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Lie-isotopic representation of stable nuclei III: Exact and time invariant representation of nuclear stability

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Abstract

In the preceding two papers, we presented exact and time invariant representations of stable nuclei via the Lie-isotopic branch of hadronic mechanics and showed the necessity for the results of lifting Heisenberg's uncertainty principle for point-like particles into the isouncertainty principle of hadronic mechanics, also called *Einstein's isodeterminism,* for extended nucleons in condition of mutual penetration. In this paper we present apparently for the first time: the treatment of protons and neutrons as *isoparticles* characterized by the isosymmetries and isorelativities of hadronic mechanics; the representation of *all* characteristics of the neutron at the nonrelativistic and relativistic levels as a hadronic bound state of an isoelectron and an isoproton; the identification of the density of the neutron in a way compatible with other experimental data; and the representation of nuclear stability despite the natural instability of the neutron and despite the extremely repulsive protonic Coulomb forces. The main implications of the above studies are: 1) The prediction of novel means for the recycling of radioactive nuclear waste by nuclear power plants via new stimulated decays. 2) The possible return to the continuous creation of matter in the universe as a consistent way to explain the $0.782\ MeV$ missing in the neutron synthesis. 3) The apparent confirmation of the historical reduction of all matter in the universe to protons and electrons.

Keywords: nuclear physics 81V35, EPR argument, hadronic mechanics, nuclear data. [1]

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1 Introduction

Inspired by the Einstein-Podolsky-Rosen view that *Quantum mechanics is not a complete theory* [1], in the preceding first paper [2] (herein referred to with the letter I) we have presented apparent insufficiencies of quantum mechanics in nuclear physics. In the preceding second paper [3] (herein referred to with the letter II) we have presenetd a systematic review and upgrade, specialized for the first time to nuclear physics, of the axiom-preserving time reversal invariant *Lie-isotopic branch of hadronic mechanics* [4]- [6] comprising the novel *isomathematics* (Sect. II-3), *isomechanics* (Sect. II-4) and *isosymmetries* (Sect. II-3.10) which are based on the axiom-preserving completion of the universal enveloping associative algebra of quantum mechanical Hermitean operators $\hat{\xi}$: $\{A, B, ...; A \times B = AB; I\}$ into the isoassociative enveloping algebra of hadronic mechanics ξ : $\{A, B, ...; A \hat{\times} B = A\hat{T}B; \hat{I} = 1/\hat{T}\}$ with consequential iso-Heisenberg's equation (Sect. II-5.4) [5] $idA/dt = A\hat{\times}H - \hat{\times}A = A\hat{T}H - \hat{T}A$ in which potential interactions are represented by the Hamiltonian H and non-potential interactions are represented by the isotopic element $\hat{T}$ at times called the Santillian.

We have shown in paper [2] that the Lie-isotopic mechanics implies:

1.1) The first known representation invariant over time of the dimension, shape and density of particles via the isotopic element $\hat{T}$ per realization (II-15) [5].

1.2) The first known quantitative representation of subluminal, continuous and instantaneous interactions of entangled particles at arbitrary distances characterized by the non-Hamiltonian interactions of the wave packets of particles [7].

1.3) The first known explicit and concrete realizations of Bohm's hidden variables λ [8] [9] in terms of the Santillian $\lambda = \hat{T}$, as being hidden in the axiom of associativity of quantum mechanics [10] [11].

1.4) The violation of Bell's inequalities [12] by a system of extended particles with spin $1/2$ under non-Hamiltonian interactions with ensuing existence of classical counterpart [13].

1.5) The EPR completion of Heisenberg's uncertainty principle for point-like particles under electromagnetic interactions into the *isouncertainrty principle* of hadronic mechanics, also called *Einstein's isodeterminism,* for extended particles under electromagnetic and strong interactions (Sect. 4.7) [14] (see [15] for an extended derivation), which principle implies a progressive recovering of Einstein's determinism [1] with the increase of the density of hadronic media (decrease of the value of $\hat{T}$) in the interior of hadrons, nuclei and stars and its full recovering at the limit of gravitational collapse due to the identity of the isotopic element with Schwartzschild's horizon, Eq. (II-34).

It should be indicated that none of the results of paper I or of this paper are possible under the validity for nuclear structures of Heisenberg's uncertainty principle for electromagnetic interactions, while the results are all possible for the completion of said principle into Einstein's isodeterminism for strong interactions (Sect. 3.3.1).

