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Problematic aspects of 20-th century antiparticles and their apparent resolution via isodual mathematics

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Abstract

In this paper, we recall Dirac's *negative energy* antiparticles, their compatibility with particle-antiparticle annihilation into light and their lack of compatibility with special relativity as well as causality laws. We then recall the 20th century *positive energy* antiparticles, their compatibility with special relativity and causality laws but their incompatibility with annihilation into light, with ensuing problematic aspects for a true antimatter character of antiprotons, anti-Hydrogen atoms and related gravity tests. We then review the *isodual branch of hadronic mechanics* whose *isodual theory of antimatter:* 1) Represents Dirac's negative energy antiparticles without causality problems. 2) Admits special and general relativities due to the invariance of quantum axioms under the isoidual map. 3) Implies matter-antimatter antigravity at all levels. 4) Predicts the existence of the negative energy *antiphoton.* 5) Is compatible with existing experimental evidence on antiparticles. We suggest the conduction of resolutory tests on the gravity of well estasblished antipartices, such as the positrons in horizontal flight in a supercooled vacuum tube. We conclude with the indication of intriguing open problems in antimatter, such as the possible *expulsion* of antiphotons by black holes following internal particle-antiparticle creation and annihilation.

Keywords: antiprotons; pseudoprotons; antimatter; hadronic mechanics [1]

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1 Dirac's negative energy antiparticles

As it is well known, the first quantitative formulation of antimatter was achieved in 1928 by P. A. M. Dirac [1] via his celebrated equation on the Minkowski spacetime $M = M(x, \eta, I)$ with coordinates $x = (x^\mu) = (x^1, x^2, x^3, x^4 = ct),\ \mu = 1, 2, 3, 4$, metric $\eta = Diag.(1, 1, 1, -1)$ and unit $I = Diag.(1, 1, 1, 1)$ over the field of real numbers $\mathcal{R}$

$$(i\eta^{\mu\nu}\gamma_\mu\partial_\nu - mc)|\psi(x)\rangle = 0, \tag{1}$$

where the Dirac's matrices $\gamma_\mu,\ \mu = 1, 2, 3, 4$ are given by

$$\gamma_k = \begin{pmatrix} 0 & \sigma_k, \\ -\sigma_k & 0 \end{pmatrix}, \quad \gamma_4 = i \begin{pmatrix} +I_{2\times2} & 0 \\ 0 & -I_{2\times2} \end{pmatrix}, \tag{2}$$

and Pauli's matrices $\sigma_k,\ k = 1, 2, 3$ represent the spin $S = 1/2$ of the particles.

According to Dirac's conception, the electron e^- is represented by the top 2 components of Eq. (1) with unit $+I_{2\times2}$, thus having *positive energy* $m_e > 0$, while the "antielectron" $\bar{e}^- = e^+$ (now called the positron), is represented by the bottom 2×2 component of Eq. (1) with unit $-I_{2\times2}$, thus having *negative energy* $\bar{m}_e = -m_e < 0$.

An important property of Dirac's conception of antimatter is that, in view of the opposite sign of the masses of the two particles, Eq. (1) allows a consistent representation of the electron-positron annihilation into light [2]

$$e^- + e^+ \ \rightarrow\ \gamma + \gamma. \tag{3}$$

More specifically, Dirac established the foundations of the notion of particle-antiparticle *annihilation*, which according to the very meaning of the word, deals with the elimination of the particle masses and ensuing transition to light.

Unfortunately, Dirac's conception of negative energy antiparticles is incompatible with special and general relativities on various counts and is afflicted by a number of problems of physical consistency that were identified by Dirac himself.

For instance, negative energies implies the possibility that the (positive energy) electron can emit a photon and pass to a negative energy state. The repetition of the process would then imply the lack of conservation of the energy. Similarly, the scattering between a positive and a negative energy particle may imply effects preceding their causes, by therefore violating causality.

Dirac attempted to resolve the above problems via his *hole theory* [3] which assumed the existence of an infinite sea of negative state positrons with occasional "holes" occupied by positive energy electrons, which theory provided a conceptual rendering of electron-positron annihilation and other events.

2 20th century positive energy antiparticles

In 1930, to achieve compatibility with special relativity (which was an understandable task for the time), J. R. Oppenheimer [4] assumed Dirac's equation as the fundamental equation of relativistic quantum mechanics, but under the theoretical assumption that *antiparticles have the same positive mass of particles* $\bar{m}_{e^+} = m_{e^-} > 0$, which assumption remains in full effect to this day.

