can be formulated for time evolution (4.10), evidently because of the loss of a well defined algebra structure in the neighborhood of the identity.

Additional problematic aspects are created by the fact that the transition from the conventional Hamiltonian H to the generalized form (4.8) generally implies the loss of direct physical significance of the algorithms at hand, i.e.: the operator "$\vec{r} \times \vec{p}$" does not represent, in general, the physical angular momentum; H_T does not represent the total energy etc. [19].

The above problematic aspects are resolved by the Lie-admissible form (4.2), as indicated in the original proposal [15]. In fact, the algebraically inconsistent brackets $A \circ B$ of Eq.s (4.10) are replaced by the brackets (A, B) of Eq.s (4.2) which verify all conditions to characterize an algebra, and that algebra results to be a nonassociative Lie-admissible algebra. Secondly, the existence of consistent right and left isotopic envelopes allows a consistent exponention into generalized group structure (4.2b) with intriguing topological properties [29]. Finally, all mathematical algorithms of the equations possess, by construction, a direct and unambiguous physical meaning, i.e.: "$\vec{r}$" represents the coordinates of the experimenter; "$\vec{p}$" represents the (nonconserved) linear momentum "$m\dot{\vec{r}}$"; "$\vec{r} \times \vec{p}$" represents the (nonconserved) angular momentum; "H" = "$T + V$" represents the (nonconserved) energy, etc.

It is also easy to see that *all conventional dissipative formulations can* *be identically formulated in the Lie-admissible form via the decompositions*

$$H_T = HS\ , \quad H_T^\dagger = RH\ , \quad H = H^\dagger\ , \quad R^\dagger = S. \tag{4.12}$$

As a matter of fact, *the Lie-admissible formulations are directly universal, in the sense that, under sufficient topological conditions, all conceivable non-conservative systems can be cast in the Lie-admissible form (universality) in the frame of the experimenter (direct universality)* [21].

The above occurrence is not a mere mathematical curiosity. In fact, the physical implications of conventional dissipative models (4.9) are far from having been investigated in sufficient depth in the existing literature. This is evidently due to the inability of conventional time evolution (4.10) to provide a quantitative characterization of the spin and of other physical characteristics. As a matter of fact, *several dissipative models in nuclear physics already characterize mutations of particles*, as the reader can verifyby using the techniques of this paper.

A primary objective of this paper is to show that the mutation of the intrinsic characteristics of particles exists already at the intermediary level of *Lie-isotopic bimodules* as per structures (4.6) and(4.4). We shall then outline the expected extension of our analysis to the full *Lie-admissible bimodules of Postulate 4.1.*

We shall therefore be dealing with mutation under preservation of the *associative* character of the envelope, although expressed in its most general possible, associative-isotopic form, and this essential structural feature shall persist at the broader Lie-admissible level. The reader should recall from Section 3 that all elaborations [20-23] of the hypothesis of mutation are based on the *nonassociative* generalization of the associative envelope of quantum mechanics.

The *limitations* of our analysis should also be pointed out from the outset, to minimize misrepresentations (or misuse) of the results. The assumption of a Lie-isotopic bi-module for the realization of the concept of eleton and related mutations implies the assumption of the following *Lie-isotopic generalization of Heisenberg's equations*, also proposed in ref. [15], p. 752.

$$\begin{aligned} i\dot{A} &= [A\hat{,}H] \stackrel{\text{def}}{=} A * H - H * A = ATH - HTA\ , \\ A(t) &= e^{iHTt} A(0)\, e^{-itTH} \\ &= \hat{I} e^{IH*t} * A(0) * e^{-it*H} \hat{I}\ , \end{aligned} \tag{4.13}$$

with corresponding *isotopic-generalization of Schrödinger's equations* [28,34]

$$\begin{aligned} i\frac{\partial}{\partial t}\Psi &= H * \Psi \stackrel{\text{def}}{=} HT\Psi\ , \\ -i\Psi\frac{\overleftarrow{\partial}}{\partial t} &= \Psi * \overleftarrow{H} \stackrel{\text{def}}{=} \Psi T \overleftarrow{H}\ . \end{aligned} \tag{4.14}$$

The underlying Lie-isotopic character is expressed by the property that the generalized brackets

$$\begin{aligned} [A\hat{,}B] &= A * B - B * A \\ &= ATB - BTA\ , \quad \forall A, B \in \hat{\mathcal{A}} \end{aligned} \tag{4.15}$$

remain Lie.

By conception, *Eq.s (4.13) represent the conservation of the energy H of the particle considered*, evidently because of the totally antisymmetric nature of the Lie-isotopic product

$$i\dot{H} = H * H - H * H \equiv 0\ . \tag{4.16}$$

This is manifestly at variance with the central role of the nonconservation of the energy, in general, for Postulate 4.1. Despite that, we shall show that rather non-trivial mutations exist for particles obeying time-evolution (4.13).

A central condition of our analysis is therefore that the energy H of the particle/state considered is conserved whenever its time evolution is characterized by Lie-isotopic equations. As a consequence, our analysis *is not* applicable to the most general physical conditions of Postulate 4.1, such as for a proton in the core of a collapsing star, or for a generic hadronic constituent. Nevertheless, our analysis is applicable to specific cases, such as: for the thermal neutrons of Rauch's experiments; or for Rutherford's electron when immersed in the proton structure [32]; or, conceivably, for the characterization of the structure of a quark under the assumption that it lies in a stable orbit.

It should be noted for completeness that Eq.s (4.13) can also represent systems with nonconserved energy, but in this case the operator H is no longer the generator of the time evolution, which is given instead by the so-called *Birkhoffian B* [19]. H is then simply a generic element of the algebra and its nonconservation is characterized by the rule

$$i\dot{H} = [H\hat{,}B] \neq 0\ . \tag{4.17}$$

These particular realizations of Eq.s (4.13) shall not be considered in this work because they generally imply the loss of the direct physical meaning of the algorithms at hand.

The operator formulation of our analysis is that compatible with the fundamental equations (4.13) and (4.14). It is given by the so-called Lie-isotopic branch of hadronic mechanics [28,30]. The fundamental algebraic structure is the enveloping associative algebra (4.6) herein referred to as *iso-envelope*, with generalized unit $\hat{I} = T^{-1}$, herein called *iso-unit*. For consistency, the algebra $\hat{\mathcal{A}}$ cannot be defined on a conventional field of complex numbers $\mathcal{C}$, but must be defined over the so-called *iso-field* [28]

$$\hat{\mathcal{C}} = \{\hat{c}|\hat{c} = c\hat{I} \; ; \; \forall c \in \mathcal{C} \; , \; \hat{I} = T^{-1}\} \; , \tag{4.18}$$

whose addition is the conventional one and whose multiplication is given by

$$\hat{c}_1 * \hat{c}_2 = \hat{c_1 c_2} = c_1 c_2 \hat{I} \; . \tag{4.19}$$

The underlying Hilbert space is of the *iso-Hilbert* form with product [30]

$$\hat{\mathcal{H}} \; : \quad \langle\phi\hat{|}\Psi\rangle \stackrel{\text{def}}{=} \langle\psi|G|\Psi\rangle\hat{I} \; , \tag{4.20}$$

where G is another fixed, Hermitean and, this time, positive- definite operator. The theory shall be restricted, for simplicity but without loss of generality, to the case

$$T \equiv G \; . \tag{4.21}$$

The operator T is therefore also assumed hereon to be positive- definite.

All conventional linear operations of quantum mechanics possess a consistent extension to operators of $\hat{\mathcal{A}}$ on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$ [28,30]. In particular, under condition (4.21), the iso-Hermitean operation coincides with the conventional one [30].

$$A^{\hat{\dagger}} \stackrel{\text{def}}{=} T^{-1} G A^{\dagger} T G^{-1} \equiv A^{\dagger} \; . \tag{4.22}$$

In summary, the Lie-isotopic branch of hadronic mechanics we shall use in this paper is characterized by the isotopies $\mathcal{A} \to \hat{\mathcal{A}}$, $\mathcal{C} \to \hat{\mathcal{C}}$ and $\mathcal{H} \to \hat{\mathcal{H}}$. Note that, in the same way as $\hat{\mathcal{A}}$ remains a universal enveloping [33] associative algebra, $\hat{\mathcal{C}}$ remains a field [28] and $\hat{\mathcal{H}}$ remains a Hilbert space [30]. Conventional quantum mechanics and its isotopic covering here considered therefore coincide at the abstract, realization-free level as a central condition of isotopy [33,19].

Under the above assumptions, *operators that are conventionally Hermitean, remain Hermitean in our isotopic theory.* It has also been proved [28] that the *eigenvalues of iso-Hermitean operators are real, although different than the original ones!* More specifically, let A be a conventional, Hermitean operator with eigenvalue $a_0 \in \mathcal{R}$ and conventional eigenvalue equation $A\phi = a_0\phi$, $A \in \mathcal{A}$. Then, the *same* operator A remains Hermitean under our isotopic lifting, but admits different real eigenvalues, say, a,according to the *iso-eigenvalue law*

$$A * \Psi = \hat{a} * \Psi \equiv a\Psi \; , \quad A \in \hat{\mathcal{A}}. \tag{4.23}$$

while the *expectation values* become

$$\langle A \rangle = \int \Psi^{\dagger} * A * d\vec{x} = \int \Psi^{\dagger} T A T d\vec{x} \; . \tag{4.24}$$

Lifting (4.23) evidently constitutes a first realization of the hypothesis of mutation, which can be expressed as follows.

LEMMA 4.1. The eigenvalues and expectation values of Hermitean operators of conventional quantum mechanics are mutated (altered) by the addition of contact non-Hamiltonian forces represented via an isotopic lifting of quantum mechanics.

Similar results are evidently expected for other characteristics of particles. This is sufficient to indicate, at this introductory level, that the isotopic lifting of quantum mechanics does indeed provide a quantitative realization of the hypothesis of mutation.

Our analysis, however, would be grossly deficient without investigating the connection between mutations and the underlying generalized symmetries, the so-called *isotopic space-time symmetries.*

In essence, the isotopic generalization of the envelope, Eq.s (4.6), implies a corresponding generalization of the entire Lie's theory, from the Poincaré-Birkhoff-Witt theorem, to Lie's Theorems, to the structure constants, to the exponentiation law and, inevitably, to the structure of Lie's groups. These aspects were studied in the original proposal of the Lie-isotopic theory [33], and then developed in monograph [19] (with the understanding that the theory is at the beginning and so much remains to be done). The following isotopic space-time symmetries have been studied until now, evidently in a preliminary way: the *iso-rotational symmetry* [38,39]; the *Galilei-isotopic symmetry* [33,19]; the *Lorentz-isotopic symmetry* [40]; and, more recently, the *Poincaré-isotopic symmetry* [41].

The understanding of this paper requires a minimal knowledge of the above generalized symmetries, as well as of the mathematical formulation of hadronic mechanics. We shall here briefly outline the iso-symmetry of primary use in this paper, the Poincaré-isotopic symmetry.

Let $M = M(x, \eta, \mathcal{R})$ be the conventional Minkowski space with local coordinates $x = (\vec{x}, c_0 t)$, where c_0 shall denote hereon the speed of light in vacuum, and metric

$$\eta = \mathrm{Diag}(1, 1, 1, -1) \tag{4.25}$$

over the field $\mathcal{R}$ of real numbers.

Let $\hat{M} = \hat{M}(x, g, \hat{\mathcal{R}})$ be an element of the infinite family of *Minkowski-isotopic spaces* [40], where

a) the local coordinates x are the same as those of M;

b) the *iso-metric* g is restricted in this paper to have the particular form

$$g = \mathrm{Diag}(b_1^2, b_2^2, b_3^2, -b_4^2)\,, \quad b_\mu \neq 0\,, \tag{4.26.a}$$

$$g = T\eta\,, \quad T = \mathrm{Diag}(b_1^2, b_2^2, b_3^2, b_4^2)\,, \tag{4.26.b}$$

which evidently preserves the topological character of the original Minkowski metric; and

c) the *iso-field* $\hat{\mathcal{R}}$ is given by

$$\hat{\mathcal{R}} = \{\hat{n} | \hat{n} = n\hat{I}\ ;\ \forall n \in \mathcal{R}\,,\ \hat{I} = T^{-1}\}\,. \tag{4.27}$$

The lifting $M \rightarrow \hat{M}$ represents the transition from motion in vacuum, to motion within a physical medium which is generally inhomogeneous and anisotropic (the understanding is that empty space remains homogeneous and isotropic in our analysis). As a consequence, the iso-metric g can have an arbitrary dependence on all possible local quantities, such as coordinates x, velocities $\dot{x}$, acceleration $\ddot{x}$, local density of the medium μ, local temperature τ, etc.

The above general dependence is compatible with the Lorentz-isotopic symmetry [40]. Nevertheless, to achieve a preliminary formulation of the Poincaré-isotopic symmetry, we require hereon that the iso-metric is independent of local coordinates, but depends on all other possible local variables, i.e.,

$$\begin{aligned} g &= T\eta\,,\ T = T(\dot{x}, \ddot{x}, \mu, \tau, \ldots) = \quad Diag(b_1^2, b_2^2, b_3^2, b_4^2) \\ \hat{I} &= \hat{I}(\dot{x}, \ddot{x}, \mu, \tau, \ldots) = T^{-1}\,. \end{aligned} \tag{4.28}$$

This implies, in particular, that the iso-spaces considered are *flat*. Gravitational interactions, therefore, have no impact in the theory of mutation considered in this paper.

The best way to illustrate iso-spaces $\hat{M}$, is by considering a concrete example. As stressed in the preceding paper [41] *all available phenomenological predictions of the behavior of the meanlife of unstable hadrons with speed show deviations from Einstein's Special Relativity*, see, e.g., ref.s [42-47]. These deviations are generally represented precisely via topology- preserving deviations from the conventional Minkowski metric, i.e., via iso-metrics of type (4.26). As an example, Nielsen and Picek [45] have identified the following generalization of the Minkowski metric for the interior of unstable mesons

$$g = \left((1 - \frac{1}{3}\alpha), (1 - \frac{1}{3}\alpha), (1 - \frac{1}{3}\alpha), (1 - \frac{1}{3}\alpha), -(1 - \alpha)\right)\,, \tag{4.29}$$

where the "Lorentz-asymmetry parameter" α has the following value for *pions*

$$\alpha = (-3.79 \pm 1.37) \times 10^{-3}\,, \tag{4.30}$$

and the *different* value for *kaons*

$$\alpha = (+0.61 \pm 0.17) \times 10^{-3}\,. \tag{4.31}$$

Realization (4.29) is sufficient to illustrate the class of systems we are interested in, both topologically as well as quantitatively.

The Poincaré-isotopic symmetry $\hat{P}(3.1)$ has been constructed in ref. [41] as the group of isometries of the infinite family of Minkowski-isotopic spaces $\hat{M}$ as characterized above. It can be defined as the set of all possible *iso-transformations* on $\hat{M}$

$$\hat{P}(3.1) : x' = \{\hat{\Lambda}, \hat{T}\} * x \stackrel{\text{def}}{=} \hat{\Lambda} * x + a\,, \quad a = \text{const.}\,, \tag{4.32}$$

which leave invariant the *iso-separation*

$$\begin{aligned} (x' - y')^{\hat{2}} &= (x'^\mu - y'^\mu) g_{\mu\nu} (x'^\nu - y'^\nu) \\ &= (x^\mu - y^\mu) g_{\mu\nu} (x^\nu - y^\nu) = (x - y)^{\hat{2}}\,, \end{aligned} \tag{4.33}$$

while constituting a *Lie-isotopic group*, i.e., a generic element $\hat{\Gamma}(w) \in \{\hat{\Lambda}, \hat{T}\}$ verifies *the isotopic group laws*

$$\begin{aligned} \hat{\Gamma}(w_1) * \hat{\Gamma}(w_2) &= \hat{\Gamma}(w_1 + w_2)\,, \\ \hat{\Gamma}(w) * \hat{\Gamma}(-w) &= \hat{I}\,, \\ \hat{\Gamma}(0) &= \hat{I}\,. \end{aligned} \tag{4.34}$$

The explicit form of $\hat{P}(3.1)$ can be expressed as follows. By construction, the isotopic lifting of a given Lie group leaves the original generators and parameters unchanged (because they directly represent physical quantities) and changes instead the structure of the group itself, although in an axiom-preserving way.

Let P_μ be the conventional generator of translations, and

$$J_1 = M_{23} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, J_2 = M_{31} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$J_3 = M_{12} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$M_1 = M_{14} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, M_2 = M_{24} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix},$$

$$M_3 = M_{34} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \tag{4.35}$$

be the conventional generators of the connected component $SO^\uparrow_+$ (3.1) of the Lorentz group, as given for instance in ref. [48] (with trivial 5×5 extension to the case of the Poincaré group). Let $\mathcal{P}$ and $\mathcal{T}$ be the conventional generators of inversions. Finally, let θ_k be the parameters of the rotation group $SO(3)$; w_k the parameters of the Lorentz boosts, $k = 1, 2, 3.$; and a^μ be the parameters for translations in M.

Then, the *iso-translation group* can be written [41]

$$\hat{T}(3.1) : \hat{T}(a) = e^{P^\mu \eta_{\mu\nu} a^\nu}|_{\hat{A}} = e^{P^\mu g_{\mu\nu} a^\nu} \hat{I}|_{A} , \tag{4.36}$$

where the reader should note the transition from exponention in $\hat{\mathcal{A}}$ to that in the original envelope $\mathcal{A}$, and where lifting (4.26b) is used (for mathematical details in this transition, the interested reader may consult ref. [28]).