We have then shown in the preceding paper I that the Lie-isotopic branch of hadronic mechanics has permitted the achievement of the first known, numerically exact and time invariant representation of the experimental data of the Deuteron conceived as a hadronic bound state of a proton and a neutron in its ground state without orbital contributions used in experimental measurements.

Based on tabulated fundamental characteristics of electrons, protons and neutrons [16]-[23], in this paper we study, apparently for the first time, nuclear stability despite the neutron natural instability and the strongly repulsive protonic Coulomb forces via the following primary steps:

1.A) The EPR completion of the 20th century notion of *particle,* as a unitary irreducible representation of the spinorial covering of the Poincaré symmetry $\mathcal{P}(3.1) = SL(2.C) \times \mathcal{T}$ into that of *isoparticle,* as an isounitary, isoirreducible isorepresentation of the isospinorial isosymmetry $\hat{\mathcal{P}}(3.1) = \hat{SL}(2.\hat{C})\hat{\times}\hat{\mathcal{T}}$ [24] [25] [26] which appears to be necessary under non-Hamiltonian interactions between extended particles due to an apparent mutation of the *intrinsic characteristics* of 20th century particles predicted in the 1978 memoir [27].

1.B) A review of extensive studies in the synthesis of the neutron from the Hydrogen atom in the core of stars as a hadronic bound state of an isoelectron $\hat{e}^-$ totally compressed and entangled inside the dense isoproton $\hat{p}^+$.

1.C) The representation of nuclear stability despite the natural instability of the neutron via the *decoupling* of the neutron into its original, *permanently stable* constituents, the isoproton and the isoelectron, resulting in a three-body structure of the Deuteron.

1.D) A representation of the nuclear stability despite the very big, protonic Coulomb *repulsion* via isoprotons.

1.E) The prediction of means for the recycling of radioactive nuclear waste by nuclear power plants via *new* stimulated decays.

1.F) The possible return to the continuous creation of matter in the universe as a consistent way to explain the 0.782 MeV missing in the neutron synthesis.

1.G) The apparent confirmation of the historical reduction of all matter in the universe to protons and electrons.

As we shall see, the primary applications of the above representation of nuclear stability are given by:

1.i) The prediction of means for the recycling of radioactive nuclear waste by nuclear power plants via new stimulated decays.

1.ii) The possible return to the continuous creation of matter in the universe a consistent way to explain 0.782 MeV missing in the neutron synthesis.

1.iii) The apparent confirmation of the historical reduction of all matter in the universe to protons and electrons.

the prediction of basically new means for the recycling of radioactive nuclear waste via their stimulated decay by the nuclear power plants themselves, the apparent reduction of all matter in the universe to protons and electrons in conditions of increasing complexity and the apparent *origin* (rather than the sole description) of gravitation in the nuclear structure.

The understanding of the main lines of this paper can be achieved via a knowledge of Ref. [2], although a technical understanding of this paper can be best achieved via a full knowledge of hadronic mechanics [4]-[6].

2 Isoparticles and isorelativities

To our best knowledge, the notion of *mutation* of elementary particles was first introduced in Sect. 5, p. 819 of the 1978 Harvard University memoir [27] to denote the expected alteration of the *intrinsic* characteristics of particles under strong interactions. To upgrade the notion of mutation with the advances occurred in the meantime, let us recall also that, according to 20th century physics, a spin $1/2$ *particle* [28] is a unitary irreducible representation of the *spinorial* covering $\mathcal{P}(3.1) = SL(2.C) \times \mathcal{T}(3.1)$ of the Poincaré symmetry $P(3.1) = SO(3.1) \times \mathcal{T}(3.1)$ (Sect. II-3.10).

The impossibility of representing the indicated mutation with conventional spacetime symmetries and relativities has suggested the following generalized notion of particles introduced at the nonrelativistic level in the 1991 paper [29] and at the relativistic level in Sect. 7.7, p. 312 on of the 1995 monograph [5].

DEFINITION 2.1: A 'relativistic isoparticle' is an isounitary isoirreducible isorepresentation of the spinorial covering of the Poincaré symmetry

$$\hat{\mathcal{P}}(3.1) = \hat{SL}(2.\hat{C}) \hat{\times} \hat{\mathcal{T}}(3.1), \tag{1}$$

formulated on the Hilbert-Myung-Santilli isospace over an isofield (Sect. II-4.3).