The above view was implemented in the particle-antiparticle conjugation via the PTC theorem [5]. As an illustration, consider the wave packet of a free particle $\psi(t,r) = exp[i(p \times r - E \times t)]$ on a Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$ with state $|p, J, \psi\rangle$ where $p = mv$, J is the spin of the particle along the third axis an $E = v^2/2m$. Then, the *charge conjugation* of the particle is given by the anti-Hermitean transformation

$$C\psi(t,r) = -\psi^\dagger(t,r), \tag{4}$$

and the *particle-antiparticle conjugation* is given by

$$\begin{gathered} PTC|p, J, \psi(t,r)\rangle = PT|p, J, -\psi^\dagger(t,r)\rangle = \\ = P|-p, -J, -\psi^\dagger(-t,r)\rangle = |p, -J, -\psi^\dagger(t,r)\rangle, \end{gathered} \tag{5}$$

according to which antiparticles have the same energy E and momentum $p = mv$ of antiparticles and are defined on the same coordinates t, r, the same Hilbert space $\mathcal{H}$ of particles over the same numeric field $\mathcal{C}$.

Recall that a particle with spin $1/2$ is technically defined as a unitary irreducible representation of the spinorial covering of the Poincare' symmetry $\mathcal{P}(3.1) = SL(2.C) \times \mathcal{T}(3.1)$ [6]. In order to render conjugation (7) compatible with the $\mathcal{P}(3.1)$ symmetry while considering Eq. (1) the fundamental equation of relativistic quantum mechanics, the physicists of the early 20th century assumed the realization of the generators of $\mathcal{P}(3.1)$ in terms of the 4×4 gamma matrices (2) [6] [7] [8] (rather than as the Kronecker product of the symmetries of the two distinct 2×2-components of Dirac's equation as done in the next section)

$$\begin{gathered} \mathcal{P}(3.1) = SL(2.C) \times \mathcal{T}(3.1), \\ \mathcal{SL}(2.C): \quad S_k = \tfrac{1}{2}\gamma_k \times \gamma_4, \quad R_k = \tfrac{1}{2}\epsilon^k_{ij}\gamma_i \times \gamma_j, \mathcal{T}(3.1): \quad P_\mu, \end{gathered} \tag{6}$$

where S_k, R_k and T_μ are the generators of spin, boosts and linear momentum, respectively, with commutation rules for $J_{\mu,\nu} = \{S_k, R_k\}$

$$\begin{gathered} [J_{\mu\nu}, J_{\alpha\beta}] = i(\eta_{\nu\alpha}J_{\beta\mu} - \eta_{\mu\alpha}J_{\beta\nu} - \eta_{\nu\beta}J_{\alpha\mu} + \eta_{\mu\beta}J_{\alpha\nu}), \\ [J_{\mu\nu}, P_\alpha] = i(\eta_{\mu\alpha}P_\nu - \eta_{\nu\alpha}P_\mu), \quad [P_\mu, P_\nu] = 0, \end{gathered} \tag{7}$$

and Casimir invariants

$$C_1 = I,$$

$$C_2 = P^2 = P_\mu P^\mu = (\eta^{\mu\nu} P_\mu P_\nu), \tag{8}$$

$$C_3 = W^2 = W_\mu W^\mu, \;\; W_\mu = \epsilon_{\mu\alpha\beta\rho} J^{\alpha\beta} P^\rho.$$

Unfortunately, the conception of antiparticles as having positive masses with conjugation (5) implies the departure from Dirac's particle-antiparticle *annihilation* into light, since positive mass particles and antiparticles can only experience a *scattering* with the production of a shower of positive mass particles, as experimentally established in the Bose-Einstein correlation [9].

The important distinction between Dirac's annihilation and 20th century particle scattering has been technically expressed in the preceding paper [10] (see also the recent comments [11]) via the following:

THEOREM 2.1: Under the spinorial Poincareé symmetry (6), particle-antiparticle conjugation (5) is incompatible with the experimental evidence of electron-positron annihilation into light (3).

PROOF: Within the class of unitary equivalence of relativistic quantum mechanics, there exists no possibility of rendering null the sum of two positive masses in the Casimir invariants of the $\mathcal{P}(3.1)$ symmetry, Eq. (8).