The connected component of the *Lorentz-isotopic group* in its fundamental representation can be written [40,41]

$$\begin{aligned} \widehat{SO}^\uparrow_+(3.1) : \hat{\Lambda}(\vec{\theta}, \vec{w}) &= \left(\prod_{k=1}^{3} {}^* e^{J_k \theta_k}|_{\hat{\mathcal{A}}}\right) * \left(\prod_{k=1}^{3} {}^* e^{M_k w_k}|_{\hat{\mathcal{A}}}\right) \\ &= \left[\left(\prod_{k=1}^{3} e^{J_k T \theta_k}|_{\mathcal{A}}\right)\left(\prod_{k=1}^{3} e^{M_k T w_k}|_{\mathcal{A}}\right)\right] \hat{I}, \end{aligned} \tag{4.37}$$

where one can note again the reformulation in terms of the conventional envelope for computational convenience. The attentive reader may note that, in the original construction of the Lorentz- isotopic group of ref. [40] there is the appearance of the full iso-metric g in the exponents, rather than the isotopic element T as in Eq. (4.37). This is due to the use in ref. [40] of the compact generators of $O(4)$ as in ref. [49], while in this paper we use the non-compact generator as in ref. [48].

The *iso-inversions* can be finally written [40,41]

$$\begin{aligned} \hat{\mathcal{P}} * x &= \mathcal{P} x = (-\vec{x}, c_0 t) , \\ \hat{\mathcal{T}} * x &= \mathcal{T} x = (\vec{x}, -c_0 t) , \\ \hat{\mathcal{P}} * \hat{\mathcal{T}} * x &= \mathcal{P}\mathcal{T} x = (-\vec{x}, -c_0 t) . \end{aligned} \tag{4.38}$$

The group structure of $\hat{P}(3.1)$ is therefore an isotope of $P(3.1)$, as expected, i.e.

$$\hat{P}(3.1) = \hat{O}(3.1) \otimes \hat{T}(3.1) . \tag{4.39}$$

The *Lie-isotopic commutation rules* of the algebra $\hat{\mathbf{P}}(3.1)$ are given by [41]

$$\begin{aligned} [M_{\alpha\beta} \hat{,} M_{\gamma\delta}] &= -g_{\alpha\gamma} M_{\beta\delta} + g_{\alpha\delta} M_{\beta\gamma} + g_{\beta\gamma} M_{\alpha\delta} - g_{\beta\delta} M_{\alpha\gamma} , \\ [M_{\alpha\beta} \hat{,} P_\gamma] &= g_{\beta\gamma} P_\alpha - g_{\alpha\gamma} P_\beta , \\ [P_\alpha \hat{,} P_\beta] &= 0 , \end{aligned} \tag{4.40}$$

as the reader can verify by using definitions (4.15), (4.26) and (4.35). The *iso-Casimir invariants* of $\hat{\mathbf{P}}(3.1)$ are the iso-unit $\hat{I}$ and the quantities

$$\begin{aligned} P^{\hat{2}} &= (P^\mu g_{\mu\nu} P^\nu) \hat{I} , \\ W^{\hat{2}} &= (W^\mu g_{\mu\nu} W^\nu) \hat{I} , \\ W_\mu &= \frac{1}{2} \varepsilon_{\mu\alpha\beta\gamma} M^{\alpha\beta} * P^\gamma , \end{aligned} \tag{4.41}$$

as well as any of their combinations.

A few comments are now in order. Under topological restriction (4.26a), *the Poincaré-isotopic group is locally isomorphic to the conventional Poincaré group* [41] and the same result occurs for the Lorentz group [40]. Thus, the statements of "broken Lorentz symmetry" for metric of type (4.29) are correct only if referred to the "simplest possible realization", that in terms of the trivial Lie product $AB - BA$. In different terms, papers [41,42] have proved that the Lorentz symmetry remains exact for generalized metrics (4.26), of course, when realized in a sufficiently general way.

The violations under consideration [42-47] must therefore be specifically referred to Einstein's Special Relativity [41].

The iso-scalar extension of the underlying field R is needed for consistency of the theory and, in particular, to preserve the iso-linearity [28]. However, due to compositions of type (4.19), only ordinary scalars enter into the practical use of the theory.

As an example, the elements $g_{\mu\nu}$ of the iso-metric in Eq. (4.36) are, strictly speaking, iso-scalars and should be written as $\hat{g}_{\mu\nu} = g_{\mu\nu}\hat{I}$. Nevertheless, this does not imply a change in the value of the exponent of Eq. (4.36) because of the identities $\hat{g}_{\mu\nu} * a^\nu \equiv g_{\mu\nu}a^\nu$.

Note that, for space-time symmetries without translations (where the iso-metric can be explicitly dependent on the local coordinates), the underlying transformations (4.32) are generally *non-linear*, although written in a formally *iso-linear* way. As a matter of fact, *any non-linear transformation can be turned into an equivalent iso-linear form*, trivially, by incorporating all the non-linear terms in the iso-unit [39].

For subsequent needs, we should also recall that the Poincaré-isotopic transformations follow the composition law

$$\{\hat{\Lambda}_1, \hat{T}_1\} * \{\hat{\Lambda}_2, \hat{T}_2\} = \{\hat{\Lambda}_1 * \hat{\Lambda}_2, \hat{T}_1 + \hat{\Lambda}_2 * \hat{\Lambda}_2\} . \qquad (4.42)$$

Note that *the sole unknown of the Poincaré-isotopic symmetry is the isotopic element* T. Also, the convergence of exponentials in $\hat{A}$ is guaranteed by the existence of a consistent generalization of the underlying methodology [33,39]. Thus, under sufficient topological restrictions, *the Poincaré-isotopic symmetry can be explicitly computed for each given isotopic element* T. For explicit cases, see ref. [40,41].

The operator formulation of $\hat{P}(3.1)$ is being investigated in a separate paper [50]. For our needs at this time it is sufficient to recall the following simplest possible *hadronization* (i.e., mapping of Birkhoffian into hadronic

mechanics) [51]

$$p_\mu^{Cl} \to p_\mu^{Op} * \Psi = p_\mu * \Psi = -i\partial_\mu \Psi ,$$
$$\hbar = 1 , \quad \partial_\mu = \partial/\partial x^\mu , \qquad (4.43)$$

which, from the independence of the iso-unit on the local coordinates, yields the substitution rule

$$p_\mu^{CL} \to p_\mu^{Op} = -i\hat{\partial}_\mu = -i\partial_\mu \hat{I} = -i\hat{I}\partial_\mu , \qquad (4.44)$$

with similar expressions for other operators.

A most important implication of hadronization (4.43) is that of requiring, for consistency, the identity of the iso-unit of hadronic mechanics with that of the iso- envelope of $\hat{\mathbf{P}}$ (3.1). To put it differently, one could start with the iso-envelope $\hat{A}$ and iso-unit $\hat{I} = T^{-1}$ as in Eq. (4.6), and then have a lifting of $P(3.1)$ characterized by a different element, say, T' of the infinite family (4.26b) with consequently different iso- unit of $\hat{\mathbf{P}}$ (3.1), $\hat{I}' = T'^{-1} \neq \hat{I}$. Under these assumptions, hadronization (4.43) would not allow the interpretations of "p" as the generator of translations of $\hat{P}$ (3.1), evidently because of the different iso-modular actions. This does not mean that such a differentiation is inconsistent. In fact, we shall encounter one in Section 8.

We assume the reader is familiar with the affine geometry underlying the transition from contravariant to covariant indices and vice versa, e.g.,

$$P_\mu = g_{\mu\nu}P^\nu , \quad P^\mu = g^{\mu\nu}P_\nu, \quad (g^{\mu\nu}) = (g_{\mu\nu})^{-1} = (g_{\mu\nu}^{-1}) . \qquad (4.45)$$

As a consequence, the fundamental iso-invariant(4.41a) can be written in any of the forms

$$P^{\hat{2}} = P^\mu g_{\mu\nu} P^\nu = P_\mu g^{\mu\nu} P_\nu = P^\mu P_\mu = P_\mu P^\mu , \qquad (4.46)$$

with similar expressions for other iso-invariants.

We also assume the reader is familiar with basic rules of isotopic theories. For instance, the square of an operator, say, $p^2 = pp$, has no mathematical meaning in our theory (actually it violates the condition of iso-linearity). The correct expression for the "square of p" is given by $p^{\hat{2}} = p * p = pTp$.

In summary, the isotopic lifting $\hat{P}(3.1)$of the Poincaré symmetry $P(3.1)$ is essentially based on the generalization of the trivial unit I which is restricted in this paper to preserve the topology of the original one (that is, of remaining positive definite)

$$I = \mathrm{Diag}(1,1,1,1,) \quad \to \quad \hat{I} = \mathrm{Diag}(b_1^{-2}, b_2^{-2}, b_3^{-2}, b_4^{-2}),$$
$$b_\mu \neq 0 , \quad b_\mu = b_\mu(\dot{x}, \ddot{x}, \mu, \tau, \ldots) . \qquad (4.47)$$

As a consequence, the isotopic and conventional Poincaré symmetries, not only are locally isomorphic, but they actually coincide at the abstract, coordináte- free level.

We are now equipped to show that, despite these mathematical equivalences, the transition from the conventional to the isotopic Poincaré symmetry has rather profound physical implications, inasmuch as it implies not only a reformulation of Einstein's Special Relativity [41] (to be compatible with phenomenological studies [42-47]), but also a mutation of the intrinsic characteristics of particles.

5 Isotopic Generalization of the Klein-Gordon Equation

Once the mathematical structure of the Lie-isotopic symmetries and of hadronic mechanics are understood, it is quite easy to identify the compatible formulation of field equations. In fact, by using iso-modular action (4.23), iso-metric (4.26) and hadronization rule (4.43), one easily obtains from iso-invariant (4.46) and the iso-kinematics of ref. [41] the following *iso-Klein- Gordon equation*

$$\begin{aligned}(\hat{g}^{\mu\nu} * p_\mu * p_\nu + \hat{k}^2) * \Psi &= (p^\mu * p_\mu + \hat{k}^2) * \Psi \\ = -(\hat{\Box} - \hat{k}^2) * \Psi &= -(\partial^\mu \partial_\mu \hat{1} - \hat{k}^2) * \Psi = 0 \\ \hat{k} &= m_0 c \hat{I} = m_0 c_0 b_4 \hat{I}\,, \quad \hat{I} = T^{-1}\,. \end{aligned} \tag{5.1}$$

The first departure from the conventional setting is due to the fact that *the above equation does not represent a free particle.* In fact, as stressed in the preceding paper [41], a *free particle must strictly obey Einstein's Special Relativity*, in which case only the conventional Minkowski metric η is admitted with isotopic element $T = 1$.

The mutation of the metric $\eta \to g = T\eta$ is per sé a representative of interactions, not of the conventional Hamiltonian-Lagrangian type, but precisely of the contact, non-Hamiltonian type under study in this paper.

As a result, Eq. (5.1) represents a massive, charged or neutral particle moving within a hadronic medium for which all other interactions are ignored.

It is easy to prove the following

LEMMA 5.1. Iso-Klein-Gordon equation (5.2) transforms as an iso-scalar under the Poincaré-isotopic group.

In fact, under transformations (4.32), we have the following invariance property

$$g^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu} - \hat{k}^2 = g^{\mu\nu}\frac{\partial}{\partial x'^\mu}\frac{\partial}{\partial x'^\nu} - \hat{k}^2\,. \tag{5.2}$$

A solution of Eq. (5.1), called *iso-plane-wave solution*, is given by

$$\Psi(x) = N\ e^{ip^\mu g_{\mu\nu} x^\nu}\,, \tag{5.3}$$

and transforms as follows

$$\Psi(x) = N e^{ip^\mu g_{\mu\nu} x^\nu} = N\ e^{ip'^\mu g_{\mu\nu} x'^\nu} = \Psi'(x')\,, \tag{5.4}$$

thus proving the iso-scalar nature of the equation.

Note that, since $\hat{k}^2$ is an iso-number, the application of operator (5.2) to solution (5.3) yields the identities

$$\begin{aligned}(\partial^\mu \partial_\mu \hat{1}) * \Psi &= -p^\mu * p_\mu * \Psi = \hat{k}^2 * \Psi\,. \\ &= m_0^2 c^2 \Psi = m_0^2 c_0^2 b_4^2 \Psi\,, \end{aligned} \tag{5.5}$$

as desired.

To identify explicitly the form of the generator of infinitesimal translations, we simply perform an isotopic lifting of the conventional case, as presented in ref. [52], p.77. We have

$$\Psi(x') = \Psi(x + \delta x) \cong (1 + \delta x \nabla)\Psi = (\hat{1} + \delta x \hat{\nabla}) * \Psi\,. \tag{5.6}$$

As a result,

$$\Psi'(x) \cong (\hat{1} - \delta a \hat{\nabla}) * \Psi = (\hat{1} + i\delta a p) * \Psi\,. \tag{5.7}$$

We confirm in this way that the operator $p_\mu = -i\partial_\mu \hat{I}$, as in hadronization rule (4.43), is indeed the generator of translations in $\hat{P}(3.1)$, provided that the isotopic elements (iso- units) of the operator algebra and of the Poincaré-isotopic symmetry coincide, as anticipated in Section 4.

In order to identify the applicable realization of the *iso-rotational symmetry* $\hat{S}0(3)$ worked out in ref. [39], we introduce the *fundamental iso-commutation rules*

$$\begin{aligned}&[p_i \hat{,} p_j] * \Psi = 0\,, \quad [x_i \hat{,} x_j] * \Psi = 0\,, \\ &[p_i \hat{,} x_j] * \Psi = -i\hat{\delta}_{ij} * \Psi = -i\delta_{ij}\Psi\,, \end{aligned} \tag{5.8}$$

where use has been made of the independence of the iso- unit from the local coordinates assumed earlier.

The generators of iso-rotations $\hat{S0}(3)$ are then given by

$$\begin{aligned}\hat{J}_i &= \epsilon_{ijk}x_j * p_k = \epsilon_{ijk}x_j T p_k \\ &= -i\epsilon_{ijk}x_j T\hat{\nabla}_k \in \hat{\mathcal{A}} ,\end{aligned} \tag{5.9}$$

and verify the iso-commutation rules

$$\begin{aligned}\hat{S0}(3): \left[\hat{J}_i \hat{,} \hat{J}_j\right] &= i\epsilon_{ijk}\hat{J}_k , \\ \left[\hat{\vec{J}}^{\hat{2}} \hat{,} J_i\right] &= 0, \quad \hat{\vec{J}}^{\hat{2}} = \sum_{k=1}^{3} \hat{J}_k * \hat{J}_k = \sum_{k=1}^{3} \hat{J}_k T \hat{J}_k ,\end{aligned} \tag{5.10}$$

thus confirming the local isomorphism $\hat{S0}(3) \approx S0(3)$ identified in ref. [39].

Note that use has been made in Eq.s (5.10) of the property identified in ref. [28]

$$[A * B \hat{,} C] = A * [B \hat{,} C] + [A \hat{,} C] * B . \tag{5.11}$$

Despite the above similarities, the reader should be aware of the rather deep physical differences between $\widehat{SO}(3)$ and $SO(3)$. In fact, *$SO(3)$ is the symmetry of the rigid sphere*

$$SO(3): \quad \vec{x}^{\hat{2}} = x_1^2 + x_2^2 + x_3^2 , \tag{5.12}$$

while *our covering $\hat{S0}(3)$ is the symmetry of the infinite family of deformed ellipsoids* [39]

$$\begin{aligned}\hat{SO}(3): \quad \vec{x}^{2} &= x_1 b_1^2 x_1 + x_2 b_2^2 x_2 + x_3 b_3^2 x_3 , \\ & b_k = b_k(\dot{x}, \ddot{x}, \mu, \tau, \ldots) \neq 0 .\end{aligned} \tag{5.13}$$

Additional physical differences will be pointed out at the end of this section.

Consider now an infinitesimal rotation $\delta\vec{x} = \vec{x} \wedge \delta\vec{\theta}_0$. Then, by lifting the conventional formulation (ref. [52]. p. 77), we have

$$\Psi'(x) = (\hat{1} + i\delta\vec{\theta} \cdot \hat{\vec{J}}) * \Psi(x) . \tag{5.14}$$

This confirms that operators (5.9) are indeed the generators of iso-rotations.

To see the compatibility with structures (4.36) and(4.37), one can perform an integration of Eq.s (5.7) and (5.14) in $\hat{\mathcal{A}}$ with respect to δ_a and $\delta\vec{\theta}$, respectively. This results in the exponentials

$$\Psi'(x) = \hat{T}(a) * \Psi(x) = e^{iap}|_{\hat{\mathcal{A}}} * \Psi(x) = e^{iaTp}|_{\mathcal{A}}\Psi(x) , \tag{5.15}$$

and

$$\Psi'(x) = \hat{\Lambda}(\theta) * \Psi(x) = e^{i\theta(\vec{\eta}\cdot\hat{\vec{J}})}|_{\hat{\mathcal{A}}} * \Psi(x) = e^{i\theta T(\vec{\eta}\cdot\hat{\vec{J}})}|_{\mathcal{A}}\Psi(x) , \tag{5.16}$$

which are manifest y in agreement with structures (4.36) and(4.37), respectively. A similar result occurs for the Lorentz-isotopic boosts.

Notice also that the above operators remain unitary as in the standard theory although with respect to the iso- Hilbert space $\hat{\mathcal{H}}$, i.e.,

$$\hat{T}^{\hat{\dagger}} = T^{-\hat{1}} , \quad \hat{\Lambda}^{\hat{\dagger}} = \hat{\Lambda}^{-\hat{1}} . \tag{5.17}$$

The *isotopic lifting of a spin-zero charged particle under an external electromagnetic field* can be r adily expressed in the form

$$\begin{aligned}&\left[\hat{g}^{\mu\nu} * (p_\mu + \frac{e}{c}A_\mu) * (p_\nu + \frac{e}{c}A_\nu) + \hat{k}^{\hat{2}}\right] * \Psi \\ &= \left[(p^\mu + \frac{e}{c}A^\mu) * (p_\mu + \frac{e}{c}A_\mu) + \hat{k}^{\hat{2}}\right] * \Psi = 0 ,\end{aligned} \tag{5.18}$$

where A_μ is the familiar (unmutated) electromagnetic potential.