The Galilean isorelativity for the non-relativistic dynamics of extended objects within resistive media has been proposed in Refs. [30] [31] and cannot be reviewed for brevity (see the excellent general review [32]). The isotopies of special relativity have been first introduced in the 1983 paper [24], then developed in papers [25] [26], in the 1991 monographs [30] [31] and upgraded in the 2021 overview [33] (see independent reviews [32] [34] [35]). A conceptual review the isorelativities appears recommendable in view of their applications in subsequent sections.

DEFINITION 2.2: The term 'isorelativities' denotes the infinite family of axiom-preserving isotopies of special relativity characterized by the infinite family of isotopic elements and related isounits of (contravariant) type (II-15), represented in terms of the characteristic quantities $n_\mu > 0$ of paper [2] as well as in terms of the characteristic quantities $b_\mu = 1/n_\mu > 0$ used in the literature of the 1990's

$$\hat{T} = 1/\hat{I} = Diag.(n_1.^2, b_2^2, b_3^2, b_4^3) \equiv Diag(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2}), \tag{2}$$

where the exponential term of Eq. (II-15) has been incorporated in the characteristics quantities and can be factorized whenever needed.

All isorelativities are formulated on an iso-Minkowski isopace $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I}$ over the isoreal isofield $\hat{\mathcal{R}}$ (Sect. II-3.5) with isocoordinates $\hat{x} = x\hat{I} = (\hat{x} - \mu)\hat{I} = (x^k, x^4 = ct)\hat{I}$, isometric $\hat{\Omega} = \hat{\eta}\hat{I},\ \hat{\eta} = \hat{T}\eta$, where $\eta = Diag.(1, 1, 1, 1)$, isoinvariant in isospace

$$(\hat{x} - \hat{y})^{\hat{2}} = (\hat{x} - \hat{y})^\mu \hat{\times} \hat{\Omega}_{\mu\nu} \hat{\times} (\hat{x} - \hat{y})^\nu \tag{3}$$

and its projection in our spacetime $M(x, \eta, I)$ over the field of real numbers $\hat{\mathcal{R}}$

$$\begin{gathered}[\tfrac{(x^1-y^1)^2}{n_1^2} + \tfrac{(x^2-y^2)^2}{n_2^2} + \tfrac{(x^3-y^3)^2}{n_3^2} - \tfrac{(x^4-y^4)^2}{n_4^2}]\hat{I} - = \\ = [b_1^2(x^1 - \hat{y}^1)^2 + b_2^2(x^2 - y^2)^2 + b_3^2(x^3 - y^3)^2 - b_4^2(x^4 - y^4)^2]\hat{I},\end{gathered} \tag{4}$$

under the universal isosymmetry (1), with ensuing unique and unambiguous characterization of relativistic isolaws (Sect. 8.4.4, p, 134 on of [33]).

Besides the admission of special relativity for the particular case $\mu = 1,\ \mu = 1, 2, 3, 4$, the most important isorelativities are given by:

2.1. Special isorelativity, which occurs for the particular case $\mu = constant \neq 1,\ \mu = 1, 2, 3, 4$, for the characterization of extended-spheroidal particles with semiaxes n_1^2, n_2^2, n_3^2 moving within a hadronic medium with density $n_4 = 1/b_4$ (see the isoaxioms of special isorelativity, p. 138 of [33] and Refs. [24]-[35]).

2.2. Exterior isogravitation, which occurs for the factorization of the isotopic element $\hat{T}_{gr}$ from any nonsingular Riemannian metric $g(x)$ for the exterior treatment of gravitation and its formulation in terms of the isounit $\hat{I} = 1/\hat{T}$ of relativistic hadronic mechanics (first presented at the 1992 *Marcel Grossmann Meeting in Gravitation,* [36] [37])

$$\begin{gathered} g(x) = \hat{T}_{gr}(x)\eta, \\ \hat{I}_{gr}(x) = 1/\hat{T}_{gr}(x), \end{gathered} \tag{5}$$