The above theorem casts shadows on the true antimatter character of the 20th century notion of antiparticles characterized by conjugation (5), because said theorem prohibits their annihilation into light, as predicted by basic axioms, available preliminary evidence (Sect. 3.5 and Fig. 1) and the following comments:

2.1. Under the condition of being represented via Dirac's equation (1), we shall call *cosmic antiprotons* the antiparticles expected to be the origin of the bright flashes detected by astronauts and cosmonauts in the dark side of our upper atmosphere [12], or detected in cosmic rays [13]. We shall then call *laboratory antiprotons* the particles $\bar{p}^-$ produced in laboratory via collision of high energy protons p^+ against a matter target t (e. g., an iridium rod) according to the reaction [15]

$$p^+ + t \;\rightarrow\; (p^+, \bar{p}^-) + p^+ + t, \tag{9}$$

for which a proton-antiproton pair is expected to be produced by *pure kinetic energy.* Under these conditions, it is natural to expect that protons and true antiprotons should annihilate into pure electromagnetic radiations (here symbolically denoted $\gamma_k, \; k = 1, 2, ...$) without massive residues

$$p^+ + \bar{p}^- \;\rightarrow\; \Sigma_k \gamma_k. \tag{10}$$

The apparent lack of true antimatter character of laboratory antiprotons $\bar{p}^-$ is then expected from the experimental evidence that proton-antiproton collisions in the Bose-Einstein correlation produce instead a shower of bosons $B_k = 1, 2, 3, ...$ [9]

$$p^+ + \bar{p}^- \rightarrow \Sigma_k B_k. \tag{11}$$

2.2. The synthesis of the neutron from an electron and a proton in the core of stars [16]

$$e^- + p^+ \rightarrow n + \nu, \tag{12}$$

has received significant theoretical [17] as well as experimental [18] verifications. The same studies have identified the subsequent apparent synthesis in the Sun and in laboratory of an electron, and this time, a neutron, into the *negatively charged* pseudoproton [19]

$$e^- + n \rightarrow \tilde{p}^- + \nu, \tag{13}$$

which is essentially a positive energy proton with a negative charge. It is therefore possible that high energy collisions of protons against a matter-target (thus, initially against the *electron clouds* of the target, rather than their nuclei) may first synthesize neutrons (11) and subsequently pseudoprotons (12), rather than true antiprotons (see [11] for additional comments).

2.3. Conjugation (5) does not allow a study of the rather general view that the antimatter world is a complete dual of the matter world [20] [21]. In fact, the representation of *particle-antiparticle* conjugation with *charge* conjugation (4) is insufficient to study as to whether the photon, the π^0 meson and other particles do have an antimatter counterpart as predictable by the extension of Dirac's negative energy antiparticles to the photon and all other particles.

Needless to say, doubts on the true antimatter character of laboratory antiprotons cast shadows on the entire antimatter character of the anti-Hydrogen atom [22], and consequently, on its claimed conventional gravity [23] (see Sect. 4 for specific comments on antigravity tests).

3 Isodual theory of antiparticles

3.1. Isodual branch of hadronic mechanics

From the analysis of the preceding sections, it appears that, whenever treated with *20th century applied mathematics,* the notion of antiparticles is not entirely compatible with physical evidence irrespective of whether the antiparticle masses are negative (Sect. 1) or positive (Sect. 2).

This author has indicated various times in his writings that the lack of resolution of century old *physical* problems is generally due to the use of insufficient *mathematics,* as illustrated by the advent of Hilbert spaces for the Hydrogen structure, Riemannian geometry for gravitation, and other historical cases.

By noting that charge conjugation (4) is anti-Hermitean, it is plausible to expect that the same conjugation applies to all other characteristics of antiparticles. This expectation lead to the construction of the *isodual branch of hadronic mechanics* [24]-[28] which can be constructed via the anti-Hermitean conjugation, called *isoduality* (and denoted with an upper or lower letter d) applied to the *totality* of quantities Q, all possible local variables $t, r, p, E, \psi, ...$ and all operations of 20th century particle physics

$$Q(t, r, p, E, \psi, ...) \;\rightarrow\; Q^d(t^d, r,^d p^d, E,^d \psi,^d ...) =$$

$$= -Q^\dagger(-t^\dagger, -r^\dagger, -p^\dagger, -E^\dagger, -\psi^\dagger, ...) = -Q^\dagger(-t, -r, -p, -E, -\psi^\dagger, ...). \tag{14}$$

A quantity Q is said to be *isoselfdual* when it coincides with its isodual, $Q \equiv Q^d$.