Note that, for consistency with the underlying $\hat{P}(3.1)$ symmetry and related kinematics [41], Eq. (5.15) must be formulated for the quantity

$$c = c_0 b_4 \gtrless c_0 . \tag{5.19}$$

A lifting, again, of the conventional formulation (ref. [52], p.80) leads to the *iso-four-current*

$$\hat{J}_\mu = \frac{1}{2m_0 i}\left[\Psi^* * \hat{\partial}_\mu * \Psi - (\hat{\partial}_\mu * \Psi^*) * \Psi\right] + \frac{e}{m_0 c}A_\mu * \Psi^* * \Psi , \tag{5.20}$$

with related conservation law

$$\hat{\partial}^\mu * \hat{J}_\mu = \partial^\mu \hat{J}_\mu = 0 . \tag{5.21}$$

The *iso-charge-density* is then given by

$$\begin{aligned}\hat{\rho} &= \frac{1}{ic}\hat{J}_4 = \frac{1}{2m_0c^2}\left[\Psi^* * \hat{\partial}_t * \Psi - (\hat{\partial}_t * \Psi^*) * \Psi\right] \\ &+ \frac{e}{m_0c^2}A_0 * \Psi^* * \Psi ,\end{aligned} \tag{5.22}$$

and, again from law (5.21), it is conserved under the conditions herein assumed (g being independent of x)

$$\frac{d\hat{Q}}{dt} = \int d\vec{x}\hat{\rho} = 0 \ . \tag{5.23}$$

The invariance of Eq. (5.15) under the connected components of the Poincaré-isotopic symmetry is the same as that of Lemma 5.1. In addition, it is easy to see that Eq. (5.18) is invariant under the *isotopic gauge transformations* worked out by Gasperini [53-55]

$$\Psi' = \hat{U} * \Psi = e^{-i\frac{e}{c}*\Lambda}\Psi \ , \tag{5.24}$$

where $\hat{U}$ is a iso-unitary operator, and the covariant derivative is lifted to the form

$$\hat{D}_\mu = (\partial_\mu - i\frac{e}{c}A_\mu * \Lambda)\hat{I} \ . \tag{5.25}$$

Eq. (5.18) is also invariant under the *iso-charge-conjugation*

$$\Psi' = \hat{\eta}_C * \Psi^* \ , \quad |\eta_c| = 1 \ ,$$

$$e' = -e \tag{5.26}$$

as well as under *iso-space-inversions*

$$\Psi'(\vec{x}', t) = \hat{\mathcal{P}} * \Psi(\vec{x}, t) = \Psi(\vec{x}, t) \ , \tag{5.27}$$

and *iso-time-reversal*

$$\Psi'(\vec{x}, t') = \hat{\mathcal{T}} * \Psi(\vec{x}, t) = \Psi(\vec{x}, t) \ . \tag{5.28}$$

We are now in a position to identify the mutations underlying the lifting of the Klein-Gordon field equation. First, one should keep in mind that the electromagnetic field A_μ, being external, cannot be mutated and therefore remains in its conventional form. Second, the spin of the particle remains zero because the particle transforms as an iso-scalar. We are therefore studying a first case of iso-field equations with minimal mutations.

Despite that, the mutations exist and are nontrivial. First, *the iso-particle has a mutated form of rest energy*

$$E_0 = m_0 c_0^2 \rightarrow E = m_0 c^2 = m_0 c_0^2 b_4^2 \gtrless m_0 c_0^2 \ , \tag{5.29}$$

which can be locally bigger or smaller than the conventional value depending on the characteristics of the medium considered. For instance, for the case of the Nielsen-Picek metric (4.29), value (5.29) is *smaller* than $m_0c_0^2$ in the interior of pions, Eq. (4.30), but *bigger* than $m_0c_0^2$ in the interior of kaons, Eq. (4.31), and, expectedly, for all other heavier hadrons [41].

The reader should recall that *mutation (5.29) has no meaning in vacuum* [41]. Thus, it strictly refers to the equivalent value of the mass of a particle when that particle is in condition of total immersion in the interior of hadronic matter, e.g., for a proton in the core of a star, or for a quark inside a hadron.

Second, the *charged iso-particle has a mutated value of the charge* which is expressed by the lifting

$$Q = \int \rho d\vec{x} \rightarrow \hat{Q} = \int \hat{\rho} d\vec{x} \ . \tag{5.30}$$

Evidently, the above lifting offers the possibility of interpreting the fractional charge of a quark as that of an iso-particle. This possibility will be touched upon in the next section.

Third, from Lemma 4.1, *all kinematical quantities of the iso-particle are generally mutated.* This is the case of the *position*

$$\langle \vec{x} \rangle = \int \phi^* \vec{x} \phi d\vec{x} \rightarrow \langle \hat{\vec{x}} \rangle = \int \Psi^* * \vec{x} * \Psi d\vec{x} \ , \tag{5.31}$$

of the *linear momentum*

$$\langle \vec{p} \rangle = \int \phi^* \vec{p} \phi d\vec{x} \rightarrow \langle \hat{\vec{p}} \rangle = \int \Psi^* * \vec{p} * d\vec{x} \ , \tag{5.32}$$

as well as, consequently, of the *angular momentum*

$$\begin{aligned} \langle J \rangle &= \int \phi^* J \phi d\vec{x} \rightarrow \langle \hat{J} \rangle = \int \Psi^* * \hat{J} * \Psi d\vec{x} \ , \\ \langle \vec{J}^2 \rangle &= \int \phi^* \vec{J}^2 \phi d\vec{x} = \int \phi^* \left(\sum_{k=1}^{3} J_k J_k \right) \phi d\vec{x} \rightarrow \end{aligned} \tag{5.33}$$

$$\rightarrow \langle \hat{\vec{J}}^{\hat{2}} \rangle = \int \Psi^* * \hat{\vec{J}}^{\hat{2}} * \Psi d\vec{x} = \int \Psi^* * \left(\sum_{k=1}^{3} \hat{J}_k * \hat{J}_k \right) * \Psi d\vec{x} \ . \tag{5.34}$$

Mutations (5.31) and (5.31) re-open intriguing problems of localizability, this time from a covering profile which are not investigated here.

Mutations (5.33) are crucial for the theory under consideration, because they avoid certain excessive approximations of conventional theories for the interior problem (at times referred as approximations of the "perpetual-motion-type"), e.g., the prediction that a proton orbits in the core of a collapsing star with a locally conserved angular momentum. In fact, under a suitable selection of the isotopic element, mutation (5.33) can represent *the decay of the orbital momentum*, say, of a proton when penetrating within hadronic matter.

In essence, mutations (5.33) represent the *deviation* from the conventional orbital motion caused by a resistive medium. Note that mutations (5.33) occur despite the preservation of the conventional commutation rules of $SO(3)$, Eq. (5.10), with consequential local isomorphism $\hat{SO}(3) \approx SO(3)$. In fact, as recalled earlier, the isotope $\hat{SO}(3)$ is, by construction [39], the group of isometries of ellipsoids (5.13). Mutation of the angular momentum is then ensured by the very variable character of invariant (5.13).

The above three types of mutations are sufficient for the limited scope of this section. Additional mutations will be considered in the next sections.

In closing, we should stress that *the invariance of the iso-field equations under the isotopic space-time symmetries does not imply that the same equations are invariant under ordinary symmetries. On the contrary, the ordinary symmetries are violated by the equations considered, and the isotopic lifting is introduced precisely to restore their exact form.*

In fact, the equations considered violate, by conception, the conventional Lorentz symmetry, evidently because constructed in generalized forms of the Minkowski space of the Nielsen-Picek type (4.29). Nevertheless, the symmetry is reconstructed as exact at the covering isotopic level. Thus, as established in ref. [40], the abstract, realization-free Lorentz symmetry remains exact for metrics (4.29).

An equally intriguing case occurs for the discrete symmetries. In fact, *the iso-space-inversions and iso-time-inversion are exact symmetries for the field equations considered, Eq.s (5.27) and (5.28). Nevertheless, the same field equations are not necessarily invariant under ordinary space and time inversions.* In fact, a violation of the latter symmetries trivially occurs when the isotopic element T is not invariant under conventional inversions.

This occurrence illustrates the conjecture submitted in ref. [56] according to which the problem of parity violation in weak interactions is still fundamentally open at this moment. What we can state correctly is that parity is violated by weak interactions "when the symmetry is realized in its simplest conceivable way", because *parity could remain an exact symmetry*

in weak interactions at the covering isotopic level, trivially, by incorporating all P-violating terms in the two degrees of freedom of the theory, the isotopic element T (or iso-unit $\hat{I}$) of the enveloping algebra (4.6), as well as the (generally different) isotopic element G of the iso-Hilbert space (4.20).

Finally, the reader should be aware of the fact that the Lie-isotopic field equations for stable orbits studied in this section are readily turned into the yet more general *Lie-admissible field equations for open/nonconservative orbits* via the mere relaxation of the Hermiticity of the isotopic element T, or the assumption that the theory is non-time-reversal invariant. In fact, under the latter conditions, Eq.s (5.1) are generalized into the forward and backward equations, respectively

$$(p^\mu \triangleright p_\mu + k^{\triangleright 2^\triangleright}) \triangleright \Psi = 0 \,,$$
$$\Psi \triangleleft (p^\mu \triangleleft p_\mu + {}^\triangleleft k^{\triangleleft 2}) = 0 \,, \tag{5.35}$$

and similar results hold for the more general iso-field equations (5.18). We shall return to this Lie-admissible aspect later on during our analysis.

In conclusion, we can state that *the isotopic lifting of the conventional Klein-Gordon field equation restores the exact connected Lorentz symmetry for generalized Minkowski metrics of type (4.29), while a similar restoration for the discrete symmetries (space and time inversions) is conceivable but requires additional specific investigations.*

6 Isotopic Generalization of Dirac's Equation

By following again the main lines of the conventional field theory [52], we are now interested in identifying the isotopic decomposition of the fundamental isoinvariant of Eq. (5.1) into first-order forms. For this purpose, we introduce 4×4 matrices $\hat{\gamma}_\mu$ such that

$$\begin{aligned} p^\mu * p_\mu + \hat{k}^{\hat{2}} &= (\hat{\gamma}_\mu * p^\mu - i\hat{k}) * (\hat{\gamma}_\nu * p^\nu + i\hat{k}) \\ &= \frac{1}{2}\{\hat{\gamma}_\mu \hat{,} \hat{\gamma}_\nu\} * p^\mu * p^\nu + \hat{k}^{\hat{2}} \\ & \qquad \hat{k} = m_0 c\, \hat{I} = m_0 c_0 b_4 \hat{I} \,, \end{aligned} \tag{6.1}$$

which hold iff

$$\{\hat{\gamma}_\mu \hat{,} \hat{\gamma}_\nu\} = \hat{\gamma}_\mu * \hat{\gamma}_\nu + \hat{\gamma}_\nu * \hat{\gamma}_\mu = 2 g_{\mu\nu} \hat{I} \,. \tag{6.2}$$

Note that the preceding relations are nothing but an isotopic lifting of the conventional properties

$$\{\gamma_\mu, \gamma_\nu\} = \gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\eta_{\mu\nu} I \,. \tag{6.3}$$

It should be stressed that lifting (6.2) is nontrivial. This can be first illustrated by noting that the trivial lifting $\hat{\gamma}_\mu = \gamma_\mu \hat{I}$ *is not* a realization of rules (6.2) because of the values

$$\hat{\gamma}_\mu = \gamma_\mu \hat{I}\,, \quad \{\hat{\gamma}_\mu \hat{,} \hat{\gamma}_\nu\} = \{\gamma_\mu, \gamma_\nu\}\hat{I} = 2\eta_{\mu\nu}\hat{I} \neq 2g_{\mu\nu}\hat{I}\,. \tag{6.4}$$

In fact, an explicit realization of the *iso-gamma matrices* $\hat{\gamma}_\mu$ is given by

$$\hat{\gamma}_\mu \stackrel{\text{def}}{=} \tilde{\gamma}_\mu \hat{I} \;=\; b_\mu \gamma_\mu \hat{I}\,, \quad \mu = 1,2,3,4\,, \tag{6.5.a}$$

$$\hat{\gamma}_k = \begin{pmatrix} 0 & \tilde{\sigma}_k \\ -\tilde{\sigma}_k & 0 \end{pmatrix} \hat{I} \;=\; b_k \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix} \hat{I} = b_k \gamma_k \hat{I}\,, k = 1,2,3, \tag{6.5.b}$$

$$\hat{\gamma}_4 = \begin{pmatrix} \tilde{I} & 0 \\ 0 & -\tilde{I} \end{pmatrix} \hat{I} \;=\; b_4 \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \hat{I} = b_4 \gamma_4 \hat{I}\,, \tag{6.5.c}$$

where, as one can see, the elements b_μ representing the mutation of the Minkowski metric (Sect. 4) enter directly into the structure of the matrices. Generalizations of the other representations of the conventional gamma matrices [52] can also be readily identified, but their explicit form is not reported here for brevity.

We are now in a position to introduce the desired *isotopic generalization of Dirac's equation (or iso-Dirac equation* for short)

$$\begin{aligned} &(\hat{\gamma}_\mu * p^\mu + i\hat{k}) * \psi\,, \\ =\; &-(i\hat{\gamma}_\mu * \hat{\partial}^\mu - i\hat{k}) * \psi = 0\,, \end{aligned} \tag{6.6}$$

which was first submitted in ref. [15], Eq. (4.20.5).

We introduce now the adjoint wavefunction $\hat{\bar{\psi}} = \psi^\dagger * \hat{\gamma}_4$. Then, each and every step of the conventional formulation of Dirac's equation (see, e.g., ref. [52], Ch. III) admits a consistent isotopic lifting. We shall point out in this section only the most salient aspects, and their primary implications. The application to Rauch's experiments will be studied in the next section.

The *iso-adjoint* of Eq. (6.6) is given by

$$(i\hat{\partial}^\mu * \hat{\bar{\psi}}) * \hat{\gamma}_\mu - i\,\hat{k} * \psi = 0\,. \tag{6.7}$$

The *iso-four-current* is then given by

$$\hat{J}_\mu = ic\hat{\bar{\psi}} * \hat{\gamma}_\mu * \psi\,, \tag{6.8}$$

and it is (conventionally) conserved

$$\hat{\partial}^\mu * \hat{J}_\mu = \partial^\mu \hat{J}_\mu = 0\,. \tag{6.9}$$

The *iso-charge-density* is given by

$$\hat{\rho} = \frac{e}{ic}\hat{J}_4 = \hat{\bar{\psi}} * \hat{\gamma}_4 * \psi = e\, b_4^2 \psi^\dagger * \psi\,. \tag{6.10}$$

The corresponding *iso-charge* can be written

$$\langle \hat{Q} \rangle = \hat{Q} = e\, b_4^2 \int \psi^\dagger * \psi d^3x\,, \tag{6.11}$$

and it is given by the inner product of the underlying iso-Hilbert space [30] times eb_4^2. Note that density (6.10) is positive-definite under the assumptions considered. Along the same lines, one can see that the iso-four-current (6.8) is iso-Hermitean.

We consider now the total angular momentum for systems (6.6), which can be defined as the sum of the orbital and intrinsic part as in the conventional case, according to the quantity in $\hat{A}$

$$J_i^T = \hat{M}_i + \hat{S}_i\,, \tag{6.12}$$

with *iso-orbital* part

$$\hat{M}_i = \int \psi^\dagger * \left(\epsilon_{irs} x_r * \frac{1}{i}\hat{\partial}_s\right) * \psi d^3x\,, \tag{6.13}$$

iso-intrinsic part

$$\hat{S}_i = \int \psi^\dagger * \left(\frac{1}{4i}\epsilon_{irs}\, \hat{\gamma}_r * \hat{\gamma}_s\right) * \psi d^3x\,, \tag{6.14}$$

and corresponding densities

$$\hat{m}_i \;=\; \epsilon_{irs} x_r * \frac{1}{i}\hat{\partial}_s\,, \tag{6.15.a}$$

$$\hat{s}_i \;=\; \frac{1}{4i}\epsilon_{irs}\hat{\gamma}_r * \hat{\gamma}_s\,. \tag{6.15.b}$$

Iso-operators (6.15a) verify commutation rules (5.9) and, therefore, the structure constants of $SO(3)$ are not lifted (although the expectation values are subjected to lifting (6.13)).

The iso-intrinsic angular momentum, operators (6.15b), provides a realization of the *isotopic generalization of the $SU(2)$ symmetry* with iso-commutation rules

$$\widehat{SU}(2) : [\hat{s}_i \hat{,} \hat{s}_i] = \epsilon_{ijk} b_k^2 \hat{s}_k \ , \tag{6.16}$$

where one can see the mutation of the structure constants. Nevertheless, the algebra characterized by Eq. (6.16) is (locally) isomorphic to the conventional $SU(2)$ algebra, as proved in ref. [39].

The conventional expression for the spin of Dirac's equation

$$\vec{s}^{\,2} \phi = \frac{1}{4}(\eta_{11}\eta_{22} + \eta_{22}\eta_{33} + \eta_{33}\eta_{11})\psi = \frac{3}{4}\phi \tag{6.17}$$

is now mutated into the value

$$\hat{\vec{s}}^{\,\hat{2}} * \psi = \frac{1}{4}(b_1^2 b_2^2 + b_2^2 b_3^2 + b_3^2 b_1^2)\psi \neq \frac{3}{4}\psi, \tag{6.18}$$

first introduced in ref. [15], p. 801.

For small deformations of the Minkowski metric, e.g., of type (4.29), Eq. (6.18) can be written

$$\hat{\vec{s}}^{\,\hat{2}} * \psi = \left(\frac{3}{4} + \epsilon\right)\psi \ . \tag{6.19}$$

We recover in this way the spin mutation originally submitted in ref. [15] and subsequently studied in ref.s [20,21] via the broader Lie-admissible algebras.

The reader familiar with Lie-isotopic symmetries will have noted that the quantity $\hat{\vec{s}}^{\,\hat{2}}$ is not an iso-Casimir invariant, trivially, because its eigenvalue is not proportional to the iso-unit as it must be for mathematical consistency [33]. In particular, $\hat{\vec{s}}^{\,\hat{2}}$ and $\hat{s}_3$ do not iso-commute in the general case $b_1 \neq b_2 \neq b_3$.