(whose positive-definite character is guaranteed by the non-singularity of the metric $g(x)$) under which realization the isoaxioms of special isorelativity, p. 138 of [33] remain valid for $n_\mu = n_\mu(x) > 0$, with particular realization given by the first Schwartzchild's paper on the exterior gravitational problem [38]

$$\hat{T}_{kk} = \frac{1}{1 - \frac{2M}{r}}, \quad \hat{T}_{44} = 1 - \frac{2M}{r}. \tag{6}$$

2.3. Interior isograviation, holding for general nonsingular, symmetric, $3{+}1$-dimensional isometrics $\Omega\mu\nu$ with an unrestricted functional dependence on all needed local variables of type (II-11), when represented via the isodifferential calculus (Sect. II-3.7) and related iso-Minkowskian geometry [39] for the study of the *origin* (rather than the sole exterior description) of gravitation in the nuclear structure [40] [45] under which assumptions the isoaxioms of p. 138, Ref. [33], remain valid for an arbitrary functional dependence of the characteristic quantities $n_\nu > 0$.

These studies are along the second Schwartzchild's paper on the interior gravitational problem [42], which paper has been vastly ignored because incompatible with general relativity, but compatible with interior isogravitation.

Note that realization (6) of the Santillian is a particular case of the nuclear analysis of [2], by therefore indicating the possible primary source of gravitation in the *electromagnetic* structure of nuclei according to the 1905 historical hypothesis by H. Poincaré [43] and its apparent realization by R. M. Santilli in the mid 1970's when at MIT [44] under the *necessary* resolution of the insufficiencies of 20-th gravitation into a form compatible with all century sciences [45].

A realization of the isoalgebra of isosymmetry (1) is given by [26]

$$\hat{R}_k = \frac{1}{2}\epsilon_{kij}\hat{\Gamma}_i \hat{\times} \hat{\Gamma}_j, \;\; \hat{B}_k = \frac{1}{2}\hat{\Gamma}_k \hat{\times} \hat{\Gamma}_4, \;\; k = 1, 2, 3, \tag{7}$$

where $\hat{R}_k$ represents the generators of *isorotations,* $\hat{B}_k$ represents the generators of the *isoboosts* and $\hat{\Gamma}_\mu = \hat{\gamma}_\mu \hat{I}$ represent the Dirac-Santilli isomatrices (II-96), while a realization of the isotranslations $\hat{\mathcal{T}}(3.1)$ is given by 4-isomomentum (II-68) $\hat{P}_\mu, \quad \mu = 1, 2, 3, 4.$

The isocommutation rules of isosymmetry (1) with generators $\hat{J}_{\mu\nu} = \{\hat{R}_k, \hat{S}_k\}$ are then given by the spinorial covering of Eqs. (II-40)

$$\begin{gathered}
\left[\hat{J}_{\mu\nu} \hat{,} \hat{J}_{\alpha\beta}\right] = \hat{\eta}_{\nu\alpha}\hat{J}_{\beta\mu} - \hat{\eta}_{\mu\alpha}\hat{J}_{\beta\nu} - \hat{\eta}_{\nu\beta}\hat{J}_{\alpha\mu} + \hat{\eta}_{\mu\beta}J_{\alpha\nu}, \\
\left[\hat{J}_{\mu\nu} \hat{,} \hat{P}_{\alpha}\right] = i\hat{\eta}^{\mu\alpha}\hat{P}_\nu - \hat{\eta}_{\nu\alpha}\hat{P}_\mu, \quad \left[\hat{P}_\mu \hat{,} \hat{P}_\nu\right] = 0, \\
\hat{\eta}_{\mu\nu} = \hat{T}\eta = (\hat{T}^\mu_\rho \eta^{\rho\nu}),
\end{gathered} \tag{8}$$

while the iso-Casimir invariants are given by the spinorial covering of isoinvariants (II-41)

$$\hat{C}_1 = \hat{I} > 0, \quad \hat{C}_2 = \hat{P}^{\hat{2}} = \Sigma_{k=1,2,3}(b_k^2 \hat{P}_k^2 - m^2 c^2 b_4^2),$$
$$\hat{C}_3 = \hat{W}^{\hat{2}} = \hat{W}_\mu \hat{\times} \hat{W}^\mu, \quad \hat{W}_\mu = \epsilon_{\mu\alpha\beta\rho} \hat{J}_{\alpha\beta} \hat{\times} \hat{P}_\rho. \tag{9}$$