In this paper, we study *point-like antiparticles in vacuum* (see Refs. [24]-[26] for extended antiparticles) treated via the *isodual mathematics* with fundamental, *negative-definite* multiplicative unit (called *isodual unit*)

$$1^d = -1^\dagger = -1, \tag{15}$$

compatible multiplication (called the *isodual product*)

$$\begin{gathered} s^d = (1^d)^{-1}, \\ 1^d \times^d n^d = 1^d s^d n^d = n^d s^d 1^d \equiv n^d, \end{gathered} \tag{16}$$

and compatible isodual images of applied mathematics, including: new numbers with negative (multiplicative) unit called *isodual numbers* [29]; *isodual spaces* [30]; *isodual Lie's theory* [24]-[26]; *isodual space-time symmetries* [31]-[34]; *isodual Minkowskian and Riemnannian geometries* [35]; and other aspects of isodual mathematics.

To prevent a prohibitive length, we limit ourselves to recall:

3.1) *Isodual fields* $\mathcal{F}^d(n^d, \times^d, 1^d)$ of *isodual real, complex and quaternionic numbers* $n^d = -n^\dagger$ [29] with isodual multiplication

$$n^d \times^d m^d = k^d \,\forall\, n^d, m^d, k^d \;\in\; F^d. \tag{17}$$

Recall that functions $f(r)$ have values on their base field. Similarly, *isodual functions* $f^d(r^d)$ have value in their isodual isofield, e.g., the isodual of e^r is given by $e_d^{r^d} = -e^{-r}$.

3.2) *Isodual space-time* [24] [25] [26] $M^d = M^d(x^{\cdot}\Omega^d, I^d)$ over $\mathcal{R}^d$ with *isodual coordinates*

$$x^d = (x^d_\mu) = (-x^{d,1}, -x^{d,2}, -x^{d,3}, -x^{d,4} = \\ = -c^d \times^d t^d = ct) \in R^d, \tag{18}$$

isodual metric

$$\Omega^d = (1^d)^{-1}\eta^d = (1^d)^{-1}Diag.(1^d, 1^d, 1^d, -1^d) = \\ = Diag(-1, -1, -1, +1), \tag{19}$$

where the isoduality $\eta^d = (1^d, 1^d, 1^d, -1^d)$ is needed for the metric η^d to be an *isodual matrix,* namely, a matrix whose elements are isodual numbers [24].

3.3) *Isodual differential calculus* of an isodual coordinate $r^d \in R^d$ [30] (see also [27])

$$d^d r^d = s^d d(r^d), \\ \frac{\partial^d f^d(r^d)}{\partial^d r^d} = 1^d \frac{\partial f^d(r^d)}{\partial r^d}. \tag{20}$$

Note that isodual mathematics is somehow "hidden" in the abstract axioms of 20th century mathematics because of the isoselfduality of the space-time invariant and of the differential

$$x^{d2d} \equiv x^2, \quad d^d r^d \equiv dr, \\ d^d F^d(r^d) \equiv dF(r^d), \tag{21}$$

as well as of other axioms, thus explaining its lack of detection until the 1993 paper [29].

3.2. Isodual theory of antiparticles

Following the achievement of maturity in the isodual mathematics, Santilli [36] proposed in 1994 the *isodual theory of antimatter* which can be defined as the isodual image of 20th century particle physics, by therefore being elaborated via isodual mathematics.

In this section, we shall attempt to verify the primary aims of proposal [36]:

1) Represent Dirac's negative energy antiparticles without its causality problems.

2) Achieve an axiomatic distinction between Dirac's particle and negative-energy antiparticle "annihilation" into light and the 20th century "scattering" of particles and positive-energy antiparticles.

3) Represent all known experimental data in antiparticles.