In fact, the correct expression of the second-order iso-Casimir invariant of $\widehat{SU}(2)$ is given by [39]

$$\tilde{\vec{s}}^{\,\hat{2}} * \psi - \left(\sum_{k=1}^{3} \tilde{s}_k * \tilde{s}_k\right) * \psi = \left(\frac{3}{4}\hat{I}\right) * \psi = \frac{3}{4}\psi \ ,$$
$$\tilde{s}_1 = b_2^{-1} b_3^{-1} \hat{s}_1 \ , \ \tilde{s}_2 = b_1^{-1} b_3^{-1} \hat{s}_2 \ , \ \tilde{s}_3 = b_1^{-1} b_2^{-1} \hat{s}_3 \ , \tag{6.20}$$

with redefined iso-commutation rules

$$[\tilde{s}_i \hat{,} \tilde{s}_j] = i \ \epsilon_{ijk} \tilde{s}_k \ , \tag{6.21}$$

which express more transparently the local isomorphism

$$\widehat{SU}(2) \approx SU(2) \ . \tag{6.22}$$

Nevertheless, whenever $b_1 = b_2 = b_3$, $\hat{\vec{s}}^{\,\hat{2}}$ is an acceptable invariant of the theory. As a result, the phenomenological studies by Nielsen and Picket [45] appear to imply the following mutation of spin

$$\hat{\vec{s}}^{\,\hat{2}} * \psi = \frac{3}{4}(1 - \frac{2}{3}\alpha + \frac{1}{3}\alpha^2)\psi \ , \tag{6.23}$$

where for pions

$$\alpha \cong -4 \times 10^{-3} \ , \tag{6.24}$$

with the corresponding value for kaons

$$\alpha \cong +10^{-3} \ . \tag{6.25}$$

In analogy with operators (6.15b), we have the following realization of the *Lorentz-isotopic boosts*

$$\begin{aligned} \hat{m}_{14} &= \frac{1}{2}\hat{\gamma}_1 * \hat{\gamma}_4 \ , \\ \hat{m}_{24} &= \frac{1}{2}\hat{\gamma}_2 * \hat{\gamma}_4 \ , \\ \hat{m}_{34} &= \frac{1}{2}\hat{\gamma}_3 * \hat{\gamma}_4 \ . \end{aligned} \tag{6.26}$$

The combination of operators (6.15b) and (6.25) then yields a realization of the covering of $\widehat{SO}_+^{\uparrow}$ (3.1), the isotopic $\widehat{SL}(2.C)$ symmetry with iso-commutation rules also given by Eq.s (4.22a), as the reader is encouraged to verify.

The symmetry of Eq. (6.6) is therefore the iso-spinoral covering of the Poincaré-isotopic symmetry (4.38) according to the structure

$$\hat{P}(3.1) = \widehat{SL}(2.C) \otimes \hat{T}(3.1) \ . \tag{6.27}$$

It is easy to prove that Eq. (6.6) and its adjoint (6.7) are invariant under iso-symmetry (6.26). In fact, the wavefunction transforms according to

$$\begin{aligned} \psi'(x') &= \hat{S}(\hat{\Lambda}) * \psi[\hat{\Lambda}^{-\hat{1}} * (x - a)] \ , \\ \overline{\psi}'(x') &= \overline{\psi}[\hat{\Lambda}^{-\hat{1}} * (x - a)] * \hat{S}^{-\hat{1}}(\hat{\Lambda}) \ , \\ \hat{\Lambda}^{\hat{t}} * \hat{\Lambda} &= \hat{\Lambda} * \hat{\Lambda}^{\hat{t}} = g^{-1} \ , \quad \det \hat{\Lambda} = \pm \det \hat{I} \ , \\ & \hat{S}(\hat{\Lambda}) \in \hat{P}(3.1) \ , \end{aligned} \tag{6.28}$$

while the iso-Dirac's equation transforms according to

$$\begin{aligned}(-i\,\hat\gamma^\mu * \hat\partial'_\mu + i\,\hat k) * \psi'(x') = 0\,,\\ \overline{\psi}'(x') * (-i\hat\partial'_\mu * \hat\gamma^\mu - i\,\hat k) = 0\,,\end{aligned} \tag{6.29}$$

under the particular rules

$$\begin{aligned}\hat\Lambda^{-\hat 1} * \hat\gamma_\mu * \hat\Lambda * \hat\partial'^\mu = \hat\gamma_\mu * \partial^\mu\,,\\ \hat\Lambda^{-\hat 1} = \hat\gamma_4 * \hat\Lambda^\dagger * \hat\gamma_4\,.\end{aligned} \tag{6.30}$$

In fact, the above transformation properties are verified by the following fundamental realization of the $\widehat{SL}(2.\mathcal{C})$ symmetry

$$\begin{aligned}\hat{\mathcal{S}}(\hat\Lambda) &= \Big(e^{\frac12\hat\gamma_2*\hat\gamma_3*\theta_1}\, e^{\frac12\hat\gamma_3*\hat\gamma_1*\theta_2}\, e^{\frac12\hat\gamma_1*\hat\gamma_2*\theta_3} \times \\ &\times\; e^{\frac12\hat\gamma_1*\hat\gamma_4*w_1}\, e^{\frac12\hat\gamma_2*\hat\gamma_4*w_2}\, e^{\frac12\hat\gamma_3*\hat\gamma_4*w_3}\Big)\,\hat I\,,\end{aligned} \tag{6.31}$$

where the last three expressions hold for speeds along the x_1, x_2, and x_3 axis, respectively.

For the invariance under *iso-space-inversions*, we have the realization

$$\begin{aligned}\hat{\mathcal{S}} &= \eta_p\hat\gamma_4\,,\ \eta_p = \pm1, \pm i\,,\\ \hat{\mathcal{S}}^{-\hat 1} * \hat\gamma_k * \hat{\mathcal{S}} &= -\hat\gamma_k\,,\quad k = 1,2,3\,,\\ \hat{\mathcal{S}}^{-1} * \hat\gamma_4 * \hat{\mathcal{S}} &= \hat\gamma_4\,.\end{aligned} \tag{6.32}$$

For the invariance under the *iso-time-inversions* we have the realization

$$\begin{aligned}\hat{\mathcal{S}} = \;&\eta_T\hat\gamma_5 * \hat\gamma_4\,,\ \eta_T = \pm1, \pm i\,,\ \hat\gamma_5 = \hat\gamma_4 * \hat\gamma_1 * \hat\gamma_2 * \hat\gamma_3\,,\\ &\hat{\mathcal{S}}^{-\hat 1} * \hat\gamma_k * \hat{\mathcal{S}} = \hat\gamma_k\,,\\ &\hat{\mathcal{S}}^{-\hat 1} * \hat\gamma_4 * \mathcal{S} = -\hat\gamma_4\,.\end{aligned} \tag{6.33}$$

It should be noted that the iso-time-reversal is equivalent to the operation of complex iso-conjugation which, for the particular form of realization of hadronic mechanics assumed in this paper (with the operators $T = G$ in Eq.s (4.20)), coincides with the conventional complex conjugation.

The reader should keep in mind that the iso-Dirac's Eq. (6.6) *does not* represent a free particle, but a particle in condition of total or partial immersion within a hadronic medium (e.g., a particle inside a pion or a kaon).

As the reader will recall from Sect. 2, the conceivable spinorial asymmetry in Rauch's experiments sees its origin in the possible mutation of the magnetic moment. In order to attempt an interpretation of Rauch's experiments, we must the efore identify the mutation of the magnetic and electric dipole moments which correspond to spin mutations of type (6.22).

For this purpose, we consider the following *isotopic generalization of Dirac's equation under an external electromagnetic field*

$$\begin{aligned}&[\hat\gamma^\mu * (-i\hat\partial_\mu + \frac{e}{c}A_\mu) - i\,\hat k] * \psi\\ &\stackrel{\text{def}}{=} (\hat\gamma^\mu * \hat\pi_\mu - i\,\hat k) * \psi = 0\,.\end{aligned} \tag{6.34}$$

In order to identify the physical implications of the above lifting, it is important to identify which physical quantity remains unchanged and which other quantity is altered.

First, *the electromagnetic field is not mutated by lifting* (6.34). This is crucial for physical consistency, because the electromagnetic field is external to the hadronic medium considered and, as such, its origin cannot be altered. In fact, the four-potentials A_μ are the conventional ones; the associated *iso-electromagnetic fields* coincides with the conventional ones

$$\hat F_{\mu\nu} = \hat\partial_\mu * A_\nu - \hat\partial_\nu * A_\mu \equiv \partial_\mu A_\nu - \partial_\nu A_\mu = F_{\mu\nu}\,, \tag{6.35}$$

and the iso-commutation rules of the $\hat\pi$-operators coincide with the conventional ones

$$[\hat\pi_\mu \hat{,} \hat\pi_\nu] = \frac{e}{c}\hat F_{\mu\nu} \equiv \frac{e}{c}F_{\mu\nu} = [\pi_\mu, \pi_\nu]\,. \tag{6.36}$$

The quantities that are mutated by lifting (6.34) in an irreducible way are the dipole moments, owing to the mutation of the γ-matrices. To see this occurrence, introduce the spin tensor of the theory

$$\hat\sigma_{\mu\nu} = \frac{1}{2i}(\hat\gamma_\mu * \hat\gamma_\nu - \hat\gamma_\nu * \hat\gamma_\mu)\,, \tag{6.37}$$

and the second-order form corresponding to Eq. (6.34)

$$\begin{aligned}&(\hat\gamma^\mu * \hat\pi_\mu + i\,\hat k) * (\hat\gamma^\nu * \hat\pi_\nu - i\,\hat k)\\ =\;& (\hat\pi^\mu * \hat\pi_\mu + \hat k^2) + \frac{e}{2c}\hat\sigma^{\mu\nu} * \hat F_{\mu\nu}\,.\end{aligned} \tag{6.38}$$

By closely following the pattern of the conventional case (ref. [52], p. 177), one can see that the second-order equation corresponding to Eq. (6.34) is Eq. (5.18) plus the term

$$\frac{e}{2c}\hat{\sigma}^{\mu\nu} * \hat{F}_{\mu\nu} = \frac{e}{2c}\hat{\gamma}^{\mu} * \hat{\gamma}^{\nu} * \hat{F}_{\mu\nu} , \tag{6.39}$$

which, by using realization (6.5), can be expressed in the form

$$\begin{aligned} & \frac{e}{2m_0 c}(\hat{\sigma}^k * H_k - i\hat{\alpha}^k * E_k) = \\ = \; & \frac{e}{2m_0 c}(\tilde{\sigma}^k H_k - i\tilde{\alpha}^k E_k) , \end{aligned} \tag{6.40}$$

where

$$\begin{aligned} \hat{\sigma}_k &= \tilde{\sigma}_k \hat{I} = b_k \sigma_k \hat{I} , \\ \hat{\alpha}_k &= \hat{\alpha}_k \hat{I} = b_k \alpha_k \hat{I} . \end{aligned} \tag{6.41}$$

We reach in this way the following *mutation of the magnetic and electric dipole moments*, respectively

$$\tilde{\mu}_k = -\frac{e}{2m_0 c}\tilde{\sigma}_k = \frac{b_k}{b_4}\mu_k , \tag{6.42.a}$$

$$\tilde{m}_k = i\,\frac{e}{2m_0 c}\tilde{\alpha}_k = \frac{b_k}{b_4}m_k , \tag{6.42.b}$$

first introduced in ref. [15], p. 803.

As an illustration, consider again metric (4.20) for which

$$b_1^2 = b_2^2 = b_3^2 = 1 - \frac{1}{3}\alpha , \quad b_4^2 = 1 + \alpha . \tag{6.43}$$

Then, we have the mutated values along the third axis

$$\tilde{\mu}_3 = -\frac{e b_3}{2m_0 c_0 b_4}\sigma_3 = \left(\frac{1-\alpha/3}{1+\alpha}\right)^{1/2} \mu_3 , \tag{6.44.a}$$

$$\tilde{m}_3 = -i\frac{e b_3}{2m_0 c_0 b_4}\alpha_3 = \left(\frac{1-\alpha/3}{1+\alpha}\right)^{1/2} \mu_3 , \tag{6.44.b}$$

where α is given by Eq. (4.30) for pions and by Eq. (4.31) for kaons.

Eq. (6.34) is evidently invariant under the full iso-spinorial space-time symmetry $\hat{P}(3.1)$, as it is the case for Eq. (6.6). In addition, Eq. (6.34) is invariant under Gasperini's isotopic gauge transformations (5.24). Finally, Eq. (6.34) is invariant under the following *iso-charge-conjugation*

$$\psi' = \eta_c^* \hat{S}^{-\hat{1}} * \psi , \; \overline{\psi}' = -\eta_c \psi^T * \hat{S} , \tag{6.45}$$

with realization for Pauli's form of the gamma matrices

$$\hat{S} = \hat{\gamma}_2 * \hat{\gamma}_4 , \tag{6.46}$$

and similar realizations for other representations.

In conclusion, *all physical characteristics of a particle, whether kinematical or intrinsic, are mutated for the iso-Dirac's equations (6.6) or (6.33), including: rest energy, charge, spin, magnetic and electric moments, etc. exactly as expected for particles in condition of total or partial immersion within hyperdense hadronic matter.*

The most direct physical application of the theory considered is a quantitative interpretation of Rauch's experiments, which will be studied in the next section. In this section we would like to indicate a most speculative, yet intriguing possibility of application to quarks theories.

As well known, the final interpretation of the intrinsic characteristics of quarks, such as their fractional charge, has remained substantially elusive until now. Yet, when assumed as physical constituents of hadrons, quarks are the ideal entities representing the physical conditions necessary for iso-field equations, inasmuch as they are in condition of total immersion within hadronic matter.

It is easy to see that *the isotopic lifting of the charge, Eq. (6.11) can indeed produce a fractional charge, of course, on purely formal grounds.* In fact, the iso-inner product (4.20) is usually normalized to the isounit $\hat{I}$, which implies that

$$\int \psi^{\dagger} * \psi d^3x = 1 . \tag{6.47}$$

As a result, Eq. (6.11) readily allows the charge mutation

$$Q = e \to \hat{Q} = e\, b_4^2 = \frac{e}{3} , \quad b_4 = \frac{1}{\sqrt{3}} . \tag{6.48}$$

It should be stressed that the above remarks have been introduced merely to illustrate the possibility, mentioned in the literature, of actually "building" quarks in the interior of hadrons as hadronic bound states of elementary constituents (e.g. the eletons). In fact, the actual construction of the model would require iso-tensorial products of iso-Hilbert spaces, resulting into a sort of granular particularization of the generalized interior metric for which values (4.29) are a sort of statistical average. This is evidently due to the fact that different fractional values of the charge would require different values of the b_4 parameter and, thus, locally varying internal metrics, exactly as functionally assumed in this work.

We hope to investigate these intriguing possibilities at some future time.

We conclude this section with the indication that the further generalization of the iso-Dirac's equations (6.6) or (6.34) to the case of unstable orbits is readily done by lifting the Hermiticity condition on the isotopic element T. This results in the *Lie-admissible generalization of Dirac's equation*

$$(\gamma^{\triangleright}_{\mu} \triangleright p^{\mu} + i\, k^{\triangleright 2^{\triangleright}}) \triangleright \psi(x) = 0\,,$$
$$\gamma^{\triangleright} : \{\gamma^{\triangleright}_{\mu}, \gamma_{\nu}\}^{\triangleright} = \gamma^{\triangleright}_{\mu} \triangleright \gamma^{\triangleright}_{\nu} - \gamma^{\triangleright}_{\nu} \triangleright \gamma^{\triangleright}_{\mu} = \gamma^{\triangleright}_{\mu} R \gamma^{\triangleright}_{\nu} - \gamma^{\triangleright}_{\nu} R \gamma^{\triangleright}_{\nu} = 2 g^{\triangleright}_{\mu\nu} I^{\triangleright}\,,$$
$$I^{\triangleright} = R^{-1}\,, \quad g^{\triangleright} = R\eta\,, \tag{6.49}$$

with adjoint

$$\overline{\psi}(x) \triangleleft \left(p^{\mu} \triangleleft \gamma^{\triangleleft}_{\mu} + i^{\triangleleft} k^{\triangleleft 2}\right) = 0$$
$$\gamma^{\triangleleft} : \{\gamma^{\triangleleft}_{\mu}, \gamma^{\triangleleft}_{\nu}\}^{\triangleleft} = \gamma^{\triangleleft}_{\mu} \triangleleft \gamma^{\triangleleft}_{\nu} - \gamma^{\triangleleft}_{\nu} \triangleleft \gamma^{\triangleleft}_{\mu} = \gamma^{\triangleleft}_{\mu} S \gamma^{\triangleleft}_{\nu} - \gamma^{\triangleleft}_{\nu} S \gamma^{\triangleleft}_{\mu} = 2 g^{\triangleleft}_{\mu\nu} {}^{\triangleleft}I\,,$$
$${}^{\triangleleft}I = S^{-1}\,, \quad g^{\triangleleft} = S\eta\,, \quad S = R^{\dagger}\,, \tag{6.50}$$

with similar extensions for the case of Eq. (6.33).

The underlying space-time symmetry is the generalization, from the Poincaré-isotopic symmetry $\hat{P}(3.1)$, to the *Poincaré-admissible symmetry* submitted in monograph [36,II], with the Galilean counterpart submitted in the original proposal [33]. We are here referring to a generalization of the isotopic $\widehat{SL}(2.C)$ symmetry, from the modular-isotopic structure (6.27) to a nontrivial, Lie-admissible, bi-modular structure which, for the case of the time evolution, assumes precisely the form of the ultimate dynamical equations of the theory, Eq. (4.2b), and with similar expressions for all other admissible transformations.