In this paper, we shall also use the explicit form of the *hadronic angular momentum* $\hat{O}(3)$ as an isosubgroup of isosymmetry (1) (Eqs. (6.4a) (6.4b) of Ref. [26] and Sect. II-5.3) and related isoeigenvaues

$$\hat{L}_k = \epsilon_{ijk} \hat{r}_i \hat{\times} \hat{p}_j, \;\; ! \left[\hat{L}_1 \hat{,} \hat{L}_j\right] = \epsilon_{ijk} n_k^2 \hat{L}_k = \epsilon_{ijk} b_k^2 \hat{L}_k,$$
$$\hat{L}_3 \hat{\times} |\psi\rangle = \pm \tfrac{1}{2} b_1^{-1} b_2^{-1} |\psi\rangle, \quad \hat{L}^{\hat{2}} \hat{\times} |\hat{\psi}\rangle = (b_1^2 b_2^2 + b_1^2 b_3^2 + b_3^2 b_1^2) |\hat{\psi}\rangle, \tag{10}$$

as well as the realization of the *hadronic spin* and related isoeigenvalues (see Eqs. (6.4c) (6.4d), p. 190 of Ref. [26] and Sect. II-5.5)

$$\hat{S}_k = \tfrac{1}{2} \epsilon_{ijk} \hat{\gamma}_i \hat{\times} \hat{\gamma}_j, \quad \left[\hat{S}_i \hat{,} \hat{S}_j\right] = \epsilon_{ijk} b_k^{-2} \hat{S}_k,$$
$$\hat{S}_3 \hat{\times} |\psi\rangle = \pm \tfrac{1}{2} b_1 b_2 |\psi\rangle, \quad \hat{S}^{\hat{2}} \hat{\times} |\psi\rangle = \tfrac{1}{4}(b_1^2 b_2^2 + b_2^2 b_3^2 + b_3^2 b_1^2) |\psi\rangle. \tag{11}$$

Some of the main features occurring for all isorelativities are given by [33]:

2.A) The *invariance* of events described by all isorelativities under isosymmetry (1) which is fundamental for the prediction of the same numeric values under the same conditions but at different times (Sect. II-3-12). By contrast, covariant theories (such as general relativity) violate this fundamental condition for experimental verifications that will pass the test of time.

2.B) The admission of arbitrary speeds of light within hadronic media with subluminal (superluminal) speeds, i.e., for media with low (high) density,

$$C = c/n_4,$$
$$C < c, \;\; n_4 > 1, \quad C > c \;\; n_4 < 1. \tag{12}$$

2.C) Under the assumption of perfect spheridicity, the isorenormalization of the energy of extended particles within a hadronic medium called the *mass-energy isoequivalence principle* (see again the isoaxioms of special isorelativity, p. 138 of [33], particularly Eq. (133), p. 139)

$$E = mc^2 \rightarrow \tilde{E} = mC^2 = \frac{n_k^2}{n_4^2} E = \frac{b_4^2}{b_k^2} E, \tag{13}$$

Figure 1: *In this figure, we illustrate some of the hadronic reactors built by Thunder Energies Corporation (now Hadronic Technologies Corporation) for the synthesis of neutrons from a commercially available Hydrogen gas.*

where $n_k^2 = 1/b_k^2$ is the semiaxis of the considered particle in the direction of its spin, and the difference $\tilde{E} - E$ is due to the *nonlocal effects* at the foundation of hadronic mechanics, namely, the effects in the mass-energy equivalence on a given mass from its hadronic medium under a state of deep EPR entanglement [7].

As we shall see in the next section, the above realization of isosymmetry (1) and related notion of isoparticles is sufficient for a consistent representation of the synthesis of the neutron in a star and for the consequential study of nuclear stability, with the understanding that progressively broader realizations of isoparticles are needed for progressively more complex physical conditions.