Figure 1: *We present a picture of the 1908 Tunguska explosion [52] that: A) Had the equivalent of hundreds of Hiroshima atomic bombs; B) Ionized the entire Earth atmosphere for days; and C) Produced on the ground no crater or solid debris. These features can only be quantitatively explained as due to an antimatter asteroid annihilating into sole electromagnetic radiations at contact with Earths's upper atmosphere, Eq. (10), thus constituting evidence of Dirac's particle-antiparticle annihilation into light (Sect. 1), while casting shadow on: the particle-antiparticle scattering into a shower of particles of the Bose-Einstein correlation [50] [51]; the true antimatter character of the nucleus of the anti-Hydrogen atom [22]; and its recent gravity test [23](Sects. 2, 3). The number of large explosions in the sky without a crater or solid debris in the ground is too large to be listed here (see, e.g. reports [53] [54]). The astrophysical origin of cosmic ray antiprotons [12] [13] and antimatter asteroids is under study for reporting in a subsequent paper.*

Isodual antiparticles are defined in the isodual Minkowski space $M^d(x^d, \Omega^d, 1^d)$ over the isodual numeric field $\mathcal{R}^d$ which space is independent but co-existing with our space-time $M(x, \eta, 1)$ thanks to identities (21) [37].

The application of isodual map (14) to charge conjugation (4) yields the *isodual map from particles to antiparticles* [26]

$$\psi(t, r) \rightarrow \psi^d(t^d, r^d) = -\psi^\dagger(-t, -r), \tag{22}$$

without any need for the *isodual parity* P^d (that would map negative into positive coordinates) and *isodual time inversions* T^d (that would map motion backward into forward in time).

The isodual theory of antiparticles resolves the problematic aspects of Dirac's negative energy antiparticles indicated in Sect 1. In fact, it is easy to see that the isodual image $\mathcal{P}^d(3.1)$ [35] of the spinorial covering of the Poincare' symmetry $\mathcal{P}(3.1$, Eq. (6), has negative-valued isodual Casimir invariants (8), by therefore bypassing Theorem II.1 and allowing an axiomatically consistent annihilation of particle-antiparticle into light.

The causality problems of Dirac's antiparticles are equally resolved by isodual mathematics because numeric values of ordinary particles are measured in terms of units (e.g., gr, sec, cm etc.) that are elements of the basic field $\mathcal{R}$. Consequently, under isoduality numeric values of isodual antiparticles must be measured in terms of units that are elements of the isodual field $\mathcal{R}^d$, thus having *negative* values $gr^d = -gr, sec^d = -sec, cm^d = -cm$, etc.

Consequently, negative energy antiparticles moving backward in time referred to negative units of mass and time are as causal as ordinary particles with positive mass moving forward in time referred to positive units of mass and time.

The problem of the conservation of the energy in particle-antiparticle scattering appears to be resolved by the accurate implementation of isodual mathematics. In fact, an antiparticle is represented in *isodual quantum mechanics* [25] [26] via the isodual Hamiltonian $H^d(r^d, p^d)$ acting on the isodual Hilbert space $\mathcal{H}^d$ with isodual states $|\psi^d(r^d)\rangle$, resulting in isodual eigenvalues E^d defined on the isodual field $\mathcal{R}^d$

$$H^d(r^d, p^d) \times^d |\psi^d(r^d)\rangle = E^d \times^d |\psi^d(r^d)\rangle, \quad E^d < 0_{R^d}, \tag{23}$$

with consequential quantitative representation of Dirac's antimatter with negative energy E^d. When projected in our Hilbert space $\mathcal{H}$ with state $|\psi(r)\rangle$ over the ordinary field $\mathcal{R}$, the same antiparticle of Eq. (23) acquires *positive eigenvalues* on $\mathcal{R}$

$$H^d(r^d, p^d) \times^d |\psi(r)\rangle = E \times |\psi(r)\rangle, \quad E > 0_R. \tag{24}$$

Similarly, a quantum mechanical particle characterized by a Hamiltonian $H(r, p)$ on $\mathcal{H}$ over $\mathcal{R}$ with *positive energy* E

$$H(r, p) \times |\psi(r)\rangle = E \times |\psi(r)\rangle, \quad E > 0_R, \tag{25}$$

when projected in the isodual space $\mathcal{H}^d$ over $\mathcal{R}^d$ acquires *negative energies* when inspected from the isodual world

$$H(r,p) \times |\psi^d(r^d)\rangle = E^d \times^d |\psi^d(r^d)\rangle, \quad E^d < 0_{R^d}. \tag{26}$$

Consequently, the total energy of the Positronium for quantum mechanics (qm) $\mathcal{P} = (e^-, e^+)_{qm}$ is positive in our space-time and negative in the isodual space-time (for brevity see Sect. 2.3.14, p. 131 of [26]).