As an example, the $SU(2)$-*admissible symmetry* can be defined via the transformation law of a given quantity A

$${}^{\triangleleft}SU^{\triangleright}(2):$$
$$A' = I^{\triangleright} \left(\prod_{k=1}^{3} e^{\frac{1}{2}\epsilon_{ijk}\gamma^{\triangleright}_{j} \triangleright \gamma^{\triangleright}_{k} \triangleright \theta_i} \right) \triangleright A \triangleleft \left(\prod_{k=1}^{3} e^{-\frac{1}{2}\epsilon_{ij\theta}\theta_i \triangleleft \gamma^{\triangleleft}_{j} \triangleleft \gamma^{\triangleleft}_{k}} \right) {}^{\triangleleft}I. \tag{6.51}$$

Similar expressions hold for the *Lorentz-admissible boosts*, as well as for the translations. We can then denote the *Poincaré-admissible symmetry* with the self-explanatory notation

$${}^{\triangleleft}P^{\triangleright}(3.1) = {}^{\triangleleft}SL^{\triangleright}(2.C) \otimes {}^{\triangleleft}T^{\triangleright}(3.1)\,, \tag{6.52}$$

and refer the interested reader to ref. [36,II] for other approaches (e.g., the derivation of the symmetry via the so-called symplectic-admissible generalization of the symplectic geometry).

Note that the invariance of the equations is maintained under this further generalization, trivially, because one given equation, say, Eq. (6.48) implies the selection of one given direction in time, with consequential activation of only the branch of symmetry (6.52) for that direction of time. This essentially reduces the problem of the symmetry of Eq. (6.49) to that of Eq. (6.34).

We are now in a position to elaborate Postulate 4.1 on the most general possible mutations, those embodied by the concept of eleton. In fact, the structure of transformation laws (6.50) clearly illustrates the maximization of the *alteration* of the intrinsic characteristics of the original particle under the most general interactions that are conceivable at this time, that is, for the most general possible nonlocal, nonconservative and unstable conditions of the particle considered.

The reader should be aware that Lie-admissible bi-modules (6.50) are some of the most complex and yet unexplored mathematical structures of the contemporary literature. Here we merely wanted to indicate their *existence*, with the understanding that we cannot possibly enter into a study of their properties at this time.

7 Applications to Rauch's Experiments

We shall now show that the iso-Dirac's equations (6.6) and its underlying Poincaré-isotopic symmetry provide a direct and quantitative interpretation of Rauch's experiments on the apparent spin-asymmetry of thermal neutrons under external nuclear fields (Sect. 2).

It appears recommendable for clarity as well as for completeness, to review first the preceding contributions in the topic [15,20,21,22,23] and then point out the intended experimental application of Eq. (6.6), as well as its differentiation with the preceding research.

As recalled earlier, the hypothesis of mutation of the intrinsic characteristics of particles was submitted in ref. [15], with particular reference to a conceivable alteration of the spin and magnetic moments of particles in the transition from motion in vacuum to motion within an hyperdense hadronic medium.

The quantitative study of the hypothesis was based on the following generalization of the conventional Dirac's equation, loc. cit., Eq.s (4.20.5)-

(4.20.8)

$$\left\{\left[\left(\gamma^{\mu}e^{i}_{\mu}+me\right)^{\rm free}_{SA}-f_{e}^{\rm Elm}\right]_{SA}-F_{e}^{\rm strong}\right\}_{NSA} \tag{7.1.a}$$

$$= \left\{\left[\gamma^{\mu}-\Gamma^{\mu}(x,\dot{x},e,\bar{e},A,\ldots)\right]e^{i}_{\mu}-f_{e}^{SA}\right\}_{NSA}=0\ ,$$

$$F_{e}^{\rm strong}=f_{e}^{SA}+\left[\Gamma^{\mu}(x,\dot{x},e,\bar{e},A,\ldots)e^{i}_{\mu}\right]^{NSA}\ , \tag{7.1.b}$$

$$f_{e}^{SA}=f_{\rm Elm}^{SA}+f_{\rm strong}^{SA}\ , \tag{7.1.c}$$

$$e^{i}_{\mu}=\partial_{\mu}e(x)\ , \tag{7.1.d}$$

where: $SA(NSA)$ represents the preservation (violation) of the *conditions of variational self-adjointness* for the existencc of a conventional representation via a first-order action principle [18]; the symbol "e" ("$\bar{e}$") represents the wave- function of an eleton (antieleton); and the Γ^{μ} are suitable 4×4 matrices.

The central contention of hypothesis (7.1) was that the conventional SA form of Dirac's equation has resolved the characterization of one, dimensionless, charged particle moving in vacuum under an *external electromagnetic field.* Conventional models for a charged particle under additional strong interactions are also self-adjoint, with consequential lack of mutation of the intrinsic characteristics, but this implies the necessary approximation of the particle and of its wave-packet as being point-like (see ref.s [18,19]). An effective way of representing the extended and therefore deformable character of hadrons is that of adding a non-self-adjoint coupling, precisely as it happens at the primitive Newtonian level. In a first-order equation, such NSA coupling must necessarily be linear in the field derivatives. Nevertheless, the new couplings now acquire a dominant role in the characterization of the spin and magnetic moments over the conventional Dirac's term. In turn, this mechanism is at the foundation of the process of mutation of the intrinsic characteristics of particles [15].

In fact, the spin of eleton (7.1) was computed in terms of the generalized spin tensor, Eq.s (4.20.9) -(4.20.12), loc. cit.,

$$\begin{aligned}\Gamma^{\mu\nu} &= \frac{1}{2}[(\gamma^{\mu}-\Gamma^{\mu})(\gamma^{\nu}-\Gamma^{\nu})-(\gamma^{\nu}-\Gamma^{\nu})(\gamma^{\mu}-\Gamma^{\mu})]\\ &= \frac{1}{2}f^{2}(x)\sigma^{\mu\nu}\ ,\end{aligned} \tag{7.2.a}$$

$$\sigma^{\mu\nu} = \frac{1}{2}(\gamma^{\mu}\gamma^{\nu}-\gamma^{\nu}\gamma^{\mu})\ . \tag{7.2.b}$$

The underlying algebraic structure was worked out in the preceding sections of the paper, via the so-called *genotopy* of the universal enveloping associative algebra $\mathcal{A}(\mathbf{SU}(2))$ of the $\mathbf{SU}(2)$ algebra into an algebra $\mathcal{U}(\mathbf{SU}(2))$ that is a *genuine Lie-admissible algebra* and, as such, generally nonassociative

$$\mathcal{A}(\mathbf{SU}(2))\to\mathcal{U}(\mathbf{SU}(2))\ , \tag{7.3}$$

(the term "genotopy" was submitted in order to differentiate the above lifting from the "isotopy", because of the alteration of the axioms of the original algebra). Genotopy (7.3) of the envelope was assumed to be the most general possible representation of non-self-adjoint interactions.

For the particular case of the *flexible Lie-admissible algebras* genotopy (7.3) was given by Eq. (4.19.10), loc. cit.,

$$\begin{aligned}&\mathcal{A}(\mathbf{SU}(2))\ :\quad J_iJ_j=\text{ASSOC.}\to\\ \to\ &\mathcal{U}(\mathbf{SU}(2))_{\lambda,\mu}\ :\quad J_i\circ J_j=\lambda(x)J_iJ_j-\mu(x)J_jJ_i=\text{NONASSOC.}\ ,\end{aligned} \tag{7.4}$$

with consequential mutation of the magnitude of the spin, Eq.(2.19.11), loc.cit.,

$$\begin{aligned}\sum_{k=1}^{3}J_kJ_k=\hbar_j^2(j+1)\quad&\to\quad\sum_{k=1}^{3}J_k\circ J_k=\hbar^2 f(x)j(j+1)\ ,\\ f(x)\ &=\ \lambda(x)-\mu(x)\ ,\end{aligned} \tag{7.5}$$

and exponential form for $J_k=\sigma_k$

$$e^{\frac{1}{2}\theta\sigma_3}\to e^{\frac{1}{2}\theta f(x)\sigma_3}\ . \tag{7.6}$$

The mutation of the magnetic and electric dipole moments was a trivial consequence of mutation (7.5) and was worked out in Eq.s (4.20.14) - (4.20.16), i.e.,

$$\hat{\vec{\mu}}=[1+f(x)]\frac{e\hbar}{2mc}\vec{\sigma}\ ,\quad \hat{m}=i[1+f(x)]\frac{e\hbar}{2mc}\vec{\sigma}\ . \tag{7.7}$$

Notice the insistence in the *concepts of spin and dipole moments as local functions* e.g., for a proton in the core of a star. Such local dependence is necessary to recover conventional values when the particle exits the hadronic medium and returns to move in vacuum under SA forces only, thus reacquiring conventional values of the spin and dipole moments [15].

As one can see, mutations (7.5) and(7.7) are equivalent to mutations (6.18) and (6.41), respectively. It is important to clarify that the fundamental equations (7.1) of ref. [15] are also equivalent to Eq. (6.6) of this paper. The problem is resolved by the Birkhoffian generalization of Hamiltonian mechanics and its underlying Pfaffian generalization of the conventional action principle [19] which, for the scope at hand, can be written in the simple relativistic form

$$\begin{aligned} \delta A &= \delta \int_{t_1}^{t_2} [p^\mu \eta_{\mu\nu} \dot{x}^\nu - H(x,p)]\, dt = 0 \rightarrow \\ &\rightarrow \delta \hat{A} = \delta \int_{t_1}^{t_2} [p^\mu g_{\mu\nu}(p,\ldots) \dot{x}^\nu - H(x,p)]\, dt = 0 \,. \end{aligned} \tag{7.8}$$

The transition from SA to NSA forces of Eq.(7.1) is precisely the transition from the canonical to the Pfaffian variational principle underlying Eq. (6.6). Such transition can be characterized by the lifting

$$\eta \rightarrow g(\dot{x},\ldots) = T(\dot{x},\ldots)\eta \,, \tag{7.9}$$

which is exactly the mechanism of deformation of the Minkowski metric at the foundation of the Poincaré-isotopic symmetry and related iso-field equations.

Regrettably, this author was unaware of Rauch's experiments at the time of writing paper [15] (1978). As a result, Eq.s (7.6) and (7.7), which are in a form ready to be applied to the data elaborations of Rauch's tests (see below), were not used for that purpose. The NSA equations were used instead to investigate the plausibility of a (generally small) violation of Pauli's exclusion principle (referred to in the title of the memoir), with the understanding that we are referring to conceivable *internal* deviations inside a nucleus or a hadron, while the conventional statistical character for the state as a whole is recovered identically (as more recently proved in ref. [31]).

The subsequent paper [20] presented a detailed proof of the mathematical consistency of the crucial exponentiation (7.6) via a reinspection of the generalization of the Poincaré-Birkhoff-Witt theorem for a flexible Lie-admissible algebra.

For instance, the following more detailed elaborations of Eq. (7.6) were worked out, Eq. (3.8)-(3.9), ref. [20].

$$e^{-\frac{i}{2}\theta\sigma_3}|_{\mathcal{A}(\mathbf{SU}(2))} = e^{-\frac{i}{2}\theta\sigma_3} = 1\cos\frac{\theta}{2} - i\sigma_3 \sin\frac{\theta}{2} \rightarrow$$

$$\begin{aligned} &\rightarrow e^{-\frac{i}{2}\theta\sigma_3}|_{\mathcal{U}(\mathbf{SU}(2))_{\lambda,\mu}} = 1 - \frac{i}{2}\theta\sigma_3 + \frac{1}{2!}\left(\frac{i\theta}{2}\right)^2 \sigma_3 \circ \sigma_3 \\ &-\frac{1}{3!}\left(\frac{i\theta}{2}\right)^3 (\sigma_3 \circ \sigma_3) \circ \sigma_3 + \ldots \\ &= \frac{(\lambda+\mu)-1}{\lambda+\mu} + \frac{1}{\lambda+\mu}\cos\left[(\lambda+\mu)\frac{\theta}{2}\right] \\ &-\frac{i}{\lambda+\mu}\sigma_3 \sin\left[(\lambda+\mu)\frac{\theta}{2}\right] . \end{aligned} \tag{7.10}$$

The authors of paper [20] were also unaware of Rauch's tests at the time of writing that paper (1979). The results of the analysis were therefore used for an elaboration of the plausibility of internal violations of Pauli's exclusion principle, from the conceivable internal variations of the statistical character of the constituents. In fact, in Table I, p. 905 [20], nuclear volumes were plotted against the ideal volumes without mutual penetration of the constituents. This diagram essentially represents the known fact (Sect. 3) that nuclear volumes imply about 1% mutual penetration of the charge distribution of the nucleonic constituents. It was then inferred that these non-local and expectedly non-Hamiltonian (that is, NSA) effects, may well result into alterations of the spin of the nucleons of type (7.5) along the hypothesis of ref. [15].

The following paper [21] was written in 1980 after the author learned about Rauch's test. The paper was then intended for the primary purpose of elaborating Rauch's data via mutations (7.3)- (7.7). After a detailed review of the underlying methodology and of Rauch's tests, paper [21] assumed the following form of the flexible Lie-admissible algebra

$$J_i J_j|_{\mathcal{A}} \rightarrow J_i \circ J_j|_{\mathcal{U}_{1,\epsilon}} = J_i J_j + \epsilon \dot{J}_j J_i \,,\ \epsilon \approx 0. \tag{7.11}$$

The exact spinorial transformation of the wave-function was then generalized into the form

$$\psi' = (1 + e^{-\frac{i}{2}\theta\sigma_3})\psi \rightarrow \psi' = (1 + e^{-\frac{i}{2}(1+\epsilon)\theta\sigma_3})\psi. \tag{7.12}$$

The *mutated intensity modulation* for the case of null original polarization, Eq. (2.6), became

$$I'|_{P=0} = \frac{I}{2}[1 + \frac{\epsilon^2}{2} + \epsilon\cos\chi + (1+\epsilon)\cos\chi\cos(1+\epsilon)\frac{\theta}{2}] \,, \tag{7.13}$$

with corresponding *mutated polarization modulation*

$$\vec{P}'|_{P=0} = \frac{\sin\chi \sin(1+\epsilon)\frac{\theta}{2}}{1+\frac{\epsilon^2}{2}+\epsilon\cos\chi+(1+\epsilon)\cos\chi\cos(1+\epsilon)\frac{\theta}{2}} . \tag{7.14}$$

The best fit for the diagram of Fig. 3 was conducted in ref. [6] and resulted in the formula

$$\begin{aligned} I' &= P_1 + P_2\cos(2\pi\frac{x}{P_3}+P_4) , \\ P_1 &= \frac{1}{2} , \ P_2 = \frac{1}{2}\cos\chi , \ P_4 = 0 , \ P_3 = \text{const} \neq 0 . \end{aligned} \tag{7.15}$$

The Lie-admissible mutation of the above best fit submitted in ref. [21] was given by

$$\begin{aligned} \hat{I}' &= \hat{P}_1 + \hat{P}_2\cos[2\pi(1+\epsilon)\frac{x}{P_3} + (1+c)P_4] , \\ \hat{P}_1 &= \frac{1}{2}(1+\epsilon\cos\chi) , \quad \hat{P}_2 . \end{aligned} \tag{7.16}$$

The experimental angle available at that time (1980) was given by Eq. (2.12). This resulted in the best fit, Eq. 4.21) of ref. [21], i.e.

$$4\pi = \frac{x}{P_3} = 716.8^\circ , \tag{7.17}$$

with numerical value

$$\epsilon \cong 4.46 \times 10^{-3} . \tag{7.18}$$

In this way, paper [21] achieved three objectives. First, it presented a direct and quantitative elaboration of Rauch's data. Second, it showed that the mutation always produces a median angle less than 720°, called *angle slow-down effect* (recall from Sect.2 that in *all* tests conducted until now on the spinor symmetry of neutrons, the median angle is always lower than the expected 720°). Third, paper [21] restored the exact spinorial symmetry at the higher $\mathcal{U}(SU))_{1+\epsilon}$ level, because the factor $(1+\epsilon)$ in the exponent compensates the angle slow-down effect, resulting in value (4.22) of Ref. [21], i.e.,

$$\begin{aligned} (1+\epsilon) &\cong \frac{P_3}{x} = \frac{720^\circ}{716.8^\circ} , \\ (1+\epsilon)\tfrac{\theta}{2}|_{\theta=716.8^\circ} &= 720^\circ . \end{aligned} \tag{7.19}$$

The value of the magnitude of the spin and of its third component corresponding to mutation (7.19) were computed in Eq.s (4.26) and (4.27), loc. cit., yielding the values

$$|s|_{\text{neutron}} = 0.50167 , \tag{7.20.a}$$
$$s_3 = 0.49777 , \tag{7.20.b}$$

with consequential loss of the exact internal applicability of Pauli's exclusion principle, Heisenberg's uncertainty principle and, of course, of Einstein's Special Relativity, in favor of suitable covering laws.

Paper [21] then concluded with the proposal of a number of additional tests for the experimental resolution of the topic (see the concluding remarks).

The subsequent paper [22] of 1981 presented a comprehensive analysis of the algebra of Pauli's matrices σ_k under the mutation of the underlying enveloping algebra of ref. [15]

$$\begin{aligned} &\mathcal{A}(\mathrm{SU}(2)) : \quad \sigma_i\sigma_j = \text{ASSOC.} \to \mathcal{U}(SU(2))_{\lambda,\mu} : \\ &: \quad \sigma_i \circ \sigma_j = \frac{1}{2}\rho(\rho+1)\sigma_i\sigma_j + \frac{1}{2}\rho(\rho-1)\sigma_j\sigma_i \\ &\lambda = \frac{1}{2}\rho(\rho+1) , \quad \mu = \frac{1}{2}\rho(\rho-1) . \end{aligned} \tag{7.21}$$

The magnitude and third component of the spin resulted to be equal to the conventional ones

$$\begin{aligned} \vec{s}^{2^0} \circ |m\rangle &= \frac{3}{4}|m\rangle , \quad s_k = \frac{1}{2\rho}\sigma_k , \ \hbar = 1 \\ s_3 \circ |m\rangle &= m|m\rangle , \quad m = \pm\frac{1}{2} . \end{aligned} \tag{7.22}$$

Nevertheless, powers higher than two exhibited deviations from conventional values, e.g.

$$(\vec{s}^{2^0})^n \circ |m\rangle = \left(\frac{3}{4}\rho^2\right)^n |m\rangle . \tag{7.23}$$

These results were interpreted as a form of short-term periodical oscillation of the spin components under an unspecified external force characterized by mutation (7.21) (called *spin fluctuation*). An illustrative classical example was also worked out.