3 Representation of the neutron synthesis from the Hydrogen

3.1. Historical notes. As it is well known, stars initiate their lives as an aggregate of Hydrogen that grows by accretion during travel in interstellar spaces. When the temperature in the core of the aggregate reaches a value of the order of $10M\ K$, E. Rutherford [46] suggested in 1920 that the Hydrogen atom is "compressed" into a new neutral particle which he called the *neutron* [16]-[23]

$$e^- + p^+ \rightarrow n. \tag{14}$$

The existence of the neutron was experimentally established in 1932, by J. Chadwick [47]. In 1933, W. Pauli [48] pointed out that synthesis (11) violates the conservation of the angular momentum. Therefore, E. Fermi [49] submitted in 1935 the hypothesis that the synthesis of the neutron occurs with the joint *emission* of a neutral and massless particle ν with spin $1/2$ which he called the *neutrino* (meaning "little neutron" in Italian)

$$e^- + p^+ \ \rightarrow \ n + \nu, \tag{15}$$

with the neutron decay

$$n \rightarrow p^+ + e^- + \bar{\nu}, \tag{16}$$

where $\bar{\nu}$ is the antineutrino.

Subsequent tests [22] established that *the neutron is naturally unstable* with a mean life of $\tau = 877$ and spontaneous decay

$$n \rightarrow e^- + p^+ + \bar{\nu}, \tag{17}$$

where $\bar{\nu}$ is the *antineutrino.*

Predictably, the synthesis of the neutron from the Hydrogen attracted attention soon following Chadwick's confirmation. According to the historical account [50] Ernest J. Sternglass conducted in 1951 the first test for the laboratory synthesis of the neutron from the Hydrogen, followed by tests in 1952 by E. Trounson and others, although none of these initial tests were reported in published papers in view of the incompatibility of the neutron synthesis with quantum mechanics and for other reasons.

To the author's best knowledge, the first published tests on the laboratory synthesis of the neutron from the Hydrogen were done in the 1960's by the Italian priest-physicist Don Carlo Borghi and his associates [51]. In essence the experimentalists constructed a cylindrical metal chamber (called *klystron*) which was filled up with a Hydrogen gas at a fraction of $1\ bar$ pressure kept mostly ionized by an electric arc with about $500\ V$ and $10\ mA$. Additionally, the gas was traversed by microwaves with the frequency of $10^{-10}\ s$. The experimentalists then placed in the exterior of the Klystron various materials suitable to be activated when exposed to a neutron flux (such as gold or silver).

Following exposures over several weeks, the experimentalists reported clear and reproducible nuclear transmutations that can only be due to a neutral hadron emitted from the Klystron. Due to insufficient evidence on neutron emission, the experimentalists conjectured that the detected nuclear transmutations were due to a new neutral particle with the mass of the neutron but spin different than $1/2$ that they called the *neutroid.*

3.2. Recent studies. In view of its fundamental character in nuclear physics, R. M. Santilli and his associates have conducted over the past four decades mathematical, theoretical, experimental and industrial research on the synthesis of the

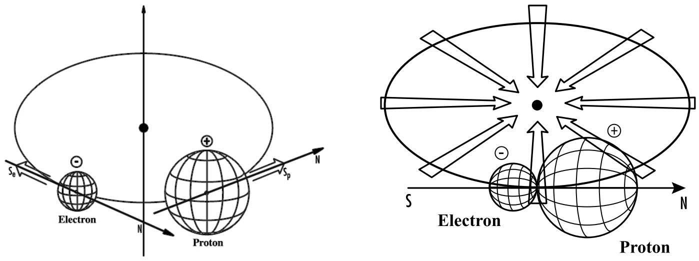

Figure 2: *In this figure, we illustrate the mechanism used by hadronic reactors (Fig. 1) for the synthesis of neutroids and neutrons via an especially designed DC electric arc between Carbon electrodes submerged within a Hydrogen gas causing the ionization of Hydrogen atoms into electrons and protons by the activation of the arc (left view) and the compression of the electron within the dense proton by the de-activation of the arc (right view).*

neutron from the Hydrogen atom in the core of stars as well as in laboratory (see theoretical studies [52]-[58], experimemtal works [59]-[65] and independent studies [66]-[71]).

The studies were initiated in the late 1970's at Harvard University under DOE support with the inapplicability of quantum mechanics for the neutron synthesis [27], followed by the proposal to construct *hadronic mechanics* in the 1978-1983 Springer-Verlag monographs [72] [73].

By far the biggest difficulty of the above studies was the representation of the spin of the neutron $S_n = 1/2$ from two particles each having spin $1/2$ as originally conceived by Rutherford [46]. This problem stimulated the construction of the Lie-Santilli isotheory (Sect. 4.4, p. 173 on of Ref. [73] and Sect. II-3.2), followed by systematic studies on the isotopies of spacetime symmetry reported in Sect. II-3.10, with particular reference on the isotopies $\hat{\mathcal{SO}}(3)$ of the angular momentum symmetry $\mathcal{SO}(3)$ at the classical and operator levels [74] [75].