Note that the isodual image of the wave function of free particles $\psi(t, lr) = exp[i(p \times r - E \times t)]$ considered in Sect. 1 is given by

$$\begin{gathered}\psi^d(t^d, r^d) = [e^{i(p\times r - E\times t)}]^d = \\ = e_d^{i^d \times^d (p^d \times^d r^d - E^d \times^d t^d)} = -e^{-i(p\times r - E\times t)},\end{gathered} \tag{27}$$

and the same rules apply for the isoduality of generic wave functions.

It should be finally indicated that the isodual map is equivalent to charge conjugation when projected in our world because it is anti-Hermitean, Eq. (14.) like charge conjugation, Eq. (4,) and admits positive energy eigenvalues, Eq. (24). Consequently, to our best knowledge, the isodual theory of antimatter verifies all existing experimental evidence on antiparticles merely subjected to the indicated re-interpretation on isodual spaces over isodual fields [26].

3.3. Isoselfdual Dirac equation

In the transition from non-relativistic to relativistic isodual theory of antiparticles with spin $1/2$, let us recall that the isodual theory of antimatter allows the *identical* reformulation of Dirac's equation (1) in the following form called *iso-self-dual Dirac's equation* because invariant under isoduality (Sect. 2.3.6, p. 118 of [26])

$$\begin{gathered}(i^d \times^d \eta^{d\mu\nu} \times^d \gamma^d_\mu \partial^d_\nu - m^d \times^d c^d) \times^d |\psi^d(x^d)\rangle = \\ = (i\eta^{\mu\nu}\bar{\gamma}_\mu \partial_\nu - mc)|\psi(x)\rangle = 0,\end{gathered} \tag{28}$$

where the 4×4 matrices $\bar{\gamma}_\mu$ are given by

$$\begin{gathered}\bar{\gamma}_k = \begin{pmatrix} 0 & \sigma_k, \\ \sigma^d_k & 0 \end{pmatrix} \equiv \begin{pmatrix} 0 & \sigma_k, \\ -\sigma_k & 0 \end{pmatrix} = \gamma_k, \\ \gamma_4 = i \begin{pmatrix} +I_{2\times 2} & 0 \\ 0 & I^d_{2\times 2} \end{pmatrix} \equiv i \begin{pmatrix} +I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix} = \gamma_4,\end{gathered} \tag{29}$$

and verify the isodual anti-commutation rules

$$\{\bar{\gamma}_\mu \bar{\gamma}_\nu\}^d = \bar{\gamma}_\mu \times^d \bar{\gamma}_\nu + \bar{\gamma}_\nu \times^d \bar{\gamma}_\mu = 2^d \times^d \eta^d_{\mu\nu}, \tag{30}$$

with total isodual spin symmetry [25]

$$S^{total}_{spin} = SU(2) \times SU^d(2), \tag{31}$$

and corresponding space-time symmetry [31]-[34]

$$S^{total}_{Dirac} = \mathcal{P}(3.1) \times \mathcal{P}^d(3.1), \tag{32}$$

both symmetries (31) and (32) being isoselfdual [26] [31]-[34].

In the author's view, a most important aspect of isodual mathematics is the axiomatic formulation of Dirac's particle-antiparticle annihilation into light via the notion of *isoselfduality* (invariance under the isoduality) with a number of important implications indicated later on.

Another important aspect is that, being a formulation for *point-like* antiparticles in vacuum, the isodual theory of antiparticles preserves special and general relativities in their entirety, since they are merely reformulated on isodual spaces over isodual fields. Intriguingly, the resulting *isodual special and general relativities* [26] are hidden in their conventional axioms, Eqs. (21), by therefore preventing a dismissal of their existence.

Finally, we should note that the isodual theory of antimatter predicts matter-antimatter *antigravity* at all levels, from Newtonian mechanics to general relativity [26]. Particularly instructive is to see the emergence of antigravity in the projection of the isodual curvature tensor in the Riemannian space of general relativity, by therefore illustrating that views on violation of general relativity by antigravity aree essentially due to insufficient mathematics along the main line of Sect. 3.1.

3.4. Isodual rules for unstable antiparticles

The isoduality of *permanently stable* particles, i.e., the electron and the proton, is done via a simple use of the above isoselfdual Dirac equation. The isoduality of *unstable* particles is conceptually and technically difficult because instability suggests a composite structure that, in turn, suggests the use of physical, i.e., experimentally detectable (rather than hypothetical and undetectable) constituents as a condition for results to have a chance of passing the test of time.