Paper [21] then passed to the identification of possible physical causes underlying Rauch's data, via a detailed study of the short range interactions of the thermal neutrons with the electric and magnetic fields of nuclei.

It essentially emerged that short range electric interactions do not produce measurable effects, as expected (the neutron being, after all, neutral). Nevertheless, short range magnetic interactions can indeed imply spin fluctuations well within the current experimental capabilities.

Consider the particular case of a particle of mass M with non-null magnetic moment $\vec{\mu}$ interacting with an external magnetic field according to the values (in self-explanatory symbols)

$$\vec{B} = (0,0,B)\,, \quad \vec{\mu} = g_s \mu_N \vec{s}/\hbar = \frac{1}{2\rho} g_s \mu_N \vec{\sigma}\,. \tag{7.24}$$

In this case the wave-functions for algebra $\mathcal{A}(\mathbf{SU}(2))$ is given by Eq. (3.15), Ref. [22],

$$\begin{aligned} \psi_m(y) &= \exp(ik_0 y)[\cos(\rho k_1 y/2 - 2im\rho \sin(\rho k_1 y))]|m\rangle \\ k_1 &= -g_s \mu_N BM/k_0\,, \quad k_0 = (2BM/\hbar^2)^{1/2}\,, \end{aligned} \tag{7.25}$$

with the oscillating amplitude

$$|\psi_m(y)|^2 = 1 - (1-\rho^2)\sin^2(\rho k_1 y/2)\,, \quad m = \pm 1/2\,, \tag{7.26}$$

and periodicity

$$\Delta y = 4\pi/\rho k_1 = 4\pi\hbar^2 k_0/\rho g_s \mu_N BM = 4\pi\hbar^2 k_0/\hat{g}_s \mu_N BM\,. \tag{7.27}$$

In this way ref. [22] reached the important result that, under genotopy (7.21), we have an alteration/fluctuation of the gyromagnetic spin factor (called *effective gyromagnetic spin factor*)

$$\hat{g}_s = \rho g_s\,. \tag{7.28}$$

Suppose now that the magnetic field $\vec{B}$ is that in the vicinity of a nucleus

$$\vec{B}_, = (z\alpha_f \hbar/e_0 c R_0^3)\vec{r} \times \vec{v}\,, \quad (r < R_0) \tag{7.29}$$

where α_f, Ze_0 and R_0 are the fine structure constant, nuclear charge and nuclear radius, respectively.

For a neutron beam with impact parameter p and velocity v, field (7.29) produces the spin fluctuation, Eq. (4.7), loc. cit.,

$$\begin{aligned} \Delta_+\vec{s} &= ZR_1(\rho^{-2} - R^{-2})^{1/2}(-s_y\cos\varphi; s_x\cos\varphi + s_y\sin\varphi; -s_y\sin\varphi)\,, \\ R_1 &= -g_s\alpha_f\hbar/M\ p\ c = 5.87 \times 10^{-18} m\,, \end{aligned} \tag{7.30}$$

where R is the radius of the unscreened nuclear field, which becomes after averaging over $-\pi/2 < e < \pi/2$, and $0 < p < R$,

$$\Delta_+\vec{s} = ZR_1R^{-1}(-s_y, s_x, 0)\,. \tag{7.31}$$

The re-elaboration of the intensity modulation under algebra $\mathcal{U}(\mathbf{SU}(2))$ was done in Eq.s (4.12)-(4.14), loc. cit., resulting in the equation

$$\begin{aligned} I(D) &= [1 - 2\epsilon\sin^2(k_2D/4)\cos^2(k_1D/4)\,, \\ &\rho = 1-\epsilon\,, \quad \epsilon > 0, \epsilon \approx 0\,, \end{aligned} \tag{7.32}$$

where D is the thickness of the phase shifter.

Ref. [22] then computed explicitly the above quantity for the case of the silicon for which

$$\begin{aligned} N &= 5.19 \times 10^{28} m^3\,, \quad Z = 14\,, \\ &R = 1.5 \times 10^{-11} m\,. \end{aligned} \tag{7.33}$$

For $B = 1$ Tesla and $2\pi/k_0 = 3 \times 10^{-10} m$, Ref. [22] gets the values

$$K_1D/4 = 347.4\ D/cm\,, \quad K_2D/4 = 51.1\ D/cm\,, \tag{7.34}$$

and concludes with the result that *a neutron beam can experience spin fluctuations (alterations of the gyromagnetic spin factor) caused by the magnetic field in the vicinity of nuclei which are indeed measurable.* In fact, for values (7.33) and (7.34) we have a measurable *decrease* of the intensity modulation characterized by the factor

$$(1-2\epsilon)^2 \approx 0.96\,. \tag{7.35}$$

The subsequent paper [23] of 1983 elaborated in great details the formulation of the spin algebra under mutation (7.21), by confirming the results achieved in ref. [22] for $s = \frac{1}{2}$ and by extending them to higher spin values. Paper [23] then re-examined the exponentiation in the nonassociative envelope $\mathcal{U}(SU(2))_{\lambda,\mu}$ resulting in expressions of type (7.6), or(7.10), i.e.,

$$U_k(\theta)e^{\frac{i}{2}\theta\sigma_3}|_{\mathcal{U}_{\lambda,\mu}} = \frac{1}{\rho^2}I + \frac{i}{2\rho^2}\sigma_j\sin(\rho^2\theta) + \frac{1}{4\rho^2}\sigma_j^2[\cos(\rho^2\theta) - 1]. \tag{7.36}$$

The transformations of the spin generators were also studied, resulting in Eq. (4.8) (loc. cit.), i.e.,

$$U_3 \circ \sigma_1 \circ U_3^t = \frac{1}{2\rho^2}[\rho^2 - 1 + (\rho^2+1)\cos(\rho^2\theta)]\sigma_1 - \frac{1}{\rho}\sin(\rho^2\theta)\sigma_2\,,$$

$$U_3 \circ \sigma_2 \circ U_3^t = \frac{1}{\rho}\sin(\rho^2\theta)\sigma_1 + \frac{1}{2\rho^2}[\rho^2 - 1 + (\rho^2+1)\cos(\rho^2\theta)]\sigma_2 ,$$
$$U_3 \circ \sigma_3 \circ U_3^t = \sigma_3 , \qquad (7.37)$$

which clearly show the departure from a pure spin system for $\rho \neq 1$, with mutated periodicity

$$\hat{\theta} = \theta[1 - (1 - \rho^2\theta^2/12 + \ldots] , \quad s = 1/2 . \qquad (7.38)$$

In this way, paper [23] confirmed an important result of paper [21] according to which *the alterations of the magnetic moment of neutrons (or spin fluctuations) caused by sufficiently intense external fields always produce a slow down of the angle of precession (the angle slow-down effect of Ref. [21]), as established by all available measures (Sect. 2).*

Paper [23] then concludes the analysis with a detailed elaboration of the electrical polarizability of the neutrons, and the result that, when such polarizability is re-inspected with algebra $\mathcal{U}(SU(2))_{\lambda,\mu}$, it should be possible to identify its upper limit.

We shall now present the applications of iso-Dirac's equation (6.33) and its underlying Poincaré-isotopic symmetry to Rauch's experiments, which can be expressed via the following points.

A: ASSOCIATIVE-ISOTOPIC ENVELOPING ALGEBRA. The first and most fundamental difference between this paper and all papers [15,20,21,22,23] is that this paper advocates the use of a generalized, yet still *associative* enveloping algebra, while all the latter papers use instead a *nonassociative* envelope. The understanding of this difference is crucial to avoid misrepresentations (or misapplications) of the results.

The roots of this selection lie in the assumed time evolution law. In this paper, we have selected the Lie-isotopic theory characterized by the fundamental time evolution law (4.13) which is still *Lie* in algebraic character. As a result, the spin must be characterized by the same algebraic structure of the time evolution, i.e., by a *Lie-isotopic algebra*. This implies that conventional quantum mechanical quantities are mutated as per Lemma 4.1, but are nevertheless *conserved.*

The fundamental algebra of our analysis is therefore the isotopic envelope

$$\mathcal{A}(\mathbf{SU(2)}) : \quad J_iJ_j = \text{ASS.} \rightarrow \hat{\mathcal{A}}(\mathbf{SU(2)}) : \quad J_i * J_j = J_iTJ_j ,$$
$$T = \text{Diag}(b_1^2, b_2^2, b_3^2, b_4^2) , \qquad (7.39)$$

with consequential isotopy of the Lie algebra

$$\mathbf{SU}(2) : \quad [J_i, J_j]_{\mathcal{A}} = J_iJ_j - J_jJ_i \rightarrow \widehat{\mathbf{SU}}(2) : \quad [J_i, J_j]_{\hat{\mathcal{A}}} = J_i * J_j - J_j * J_i . \qquad (7.40)$$

It is important to know that *the use of a non-associative envelope is unnecessary under the above assumptions.* This is due to a property identified at the time of the proposal of the Lie-isotopic theory in ref. [33] (see also Ref. [15],Sec. 4.14) which can be expressed with the following

LEMMA 7.1. A Lie-isotopic algebra attached to a non-associative Lie-admissible envelope always admits an identical representation via the algebra attached to an associative-isotopic envelope (and vice versa).

Proof. Consider a general non-associative, Lie-admissible algebra $\mathcal{U}$ with product $(A, B) = ARB - BSA$ of Eq. (4.2). Its attached antisymmetric algebra $\hat{L}$ is Lie-isotopic algebra (4.4), i.e.

$$\hat{L} = \mathcal{U}^- : [A, B]_{\mathcal{U}} = (A, B) - (B, A) = ATB - BTA ,$$
$$T = R + S . \qquad (7.41)$$

Consider now the associative-isotopic algebra $\hat{\mathcal{A}}$ with product $A * B = ATB$ of Eq.(4.6). Then its attached antisymmetric algebra $\hat{L}$ is given by

$$\hat{L} = \hat{\mathcal{A}}^- : [A, B]_{\hat{\mathcal{A}}} = ATB - BTA , \qquad (7.42)$$

and *coincides* with the algebra of Eq. (7.35) for $T = R + S$. We thus have the identity

$$\mathcal{U}^- \equiv \hat{\mathcal{A}}^- : \quad [A, B]_{\mathcal{U}} \equiv [A, B]_{\hat{\mathcal{A}}} = ATB - BTA . \quad \text{Q.E.D.} \qquad (7.43)$$

As a consequence of the above property, the results of Ref.s [15,20,21,22,23] can be *identically* reformulated in terms of a scalar-isotopy of the associative envelope [33], for instance, according to particularization of rule (7.43) for Refs. [22,23]

$$\mathcal{U} : \quad (A, B) = \frac{1}{2}\rho(\rho+1)AB + \frac{1}{2}\rho(\rho-1)BA ,$$
$$\hat{\mathcal{A}} : \quad A * B = \rho AB , \quad \rho \in \mathcal{R} ,$$
$$\mathcal{U}^- \equiv \hat{\mathcal{A}}^- ; [A, B]_{\mathcal{U}} \equiv [A, B]_{\hat{\mathcal{A}}} = \rho AB - \rho BA . \qquad (7.44)$$

After the above isotopic reformulation, another difference between the analysis of this paper and those of the preceding ones is the transition from scalar isotopy (7.44) to the operator one (7.39), i.e., the generalization $\rho \rightarrow T$. Under the assumed diagonal form of T, this implies the generalization from one parameter to four functions. This generalization is crucial to understand the behavior of the spin.

In fact, the preceding one-parameter theory cannot describe the deformation of the shape of the particles (see below). As a result, we can at best have fluctuations of the spin as per Eq.s (7.30) and (7.31). For a four-parameter theory the situation is different. In fact, besides the direct representation of the deformation of the shape of the particle, we can indeed have an alteration of the value of the spin, as per Eq.(6.23) (see next section for more insight).

The latter occurrence is due to the *differentiation between the eigenvalues of an operator and its expectation values* reviewed in Sect. 4. In fact, as clearly shown by Eq. (6.20), the iso-eigenvalues of the spin operators can indeed be the conventional ones, trivially, from the basic isomorphisms $SU(2) \approx \widehat{SU}(2)$, and we shall write for $s = \pm 1/2$

$$\begin{aligned} \hat{\vec{s}}^{\hat{2}} * |m\rangle &= \frac{3}{4}|m\rangle\,, \\ s_3 * |m\rangle &= m|m\rangle,\ m = \pm\frac{1}{2}\,. \end{aligned} \tag{7.45}$$

Nevertheless, the expectation values of the above operators can be different, and we shall write

$$\begin{aligned} \langle \hat{\vec{s}}^{\hat{2}} \rangle &\neq 3/4\,, \\ \langle s_3 \rangle &\neq 1/2\,. \end{aligned} \tag{7.46}$$

To state it differently, in conventional quantum mechanics, physical quantities are directly characterized by the eigenvalues of a Hermitean operator. Within the context of the covering hadronic mechanics, the situation is no longer that simple. In fact, the identification of the isotopy of the enveloping algebra, i.e., the identification of the isotopic element T of Eq. (4.26), is not sufficient to characterize physical quantities because of the existence of the independent degree of freedom in the definition of the inner product of the Hilbert space, i.e., the second isotopic element G of Eq.(4.20). The selection of whether or not eigenvalues and expectation values coincide is not provided by the theory, but rather by the physical conditions at hand.

This occurrence was illustrated first via the angular momentum. The $\mathbf{SO}(3)$-isotopic algebra (6.22) coincides with the conventional $\mathbf{SO}(3)$ algebra. Nevertheless, the former may describe a continuously decaying angular momentum, while the latter can only describe stationary orbits. The corresponding situation for the spin, Eq.s (7.45) and (7.46) is exactly the same.

Besides Lemma 7.1 and the use of the largest possible degrees of freedom offered by the abstract axioms of quantum mechanics, there are additional reasons emerged in more recent times for the abandonment of a non-associative envelope in favor of an associative-isotopic one, such as:

a) The use of a non-associative envelope generally implies the loss of the unit [33]. This is the case already for the flexible Lie-admissible algebras $\mathcal{U}(SU(2))$ of Eq.s (7.15), and the same situation evidently occurs for general Lie-admissible algebras $\mathcal{U}$. In turn, the lack of the unit creates rather serious problematic aspects, such as: the inability to quantize (remember that Planck's quantum of energy is the algebraic unit of quantum mechanics); etc.

b) The use of a non-associative envelope generally implies the loss of a consistent formulation of the Poincaré-Birkhoff-Witt theorem (because of the loss of ordering caused by the non-associativity of the product and other reasons) [33]. In turn, this implies the general loss of a consistent "exponentiation",with consequential, rather serious problematic aspects in identifying the applicable space-time symmetries.

c) The attempts at constructing a generalization of quantum mechanics based on a non-associative envelope meet rather serious problems of consistencies in the formulation of the Schrödinger's equations [58].

For these and other reasons, utmost priority has been given in the construction of a generalization of quantum mechanics for which the underlying enveloping algebra is still *associative*, as stressed throughout this paper.

All the above comments apply for the case when the fundamental equations of the theory are the *Lie-isotopic* forms (4.13). For the case when the more general *Lie-admissible* equations (4.2) apply, the situation is different. In fact, as indicated earlier, *the brackets characterizing the spin and other physical characteristics of particles must coincide with the brackets of the time evolution law* for physical and mathematical consistency. As a consequence, when an open non-conservative system with law (4.2) is considered, the spin algebra must necessarily be the Lie-admissible algebra $\mathcal{U}$, e.g., of type (7.3).

In this case, the conventional spin algebra $SU(2)$ is already nonassociative, as well known, and such a character persists under the generalization. The point that should be stressed is that the envelope remains fully associative as in structures (4.7), thus admitting a fully consistent, right and left unit, a consistent exponentiation into forms of type (6.51), and avoiding the problematic aspects of a non-associative extension of Schrödinger's equation, as clearly shown by Eq.s (4.3).

In this case, one can still define a Lie-isotopic algebra $\hat{L} = \mathcal{U}^-$ that is attached to the algebra $\mathcal{U}$. The point is that the Lie algebra $\hat{L}$ now has no physical meaning at all because its brackets are absent in the basic time evolution law. It is important to clarify that papers [15,20,21] were conceived for the above most general possible *non-conservative, Lie-admissible conditions* and, thus, the algebra characterizing the spin had to be non-associative, Lie-admissible and non-Lie.

B: QUANTITATIVE INTERPRETATION OF THE EXPERIMENTAL DATA. For the case of the conventional Dirac's equation, Rauch's experiments test the spinorial property of the wave-function. For spin flips around the third axis, this property can be expressed in 4-dimension by the familiar expression

$$SU(2): \quad \psi' = \hat{R}(\theta_3)\psi = e^{\gamma_1\gamma_2\frac{\theta_3}{2}}\psi. \tag{7.47}$$

The data of the experiments have been reviewed in Sect. 2. Besides conceivable violations with insufficient experimental information at this time (e.g., the apparent loss of the sinusoidal behavior of the intensity modulation), all experimental measures exhibit a median angle consistently lower than the expected 720° of the exact symmetry, while the best available measures are given by Eq. (2.13) which *do not* include 720° in their minimal-maximal values.