As a result of these preparatory studies, Santilli was able to achieve a numerically exact and time invariant representation of *all* characteristics of the neutron at the nonrelativistic level in the 1990 paper [52] and at the relativistic level in the 1995 paper [56] with a more extensive study in monograph [57].

Following, and only following, the achievement of a consistent representation of the neutron synthesis via the Lie-isotopic branch of hadronic mechanics, Santilli initiated in 2007 experimental tests on the laboratory synthesis of the neutron from the Hydrogen [59]-[63]. According to these studies, the neutron synthesis from the Hydrogen can be generated by *hadronic reactors* (Fig. 1) consisting of a metal vessel containing in its interior a commercially available Hydrogen gas at pressure and a pair of submerged carbon electrodes powered by a specially

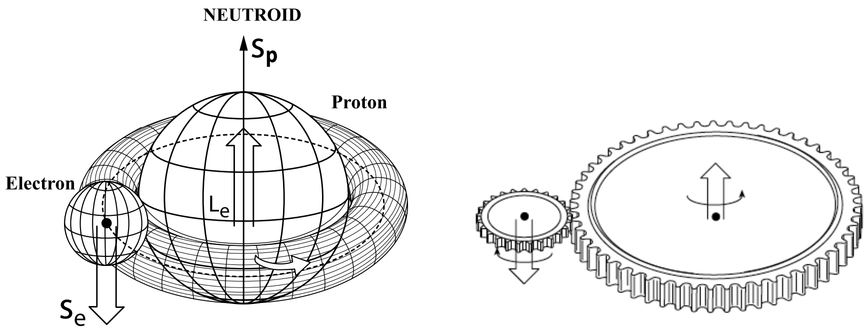

Figure 3: *In the left of this figure, we illustrate the predicted structure of the neutroid in its ground state as a hadronic bond of electrons and protons in singlet couplings with null eigenvalues of the angular momentum and of the spin under their very big Coulomb attraction. In the right of this figure, we present a conceptual gear equivalent of the left view to illustrate the reason for the half life of neutroids as being about* 10% *that of neutrons, i.e., of about* $8\ s$ *(which is extremely big for particle standards).*

designed, external DC source via a gap controllable from the outside. During operations, the DC arc is continuously connected and disconnected because of the consumption of the carbon electrodes. During its activation (left of Fig. 2), the special form of the DC arc ionizes the Hydrogen gas by creating a plasma mostly composed by protons and electrons in its cylindrical surroundings, while during its deactivation (right of Fig. 2), the DC arc compresses the ionized gas from all directions toward its symmetry axis, by realizing in laboratory Rutherford compression of the electron inside the proton.

Tests [59]- [65] have confirmed: 1) The production of Don Borghi's neutroids (Fig. 3) for DC power of the order of $5\ kw$, gas pressure of $5\ psi$ and electrode gap of $2\ mm$. 2) The production of neutrons (Fig. 4) for powers of the order of $50\ kw$, gas pressure from $10\ psi$ on and electrode gas of $5\ mm$. In particular, the synthesis of neutroids resulted to be an unavoidable step prior to the synthesis of the neutron.

Following, and only following, sufficient experimental evidence on the laboratory synthesis of the neutron from a Hydrogen gas, Santilli founded in 2012 the U. S. publicly traded company *Thunder Energies Corporation* (now the privately held Hadronic Technologies Corporation, www.hadronictechnologies.com) for the production and sale of *Directional Neutron Source* (DNS), namely, equipment producing a beam of low energy neutrons (with energy less than $100\ eV$) in a preferred direction and in the desired intensity for the detection of fissionable material that may be hidden in baggage (lower left view of Fig. 1).

Interested readers should be aware that commercially available AC-DC power units to create electric arcs between carbon electrodes submerged within a Hydrogen gas may synthesize neutroids (Fig. 3) and other unstable states under the big electron-proton Coulomb attraction (Eq. I.1), but said power units are not designed to synthesize neutrons via the compression of electron inside the proton.