According to the author's experience in the field dating back to the 1990's, the *numerically exact and time invariant* representation of *all* characteristics of unstable particles, including the mechanism for the spontaneous decay, the value of its mean life and the remaining characteristics could only be possible via the assumption that the physical constituents of unstable particles are ordinary particles are bonded together by nonlinear, nonlocal and nonpotential interactions due to deep mutual penetration and entanglement [38] of the wave packets according to the laws of *hadronic mechanics* (hm) [24] [25] [39] and produced free as a tunnel effect from hadronic to quantum mechanics in the spontaneous decays, generally those with the lowest modes.

As an example, the numerically exact and time invariant representation of all characteristics of the π^0 meson with physical constituents (which is needed for a consistent isodual conjugation) could only be reached by assuming that the π^0 is a *compressed positronium,* namely, the known quantum mechanical positronium [40] compressed down to mutual distances of the order of 10^{-13} cm under the laws of hadronic mechanics (see Table 5.1., p. 827 on of the 1978 Harvard University memoir [39] and Sect. 2.5.4 of the 2021 update [41])

$$\pi^0 = (e^-_\downarrow, e^{-d}_\uparrow)_{hm} \equiv \pi^{0d}. \tag{33}$$

The presence of the positron in the pi^0 structure then allowed the understanding of the mechanism of its spontaneous decay via electron-positron annihilation, the numeric value of its mean life, the particle produced in the spontaneous decays and the remaining characteristics.

Under the above conditions, the π^0 meson is isoselfdual, namely, a particle that exists identically in our world as well as in the antiparticle world. Note that representation (33) is impossible for quantum mechanics due to the lack of admission of contact non-Hamiltonian interactions between the wave packets of the electron and positron in deep EPR entanglement.

The exact and invariant representation of *all* characteristics of the $\mu^\pm$ lepton, including its anomalous magnetic moment [42], has been achieved via the structure [39] [41]

$$\mu^\pm = (e^-_\downarrow, e^\pm_\uparrow, e^+_\downarrow)_{hm}, \tag{34}$$

in which case μ^+ can be interpreted as the antiparticle image of μ^-. The antiparticle interpretation of the remaining hadronic structure models of unstable leptons, mesons and baryons is left to the interested reader [41].

3.5. Intriguing predictions

The isodual theory of antimatter identifies the violation of the isoselfdual symmetry by annihilation (3) (because its l.h.s. is isoselfdual while the r.h.s. is not), but restores said symmetry via the prediction of the *antiphoton* (technically identified as the *isodual photon* and denoted γ^d) [43] consisting of photon with all characteristics opposite those of ordinary photons, with consequential reformulation of annihilation (3) into the isoselfdual firm on both sides

$$e^- + (e^-)^d \rightarrow \gamma + \gamma^d. \tag{35}$$

An intriguing prediction of the isodual theory of antimatter is that, contrary to the rather popular belief for which nothing can escape from black holes, antiphotons are predicted to be *expelled* by black holes due to antigravity following internal particle-antiparticle creation and subsequent annihilation in the isoselfdual form (35).

The apparent lack of true antimatter character of laboratory antiprotons appears to be confirmed by the 1908 Tunguska explosion [52] in Siberia which (Fig. 1): had the power equivalent to hundreds of Hiroshima atomic bombs; ionized the entire Earth's atmosphere to the point that, the day after, people would read newspapers at midnight without artificial light; left no crater or debris in the ground. Since atomic bombs did not exist in 1908, the sole quantitative representation of the Tunguska explosions is that it was due to the entire annihilation into electromagnetic radiations of an antimatter asteroid of unknown astrophysical origin (as it is the case for cosmic antiprotons) when in contact with our matter atmosphere.

Therefore, the Tunguska explosion appear to confirm Dirac's particle-antiparticle annihilation into light and casts shadows on the antimatter character of laboratory antiprotons as well as of the nucleus of the anti-Hydrogen atom. Opposing interpretations of the Tunguska explosion are encouraged provided that they reach a numerical representation of the origin of the produced extreme energy.

Evidently, for the proton-antiproton annihilation (10) to be compatible with the lack of any debris in the Tunguska and other explosions (Fig. 1) it shoud have the isoselfdual form

$$p^+ + (p^+)^d \;\rightarrow\; \Sigma_K \gamma_k + \Sigma_k \gamma_k^d. \tag{36}$$

The number of similar large explosions, that also occurred without solid debris in the ground and caused the local termination of radio communications, is too long to be reported here (see, e.g., the 2017 explosion [53]). Among older similar events, we mention the 1871 Chicago fire [54] also without a crater on the ground but the electromagnetic radiations were such to instantly melt bricks. It is unfortunate for our own safety that, despite their magnitude and anomalous character, the study of these events is dismissed by mainstream academia because not predicted by special and general relativities.