Under the covering Poincaré-isotopic symmetry for iso-space $\hat{M} = \hat{M}(x, g, \hat{\mathcal{R}})$, $g = T\eta$, and iso-Hilbert space (4.20) with $T = G > 0$, property (7.41) is replaced by the more general expression

$$\begin{aligned} \widehat{SU}(2): \quad \psi' = \hat{R}(\theta_3) * \psi &= e^{\hat{\gamma}_1 * \hat{\gamma}_2 * \frac{\theta_3}{2}}\psi \\ &= e^{\gamma_1\gamma_2(b_1 b_2)\frac{\theta_3}{2}}\psi, \end{aligned} \tag{7.48}$$

which allows a direct and quantitative interpretation of data (2.13). In fact, for $\theta_3 = 714°$, we have

$$b_1 = b_2 \cong 1.005\,. \tag{7.49}$$

This essentially shows that the thermal neutrons experience a small deformation of their charge distribution, from an ideal perfect sphere for the exact $SU(2)$ symmetry, to an *oblate spheroidal ellipsoid.* This is precisely the type of shape of the neutrons which is expected in the physical reality as illustrated in Fig. 1 (recall that Earth and all known spinning objects have a shape precisely of that type).

Also, under deformation (7.49), we have a necessary *decrease* of the angle of precession, as identified in ref. [21] and confirmed in papers [22,23]. In this way we interpret again the angle slow-down effect, but this time with a theory with a sufficient number of parameters to actually identify the shape of the deformed charge distributions.

At a deeper analysis, we have to expect from the anomalous value of the magnetic moments and other properties, that protons and neutrons have charge distributions which are of oblate spheroidal ellipsoidical nature already in the absence of external forces. In fact, specific investigations along these lines conducted within the context of hadronic mechanics [59] have identified the following possible shape of the charge distribution of the proton

$$\begin{aligned} \vec{r}^{\hat{2}} &= x b_1^2 x + y b_2^2 y + z b_3^2 z\,, \\ & b_1^2 = b_2^2 = 1\,, \quad b_3^2 = 0.60\,, \end{aligned} \tag{7.50}$$

with a similar shape expected for the neutron. Sufficiently intense external forces are then expected simply to increase the above oblate character, resulting in a further modification of the value of the magnetic moment.

Next, we must identify the value of the mutated magnetic moment which corresponds to mutation (7.49). Under the conditions assumed, the magnetic moment is directed along the third axis. Then the mutated value of the magnetic moment is given by Eq. (6.41), i.e.

$$\tilde{\mu}_N = \frac{b_3}{b_4}\mu_N\,. \tag{7.51}$$

The value of the parameter b_3 can be computed under the assumption of preservation of the density (i.e., volume) of the neutron for deformation (7.49), resulting in the value for an originally spherical charge distribution

$$b_3 \cong 0.995 \tag{7.52}$$

and in the value

$$b_3 \cong 0.597 \tag{7.53}$$

for an originally oblate charge distribution (7.50).

Assume now that the value $7,496\ G$ for the external magnetic field, as in Rauch's experiments (Sect. 2) yields 720° spin precession for a neutron beam in vacuum, while the same magnetic field yields 714° for a beam in interactions with the Mu-metal nuclei filling up the magnet gap. Then, we have

$$\frac{\tilde{\mu}_N}{714} = \frac{\mu_N}{720} , \tag{7.54}$$

from which

$$b_4 \cong 1.0033 . \tag{7.55}$$

The mutated magnetic moment, for (7.52), has the value

$$\tilde{\mu}_N = 0.991\mu_N , \tag{7.56}$$

while ascending the value for case (7.53)

$$\tilde{\mu}_N = 0.595\mu_N \tag{7.57}$$

thus resulting in a mutated value of the magnetic moment smaller than the conventional angle, as expected from the angle slow-down effect.

C: RESTORATION OF THE EXACT SPINORAL SYMMETRY. The *conventional* spinorial symmetry is manifestly broken for data (2.13). In particular, the symmetry is broken in its essential part, the $SU(2)$-transformation law of the spinoral wave-function, Eq. (7.47).

Nevertheless, the isotopic lifting restores the spinoral symmetry to an exact form although at the covering $SU(2)$-isotopic level. In fact, for the iso-Dirac's equation, we have transformation law (7.48) for which Rauch's angle of 714° does indeed characterize an *exact* symmetry because

$$b_1 b_2 \frac{\theta_3}{2}|_{\theta_3=714^\circ} = 720^\circ . \tag{7.58}$$

In this way, *the iso-Dirac's equation reconstructs as exact at the isotopic level, the spinorial symmetry which is broken at the conventional level.*

This occurrence was expected. In fact, all conventionally broken space-time symmetries investigated until now have resulted to be exact at the covering isotopic level, such as:

- the conventional rotational symmetry $SO(3)$ is manifestly broken for ellipsoid of type (7.50). Nevertheless, such ellipsoids are left invariant by the isotopic symmetry $\widehat{SO}(3)$ [39].

- The conventional Lorentz symmetry $SO(3.1)$ is manifestly broken by the deformation of the Minkowski metric of the Nielsen-Picek type (4.29). Nevertheless the latter metrics are left invariant by the covering, Lorentz-isotopic symmetry $\widehat{SO}(3.1)$ [40].

- The conventional Poincaré symmetry $P(3.1)$ is manifestly broken by the class of metrics (4.26). Nevertheless, these metrics are all left invariant by the covering, Poincaré-isotopic symmetry $\hat{P}(3.1)$ [41].

with similar results expected for the discrete symmetries [56].

Most importantly, in all the above cases the covering isotopic symmetries have been proved to be locally isomorphic to the conventional symmetries because of the preservation of the topological character of the original metrics.

In conclusion, we can state that *Rauch's fundamental experiments on the spinorial symmetry of neutrons in their currently available form provide evidence for the possible violation of the conventional realization of the $SU(2)$-spin symmetry, with the understanding that the same symmetry remains exact when realized in a sufficiently general way, for instance, in the isotopic realization submitted in this paper.*

8 Dirac's Mutation of Dirac's Equation

We shall now review two of the last papers written by P.A.M. Dirac [60,61]. They present an intriguing generalization of his celebrated equation which characterizes a mutation of the spin from $\frac{1}{2}$ down to zero. We shall then show that this generalized equation possesses an essential isotopic structure, by therefore providing an illustration of our theory of mutation of the intrinsic characteristics of particles, this time, for a finite mutation of the spin.

In the subsequent paper [32] of this series, we shall show that Dirac's generalized equation apparently allows a consistent formulation of the historical hypothesis by Rutherford according to which the neutron is a compressed hydrogen atom (i.e., a bound state of one conventional proton and one mutated electron). In the same paper [32] we shall review Barut's model on the neutron structure [62,63,64] as a bound state of one proton, one electron and one antineutrino (as unmutated particles). We shall then show that, when Barut's model is reinspected within the context of hadronic mechanics, it appears to be at the foundation of the mechanism of creation of the neutrino.

By preserving its original notation, Dirac's paper [60] can be summarized as follows. Consider two harmonic oscillators in one dimension with dynamical variables

$$(q) = (q_a) = (q_1, p_1; q_2, p_2) , \quad a = 1,2,3,4 ,$$
$$[q_a, q_b] = q_a q_b - q_b q_a = i\beta_{ab} , \tag{8.1}$$

where

$$\beta = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} , \quad \beta^T = -\beta , \quad \beta^2 = -1 , \quad \beta^{-1} = \beta^T . \tag{8.2}$$

The generalized form of the conventional Dirac's equations proposed by Dirac himself at the very beginning of the paper [60] (Eq. 1.3) is given by

$$\left(\frac{\partial}{\partial x_0} + \alpha_r \frac{\partial}{\partial x_r} + \beta\right) q\psi = 0 , \tag{8.3}$$

where: ψ is a scalar (one-dimensional) wavefunction with the dependence $\psi = \psi(x,q)$; the x's are the space-time coordinates of a (conventional) Minkowski space; q is a column matrix with the four elements (8.1); the α_r are 4×4 matrices that anticommute with each other and with β; and the product is the conventional associative product. One of the various possible realizations of the α_r presented by Dirac is given by

$$\alpha_1 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} . \tag{8.4}$$

Assume $\alpha_0 = 1$ and $\partial_\mu = \partial/\partial x^\mu$. Then Eq. (8.3) was rewritten by Dirac in the form

$$(\alpha_\mu \partial^\mu + \beta) q\psi = 0. \tag{8.5}$$

Next, Dirac identified the main law of the α matrices which resulted to be of the form

$$\alpha_\mu \beta \alpha_\nu + \alpha_\nu \beta \alpha_\mu = 2\beta\eta_{\mu\nu} , \tag{8.6}$$

where $\eta_{\mu\nu}$ is the conventional Minkowski metric.

The *conventional* Lorentz covariance of the equation was studied via the infinitesimal transformations

$$x^*_\mu = x_\mu + a_\mu{}^\nu x_\nu , \tag{8.7}$$

which resulted in the transformed equation

$$\{(\alpha_\mu + a_\mu{}^\nu \alpha_\nu)\partial^\mu + \beta\} q\psi = 0 . \tag{8.8}$$

By putting

$$N = \frac{1}{4} a^{\rho\sigma} \alpha_\rho \beta \alpha_\sigma , \tag{8.9}$$

Eq. (8.8) can be rewritten

$$\{\alpha_\mu (1 - \beta N)\partial^{\mu *} + \beta(1 + N)\} q\psi = 0 , \tag{8.10}$$

thus resulting in the final form

$$(\alpha_\mu \partial^{\mu *} + \beta) q^* \beta = 0 ,$$
$$q^* = (1 - \beta N) q . \tag{8.11}$$

Dirac (loc. cit.) then passes to prove that his new equation (8.3) has only *positive energy* (normalizable) solutions. Assume $i\partial_\mu = p_\mu$. The equation can then be written

$$\{(p_0 - p_3)q_1 + (i + p_1)q_3 - p_2 q_4\}\psi = 0 ,$$
$$\{(p_0 - p_3)q_2 - p_2 q_3 + (i - p_1)q_4\}\psi = 0 ,$$
$$\{(p_0 + p_3)q_3 + (p_1 - i)q_1 - p_2 q_2\}\psi = 0 ,$$
$$\{(p_0 + p_3)q_4 - p_2 q_1 - (p_1 + i)q_2\}\psi = 0 . \tag{8.12}$$

By applying a Lorentz transformation to the rest frame with $\vec{p} = 0$, the above equations show that p_0 can be either $+1$ or-1. Of these two possibilities only the first is normalizable because the underlying wave function has the form

$$\psi = k \exp\{-\frac{1}{2}\left[q_1^2 + q_2^2 a + ip_1(q_1^2 - q_2^2)0 - 2ip_2 q_2\right] / (p_0 + p_3) \exp(-ip^\mu x_\mu)\}. \tag{8.13}$$

This established the first significant difference between the new equation and the conventional one (for which, as the reader knows well, both positive and negative energies are admitted).

But by far the biggest differences emerged for the spin. Consider the familiar total angular momentum

$$J_{\rho\sigma} = x_\rho \partial_\sigma - x_\sigma \partial_\rho - i s_{\rho\sigma} \ . \tag{8.14}$$

To identify the explicit form of $s_{\rho\sigma}$, Dirac considered the equation

$$\begin{aligned} &[(\alpha_\mu \partial^\mu + \beta)q, a^{\rho\sigma}(x_\rho \partial_\sigma - x_\sigma \partial_\rho)] = 2a^{\mu\sigma}\alpha_\mu \partial_\sigma q \ , \\ &[(\alpha_\mu \partial^\mu + \beta)q, w] = 2i(\alpha_\mu \partial^\mu + \beta)\beta N q \ , \end{aligned} \tag{8.15}$$

which can be written

$$\begin{aligned} &[(\alpha_\mu \partial^\mu + \beta)q \ , \ a^\rho(x_\rho \partial_\sigma - x_\sigma \partial_\rho) + iW] = \\ &= -2N\beta(\alpha_\mu \partial^\mu + \beta)q \ , \quad W = -q^T N \ , \end{aligned} \tag{8.16}$$

thus yielding the form

$$a^{\rho\sigma} s_{\rho\sigma} = -W = -q^T N q = -\frac{1}{4} a^{\rho\sigma} q^T \alpha_\rho \beta \alpha_\sigma q \ . \tag{8.17}$$

As a consequence

$$s_{\rho\sigma} = -\frac{1}{8} q^T (\alpha_\rho \beta \alpha_\sigma - \alpha_\sigma \beta \alpha_\rho) q \ , \tag{8.18}$$

which becomes via Eq. (8.6)

$$s_{\rho\sigma} = -\frac{1}{4} q^T \alpha_\rho \beta \alpha_\sigma q + \frac{1}{4} g_{\rho\sigma} q^T \beta q = -\frac{1}{4} q^T \alpha_\rho \beta \alpha_\sigma q + \frac{1}{2} i \eta_{\rho\sigma} \ . \tag{8.19}$$

For $\rho, \sigma = 1, 2, 3$, $M_{\rho\sigma}$ can be interpreted as the angular momentum while $s_{\rho\sigma}$ is the spin. By using expression (8.4) for the α's, the spin components can be computed explicitly,

$$\begin{aligned} s_{23} = \frac{1}{2}(q_1 q_2 + q_3 q_4) \ , \qquad s_{31} &= \frac{1}{4}({q_1}^2 - {q_2}^2 + {q_3}^2 - {q_4}^2) \ , \\ s_{12} &= \frac{1}{2}(q_2 q_3 - q_1 q_4) \ , \end{aligned} \tag{8.20}$$

and also

$$\begin{aligned} s_{01} = \frac{1}{4}(q_1^2 q_2^2 - q_3^2 + q_4^2 \ , \quad s_{02} &= \frac{1}{2}(q_3 q_4 - q_1 q_2) \ , \\ s_{03} &= \frac{1}{2}(q_1 q_3 + q_4 q_2) \ . \end{aligned} \tag{8.21}$$

As a consequence,

$$\bar{s}^2 = s_{23}^2 + s_{31}^2 + s_{12}^2 = \frac{1}{16}(q_1^2 + q_2^2 + q_3^2 + q_4^2)^2 - \frac{1}{4} = s(s+1), \tag{8.22}$$

and, finally,

$$\begin{aligned} s = \frac{1}{4}(q_1^2 + q_2^2 + q_3^2 + q_4^2) - \frac{1}{2} &= \frac{1}{2}(n + n') \ , \\ n, n' &= 0, 1, 2, 3, \ldots \ . \end{aligned} \tag{8.23}$$

In this way, Dirac (loc. cit.) reached the remarkable conclusion that *modification (8.5) of his celebrated equation can have only even values of spin beginning with the zero value.*

Dirac then passed to the identification of the four-current

$$J_\mu = \int \psi^\dagger q^T \alpha_\mu q \psi d^2 \ , \tag{8.24}$$

which verifies the usual conservation law

$$\partial^\mu J_\mu = 0 \ , \tag{8.25}$$

and transforms under the (conventional) Lorentz transformations

$$J'_\mu = \int \psi^\dagger q^T (\alpha_\mu + {a_\mu}^\sigma \alpha_\sigma) q \psi d^2 q = J_\mu + {a_\mu}^\sigma J_\sigma \ . \tag{8.26}$$

The charge density

$$J_0 = \int \psi^\dagger q^T q \psi d^2 q \ , \tag{8.27}$$

is positive definite while the underlying wavefunction is normalized to unity according to the rule

$$\int \psi^\dagger q^T q \psi d^3 x = 1 \ . \tag{8.28}$$

Generalization (8.28) of the charge density of the conventional Dirac's equation shows another departure from orthodox values. In fact, value (8.28) is manifestly different than the corresponding value for the conventional equation.

Dirac concluded paper [60] with the calculation of the Fock representation of his new equation and additional developments not reviewed here for brevity.

The subsequent paper [61] was primarily devoted to the physical interpretation of the new equation. It turns out that the theory describes a collection of random circles covered by the point x on a sphere. This results in a random motion on said sphere. In particular, its radius is not constant but pulsates in time within given boundaries.

Paper [61] then concludes with the proof that *the particle in its ground state has a zero spin for all possible values of the linear momentum.*

The reinterpretation of Dirac's pioneering paper [60,61] within the context of the Lie-isotopic theory is quite instructive. First, the isotopic element of the iso-envelope is *not* q (trivially, because this quantity is a column matrix), but β. Thus, *all* associative products must be formulated in the isotopic form, say,

$$\hat{\mathcal{A}} \ : \ A * B \stackrel{\text{def}}{=} A\beta B \ , \quad \hat{I} = \beta^{-1} \ . \tag{8.29}$$

It is easy to see that Dirac's new equation does indeed verify this fundamental requirement at all levels.

Introduce the four-component column wave-function $\phi = q\psi$ (as in the conventional equation). Then, by recalling that $\beta^{-1} = \beta^T = -\beta$, Eq. (8.5) can be readily written in the isotopic form

$$\begin{aligned}(\alpha_\mu \partial^\mu + \beta) q\psi &= (\hat{\alpha}_\mu * \hat{\partial}^\mu + 1) * \phi = 0 \ , \\ \hat{\alpha}_\mu = \alpha_\mu \hat{1} \ , &\quad \hat{\partial}^\mu = \partial^\mu \hat{1} \ ,\end{aligned} \tag{8.30}$$

which, in particular, verifies hadronization rule (4.43) (but not the conventional quantization rule).

To identify the properties of the $\hat{\alpha}$ matrices, we consider the expression for the characterization of the second- order equation

$$\begin{aligned}&(\hat{\alpha}_\mu * \hat{\partial}^\mu + 1) * (\hat{\alpha}_\nu * \partial^\nu - 1) \\ &= \frac{1}{2}\{\hat{\alpha}_\mu \hat{,} \hat{\alpha}_\nu\} * \hat{\partial}^\mu * \hat{\partial}^\nu - \beta \\ &= -(\partial_\mu \partial^\mu + 1)\beta \ ,\end{aligned} \tag{8.31}$$

which holds iff

$$\{\hat{\alpha}_\mu \hat{,} \hat{\alpha}_\nu\} = \hat{\alpha}_\mu * \hat{\alpha}_\nu + \hat{\alpha}_\nu * \hat{\alpha}_\mu = -2\eta_{\mu\nu}\hat{I}. \tag{8.32}$$

Note that the above equations *coincide* with Eq. (8.6) because of the property $\beta = -\hat{I}$. Next, since all operators belong to isoenvelope $\hat{\mathcal{A}}$, so must be

the case also of the angular momentum and spin. Again, it is easy to see that quantity (8.14) can be readily rewritten in the isotopic form

$$J_{\rho\sigma} = x_\rho \partial_\sigma - x_\sigma \partial_\rho - i s_{\rho\sigma} \equiv x_\rho * \hat{\partial}_\sigma - x_\sigma * \partial_\rho - i\hat{s}_{\rho\sigma} \ , \tag{8.33}$$

while the spin is in full isotopic form already as written by Dirac. In fact, Eq.(8.18) can be written

$$s_{\rho\sigma} = -\frac{1}{4}\overline{\alpha}_\rho * \overline{\alpha}_\sigma + \frac{i}{2}\eta_{\rho\sigma} \ , \quad \overline{\alpha}_\rho = \alpha_\rho q \ . \tag{8.34}$$

The underlying Hilbert space was equipped by Dirac with the *conventional* inner product, i.e.,

$$\mathcal{H} : \langle \phi | \phi \rangle = \int \phi^\dagger \phi d^3x = 1 \ . \tag{8.35}$$

This is fully compatible with isoenvelope $\hat{\mathcal{A}}$ (see Sect. 4). In this case, the current is given by Dirac's expression (8.25) without need of any reformulation. The same happens for other calculations based on Hilbert space formulations.

The reader should however recall that, in the above framework, the definition of conventional and iso-Hermiticity are different. Thus, it can be proved that operators such as angular momentum and spin are not Hermitean in Dirac's generalized theory. Nevertheless, conventional and iso-Hermiticity can be made to coincide with an isotopic lifting of the Hilbert space with the same isotopic element β, as in structure (4.20) with $T = G$. This reformulation can be readily achieved by introducing, e.g. the new wavefunction

$$\hat{\phi} = \beta^{1/2}\phi \ , \tag{8.36}$$

under which the conventional inner product (8.36) can be reformulated into the isotopic form

$$\hat{\mathcal{H}} \ \langle \phi | \phi \rangle = \int \phi^\dagger \phi d^3x = 1 \to \hat{\mathcal{H}} \ : \ \langle \hat{\phi} \hat{|} \hat{\phi} \rangle = \hat{I} \int \hat{\phi}^\dagger * \hat{\phi} d^3x = \hat{I} \ . \tag{8.37}$$

Under the above modified assumptions, the Hermiticity of fundamental physical quantities, such as spin and angular momentum, is preserved because of the identity of the conventional Hermiticity with the iso-Hermiticity. The four-current (8.25) must be in this case rewritten in the different form

$$\hat{J}_\mu = \int \hat{\phi}^\dagger * \alpha_\mu * \phi d^2q \ . \tag{8.38}$$

It is easy to see that the above four-current verifies too all essential requirements of J_μ and it is therefore a fully acceptable expression.

Finally, the covariance under Lorentz-isotopic transformations can be readily reached via the trivial isotopy

$$x'_\mu = x_\mu + \hat{a}_\mu{}^\nu * x_\nu \ , \quad \hat{a}_\mu{}^\nu = a_\mu{}^\nu \hat{I} \ , \tag{8.39}$$

as the reader is encouraged to verify. It should be indicated that, in the theory under consideration here, the trivial lifting $a_\mu{}^\nu \rightarrow a_\mu{}^\nu \hat{I}$ is necessary because the theory is formulated in a *conventional* Minkowski space without any deformation of the space-time metric. On the contrary, such a trivial isotopy of the Lorentz group would be inconsistent for the content of Sect.s 6,7 owing to the necessary presence there of a nontrivial modification of the Minkowksi metric.

It should be stressed that the trivial realization (8.39) of the Lorentz group has been introduced here as a mere illustration of the consistency of Dirac's generalized equation. The next logical step is to re-inspect the covariance of Eq. (8.30), and show that the underlying symmetry is a bona-fide lifting of the Lorentz groups for the invariance of the iso-metric

$$g = \beta\eta = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} ,$$

$$x^{\hat{2}} = x^\mu g_{\mu\nu} x^\nu = x^1x^3 - x^2x^4 - x^3x^1 - x^4x^2 \ , \quad x^4 = c_0 t. \tag{8.40}$$

Since the above iso-metric preserves the topological properties of $-\eta$, it is possible to prove that the emerging Lorentz-isotopic group is locally isomorphic to the iso-dual [39] of the conventional Lorentz group. This study is not conducted here for brevity.

As a final remark, the reader should be aware that in the final part of paper [60] Dirac points out certain inconsistencies that emerge in his generalized equation whenever an interaction term is included. These inconsistencies are a direct consequence of the unawareness by Dirac of the essential isotopic structure of his theory. (In fact, interactions terms, when written in the conventional associative form for an essential isotopic structure, violate the linearity conditions, as we now know [28]). However, as shown in ref. [32], these inconsistencies are absent when the interactions are treated in a mathematically consistent way, that is, as elements of the iso-envelope.

In summary, the isotopic lifting from the conventional to the generalized Dirac's equation reviewed in this section

$$\begin{aligned} (\gamma_\mu \partial^\mu + 1)\psi &= 0 \rightarrow (\alpha_\mu * \hat{\partial}^\mu + 1) * \phi = 0 \ , \\ T &= \beta \end{aligned} \tag{8.41}$$

provides an intriguing illustration of the notion of mutation of spin, charge, and other intrinsic characteristics of particles.

These mutations are not mere mathematical curiosities. On the contrary, they can be of assistance in understanding the origin of fractional charges of quarks, as well as in achieving a consistent identification of the constituents of hadrons (or of quarks) with physical, ordinary particles, merely existing in a mutated state due to their total immersion within hyperdence hadronic matter.

9 Concluding Remarks

As is well known, quantum mechanics was conceived and constructed, in both its non-relativistic and relativistic forms, for the quantitative treatment of the atomic structure, for which it subsequently resulted to be exactly valid according to a simply overwhelming volume of experimental evidence.

Nevertheless, the advent of nuclear physics and the discovery of newer and newer physical conditions, stimulated numerous studies on the possible violation of quantum mechanics outside the arena of its original conception (the interested reader may consult, as representative recent articles, ref.s [65-69] and quoted articles).

As equally well known, these latter studies have been essentially inconclusive, in the sense that no experimentally measurable violation of quantum mechanics has emerged in a conclusive way until now.

This paper supports the view that, rather than being resolved, the problem of the experimental detection of the limits of exact validity of quantum mechanics is just at the beginning, for ***physics is a discipline which does not admit terminal theories.*** In particular, the lack of experimental resolution of the issue appears to be due to a number of intrinsic insufficiencies of studies [65-69] which have resulted in the lack of deviations, not from quantum mechanics itself, but more appropriately from the physical conditions of its original conception. This is no criticism for studies [65-69] and quoted research, for ***physics is also a discipline that proceeds by trial and error.*** As a matter of fact, the path covered by these studies was mandatory and, because of their negative results, we are now in a better position to identify different avenues.

The different line of research considered in this paper contends that, *the possible experimental detection of deviations from quantum mechanics demands the maximization of the departures from the original physical conditions of quantum mechanics*, while such departures are not sufficiently emphasized in ref.s [65-69] and quoted papers.

To begin, none of the studies [65-69] focus the attention on the true, most important limitation of (non-relativistic and relativistic) quantum mechanics: the representation of particles as being point-like. We can therefore express one first condition as follows:

> *CONDITION A: A first necessary condition for the experimental detection of possible deviations from quantum mechanics, is that the tests must be fundamentally dependent on the limitations of the theory caused by its point-like approximation of particles, and deal with physical conditions where the extended and therefore deformable character of the particles is expected to produce measurable effects.*

The above limitations are so evident and so fundamental, that the problem of the construction of a covering of quantum mechanics for the representation of the extended deformable-character of particles has scientific value irrespective of, and prior to, the experimental resolution of the issue.

In fact, on one side, the point-like abstraction of particles is embedded in the very mathematical structure of the theory, owing to its local-differential character. On the other side, the extended character of the charge distributions (and of the wave-packet) of hadrons is an experimental fact. But, whenever physical dimensions are admitted for charge distributions (or for their wave-packets), their deformability becomes consequential, because perfectly rigid bodies do not exist in the physical reality. Still in turn, the moment the deformability of extended charge distributions is acknowledged as an expected fact of physical reality, the applicability of the very foundations of quantum mechanics becomes questionable. After all, we should not forget that the rotational symmetry is known to be inapplicable to deformable bodies since its inception.

Despite the incontrovertible plausibility of the above arguments, the experimental detection of deviations from quantum mechanics has remained elusive. Another central contention of the line of studies of this paper is that the virtual totality of the experiments considered in ref.s [65-69] essentially

deal with closed systems in their center-of-mass treatment. These systems are isolated and, as such, their total conservation laws hold exactly. The exact validity of quantum mechanics is then expected to be consequential, because stable isolated states must be conventionally quantized.

At a deeper analysis, the experiments considered in ref.s [65-69] are at variance with the historical process that lead to the establishing of quantum mechanics itself. In fact, the discipline was established via measures under *external* electromagnetic interactions. The second central condition of this paper can be expressed as follows.

> *CONDITION B: A second necessary condition for the experimental detection of possible deviations from quantum mechanics is that measures are conducted under external electromagnetic and short range nuclear/strong interactions, so as the maximize the non-conservative conditions of the system considered.*

Irrespective of whether the mechanics which is applicable in the interior of hadrons is the conventional quantum mechanics or a possible covering discipline, the physical characteristics of isolated hadrons obey quantum mechanics exactly. If the experimenter maximizes the conservative conditions of its physical framework, the exact applicability of quantum mechanics is maximized. On the contrary, in order to effectively search for possible deviations, one must maximize the deviations from the original conditions of the theory, that is, maximize the departures from the stable orbit of the atomic structure.

At this point, the necessary conditions for a truly effective experimental study become specialized to the physical law considered. For instance, if a test of Pauli's principle is desired, it becomes mandatory, for credibility, to monitor the dependence of the experimental results from the assumption of the principle in the data elaboration. In different terms, if the data are elaborated with a theory that is centrally dependent on the exact validity of Pauli's principle, the "experimental results" will likely be compatible with such an assumption, and no true test of Pauli's principle has actually occurred. This aspect and others (e.g., the necessary experimental condition to truly claim the validity of the T-symmetry in particle physics [70]) will be studied elsewhere.

In this paper we are interested, specifically, in the problem of the spinorial symmetry of extended-deformable charge distributions. Our third condition can then be expressed as follows.

CONDITION C: A sufficient condition for the experimental detection of possible deviations from quantum mechanics is to measure the intrinsic characteristics of hadrons under sufficiently intense external, electromagnetic and nuclear/strong interactions.

Points are eternal and immutable geometrical objects. Therefore, the intrinsic characteristics of particles are rigidly unchangeable for quantum mechanics, as well known. Extended charge distributions, instead, are deformable. For the covering hadronic mechanics, the intrinsic characteristics of particles therefore depend on the local physical conditions at hand.

Experiments that meet all the above criteria are the tests of the spinorial symmetry of neutrons conducted by Rauch and his collaborators [4-9]. In fact: the experiment is centrally dependent on the expected alteration of the magnetic moment of neutrons caused by a deformation of their shape (Condition A); the experiment verify the central condition of conducting measures under the *external* nuclear field of the Mu-metal sheets (Condition B); and the experiments measure an intrinsic characteristic of particles, the spinorial symmetry (Condition C).

Rauch's experiments are of truly fundamental character because they have been the first to perform the transition from the traditional measures under external electromagnetic interactions, to external electromagnetic and nuclear interactions.

Nevertheless, Rauch's experiments are inconclusive at this time. For this reason, a series of experiments were suggested in Ref. [21], in orderto achieve the final experimental resolution of the ultimate validity or invalidity of the spinorial symmetry in particle physics. Let us review them below:

EXPERIMENTS I: Repeat experiments[4-9] with a better accuracy. The technology of neutron interferometers has advanced considerably since the time of the last run of Rauch's experiments (1978). The mere improvement of the experimental error could provide now the final resolution of the fundamental issue here considered.

EXPERIMENTS II: Repeat experiments [4-9] in vacuum, and compare the results with those of Experiments I above. Recall that the spinorial-asymmetry is expected from the interactions of neutrons with nuclear fields in the magnet gap. It is important to confirm this expectation by isolating its contribution from other possible effects.

EXPERIMENTS III: Repeat experiments [4-9] for multiple spin flips. We understand that neutron interferometer techniques today allow the achievement of up to 50 spin flips and more, with the same experimental error. It is then evident that, without any improvement of the error, the fundamental issue considered here could be resolved via a mere increase of the number of spin flips.

EXPERIMENTS IV: Repeat experiments [4-9] for increased nuclear interactions, e.g. with heavier matter within the magnetic gap. The spinorial asymmetry studied in this paper is iso-linear, but ultimately non-linear in conventional formulation (recall that the isotopic element *multiplies* the Hamiltonian). An increase of the external nuclear interactions is then expected to produce a spinorial asymmetry proportionately higher than the corresponding increase of the nuclear mass.

EXPERIMENTS V: Repeat experiments [4-9] with particles other than neutrons. In fact, the property studied in this paper is expected to be universal for all extended-deformable charge distributions, and a verification of this expectation is evidently important.

Also, in each of the above five groups of experiments, the following different quantities were suggested for accurate measures [21]:

1. *Measure the most probable median angle.* As recalled earlier, all investigations of the expected spinorial asymmetry of tests [4-9] lead to the *angle slow-down effect*, i.e., a decrease of the magnetic moment of neutrons, with consequential decrease of the angle of spin flip over that predicted. It is therefore essential that experiments focus their attention on such a median angle.

2. *Identify the shape of the intensity modulation as well as that of the polarization modulation.* In fact, the shape of the curves of Fig. 3 does not appear to be a sinusoid, as predicted by the exact $SU(2)$ symmetry. The final experimental resolution of this additional point, in addition to data 1) is important because of the possibility that the median angle might indeed result to be 720°; yet the shape of the curve of the intensity modulation is not sinusoidal. In turn, this occurrence is important to differentiate between the spin fluctuations of ref.s [22,23] and the spin mutation of ref.s [15,20,21].

3. *Measure the phase between the intensity and polarization modulations.* This information may provide additional valuable input for deeper studies of the problem (recall that this phase is unchanged by the mutation of this paper, although more general models in which this phase too is mutated are conceivable).

We closed the preceding paper [41] with comments on the unreassuring current conditions of theoretical physics caused by the excessively overdue tests of the apparent violation of the local validity of Einstein's Special and General relativities in the interior of hadrons, via the measures of the behavior of the mean-life of unstable hadrons with speed. In fact, these manifestly elemental tests continue to be ignored decade after decade, while the totality of the phenomenological studies (e.g. ref.s [42-47]) show violation.

We cannot close this article without comments regarding the unreassuring current condition of nuclear physics caused by the lack of experimental resolution one way or the other of Rauch's fundamental tests (the last one actually run being that of 1978 [7]).

This unreassuring condition emerges quite clearly if the reader meditates a moment on the following facts.

1. *The proposal essentially dates back to the early 40's.* In fact the tests deal with the historical open legacy of quantum mechanics on the expected alteration of the magnetic moments of nucleons (Sect. 1).

2. *The plausibility of the underlying hypothesis is simply incontrovertible.* In fact, the alteration of the magnetic moments of nucleons is expected as a necessary consequence of the deformation of their extended charge distribution. The *amount* of the deformation for given external fields is in question. But the *existence* of the deformation is beyond any credible doubt (Fig. 1).

3. *The far reaching implications of the experiments for the human society are also incontrovertible.* As stressed from the beginning, the experimental detection of alterations of the magnetic moments of nucleons may void most of the multi-billion dollars efforts on the controlled fusion, trivially, because it implies an alteration of the intrinsic characteristics of the particles exactly at the instant of initiation of the fusion process.

4. *The existing experimental data by Rauch [4-9], even though inconclusive, show clear violations.* In fact, not only the value of 720° needed to establish the exact spinorial symmetry is not contained within the current experimental data (2.13), but the median angle of all experiments has been consistently below the value 720°, and the shape of the curve representing the intensity modulation is convincingly at variance with a sinusoid.

5. *The cost of the experiments is by far, modest, when compared with other, manifestly lesser relevant experiments currently preferred by the physics community.*

6. *The experimental apparata have been available since several decades at numerous nuclear physics laboratories throughout the world.*

7. *The implications of the results, whether in favor or against old doctrines are truly of historical proportions.* After all, we are referring to the test of the true, ultimate foundations of contemporary physics: the rotational symmetry.

Despite all the above aspects, the fundamental experiments under consideration here continue to be ignored, thus providing credibility to the unreassuring conditions of nuclear physics indicated earlier.

As stressed in Ref. [41], physics is a discipline with an absolute standard of value: the experimental verification. Experiments themselves have their own standard of value: the more fundamental the test, the more relevant its conduction as compared to lesser fundamental tests.

The excessively protracted ignorance of the final resolution of Rauch's fundamental tests despite the well known aspects 1)-7) above, is a *de facto* departure from the above sound structure of physics, thus providing credibility to the recent view of a number of observers according to which *the most a-scientific process of contemporary human society is the current scientific process.*

NOTE ADDED: This paper was written back in 1985 but left unpublished because of the lack of interest by the scientific community at that time on the problem considered. It has been recently revised and released for publication following the warm recommendations by Prof. A. Jannussis of the University of Patras, Greece, Prof. R. Mignani of the University of Rome, Italy, and other colleagues I consider most highly. In the final revision of the paper I specifically attempted to preserve its original conception of 1985. Nevertheless, the reader should be aware that numerous advancements in Lie-isotopic theories have occurred since the that time, as outlined in review [71]. In particular, I feel obliged to point out that the first generalization of Dirac's equation, truly written in the language of the Lie-isotopic theory, has been achieved by A. Jannussis and his collaborators [72] via an isotopy of the conventional Langrangian formulation. Also, a rather intriguing and though provoking application to the quasar red-shift has been recently formulated by R. Mignani [73].

Preprint of the Institute for Basic Research No.IBR-TP-894, 15 July 1989.
First revision 9 October 1990. Second revision 5 November 1990.

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