4 Antigravity tests

As indicated the preceding paper [10], Theorem 2.1 casts shadows in the true antimatter character of labratory antipritons [14] and, consequently, on the full antimatter character of anti-Hydrogen atoms [22] with ensuing shadows on their conventional gravity [23].

In the author's view, these results are a consequence of this axiomatic representation of particle-antiparticle annihilation via isoselfduality and, therefore, are a consequence of its violation in the proton-antiproton annihilation (11) of the Boise-Einstein correlation [50] [51].

In view of the above problematic aspects we suggest that, prior to any final claim, the gravity between matter and antimatter should be tested via antiparticles of clear and independently proved antimatter character, as it is the case for R. M. Santilli [44] [45] 1994 comparative test of the gravity of very low (thermal) energy electron and positron beams in horizontal flight in a supercooled vacuum tube in which the length of the tube can be selected for given eV in such a way to allow a *visible* deviation due to gravity when the beam hits a terminal scintillator, and the radius of the tube can be selected to render stray field fluctuations smaller than the expected terminal deviation due to gravity.

Test [44] [45] was presented at the 1996 international Workshop on Antimatter, Sepino, Italy, the 2011 San Marino Workshop on Antimatter, the 2016 SIPS Conference, Hainan Island, China, the 2023 SIPS Conference in Panama and other meeting. The feasibility, low cost and resolutory character of test [44] [45] have been confirmed by A. P. Mills [46] [47], V. de Haan [48] [49] and others.

5 Concluding remarks

In the first section, we recalled that Dirac's [1] characterizations of *negative energy* antielectrons (positrons) and antiprotons (called in this paper cosmic antiprotons [12] [13]) verify particle-antiparticle annihilation into light (3) but violate special relativity and have a number of causality problems.

In the second section, we recalled that, to achieve compatibility with special relativity, the physicists of the early 20th century [4] made the theoretical assumption (still valid nowadays) that antiparticles carry *positive energy* (called in this paper laboratory antiprotons [14]); we introduce, apparently for the first time, a theorem proving that positive energy antiparticles cannot verify Dirac's annihilation into light; and we present a number of arguments casting shadows on the true antimatter character of laboratory antiprotons [14], anti-Hydrogen atoms [22] and the recent claim of their conventional gravity [23].

Consequently, when treated with 20th century applied mathematics, the notion of antiparticles as not achieved compatibility with experimental evidence for both Dirac's and 20th century conception, by therefore warranting the use of a basically *new mathematics.*

Along the latter lines, we have reviewed in the third section decades of mathematical, theoretical and experimental studies on antiparticles based on *isodual mathematics* which is as an anti-Hermitean image of applied mathematics (called isodual and denoted with the letter d) characterized by the *negative multiplicative unit* $1^d = -1$ at all levels, beginning with basic numeric fields.

With the understanding that a technical knowledge of the paper requires the study of the original references due to decades of mathematical, theoretical and experimental studies on antimatter, we have shown that the ensuing isodual theory of antimatter provides:

5.1) A causal representation of Dirac's negative energy antiparticles and their annihilation into light via the isoselfduality (invariance under isoduality).

5.2) A full admission of special and general relativities, only defined on isodual spaces over isodual numeric fields.

5.3) The prediction of the negative energy *antiphoton*[43]

5.4) Positive eigenvalues when projected on conventional spaces over conventional fields.

5.5) The representation of available experimental data on antiparticles.

The main scope of this paper is to indicate the existence of a number of intriguing open problems on antimatter whose study requires a collegial participatoin, among which we mention

5.A) The astrophysical origin of cosmic antiprotons as well as of large explosions in our atmosphere without debris on the ground.

5.B) The comparative test of the gravity of thermal electrons and positrons [44]-[49].

5.C) The possible *expulsion* of antiphotons by black holes following internal particle-antiparticle creation and annihilation into photon-antiphoton pairs.

All in all, a serious scientific process for our own safety (Fig. 1) requires the admission that our current knowledge on antiparticles, as well as antimatter at large, is quite limited, if not misleading, due to one century of restriction of the studies to be compatible with theories, such as special and general relativities, that were not conceived for antimatter.

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