3.3. Nonrelativistic representation of the neutron synthesis from the Hydrogen. Studies [52]-[56] were conducted under the assumptions (dating back to the 1978 memoir [27]) that:

3.3.A) The angular momentum of the electron compressed inside the proton is *constrained* to be equal to the spin of the proton to prevent big resistive forces caused by the motion of its extended wave packet against the dense medium in the interior of the proton.

3.3.B) When compressed inside the dense proton, the electron e is mutated into a new particle called the *eleton* in Sect. of [27] and hereon called the *isoelectron* $\hat{e}^-$, while the proton is mutated into the *isoproton,* although in first approximation the proton can be assumed to be unmutated, $\hat{p}^+ \approx p^+$ since it is about 1800 times heavier than the electron.

3.3.C) While the numeric value of the mean life of the resulting hadronic state may be subject to scientific debates, the existence of a bound state between the electron and the proton at short distance should be beyond scientific doubt in view of their extremely big Coulomb attraction.

The above assumptions imply the following *structure model of the neutron according to hadronic mechanics* (hm) which can be constructed via isounitary transformations (Sect. 3.12) of the quantum mechanical (qm) model of the Hydrogen atom

$$n = (\hat{e}^-_\downarrow, \hat{p}^+_\uparrow)_{hm} = \hat{U} \hat{\times} (e^-_\downarrow, p^+_\uparrow)_{qm} \hat{\times} \hat{U}^\dagger, \tag{18}$$

under the Coulomb *attraction* in the macroscopic value of 230 Newton,

$$F = -\frac{e^2}{r^2} = -(8.99 \times 10^9)\frac{(1.60 \times x10^{-19})^2}{(10^{-15})^2} = -230 \; Newtons. \tag{19}$$

3.3.1. The fundamental role of Einstein's isodetermiunism. The most serious insufficiency of quantum mechanics in nuclear physics identified in [2] has been the prohibition by Heisenberg's uncertainty principle to represent the synthesis of the neutron from the hydrogen in the core of stars, because when said principle is applied to the coordinate r_e and momentum p_e of an electron in the core of stars,

$$\Delta r \Delta p = \frac{1}{2} | < \psi(r_e)| \times [r_e \hat{,} p_e] \times |\psi(r_r) > \geq \frac{1}{2}\hbar, \tag{20}$$

it implies standard deviations of the coordinate bigger than the neutron radius with consequential kinetic energy of the electron bigger than the neutron mass,

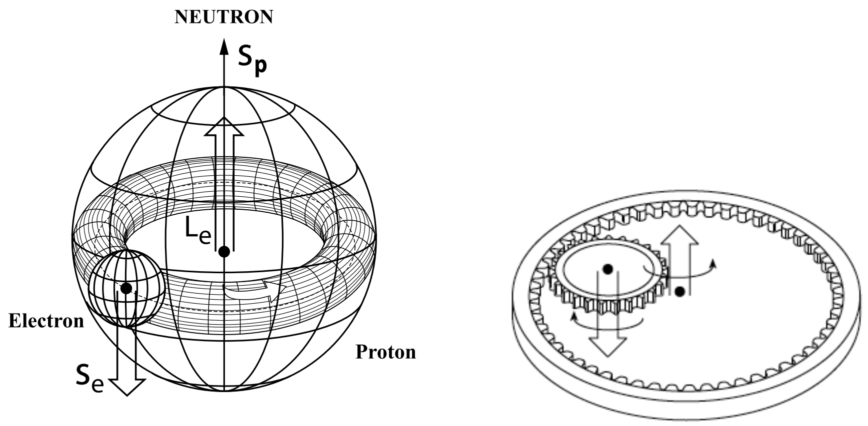

Figure 4: *In the left of this figure, we illustrate the compression of the neutroid (Fig. 3) into the neutron (Fig. 4), resulting in a constrained hadronic angular momentum of the electron within the dense medium inside the proton that, to avoid extreme resistive forces, has to be equal to the proton spin with ensuing total angular momentum* $1/2$.

Eq. (I-3), i.e.,

$$\Delta r_e > R_n = 0.87 \times 10^{-13}\ cm,$$
$$\Delta v_e > \tfrac{\hbar}{\Delta r_e \times m_e} > 10^{10}\ m/s, \tag{21}$$
$$\Delta K_e = \tfrac{1}{2m_e} \times (\Delta p_e)^2 > m_n = 939.56\ MeV/c^2.$$

By recalling that the validity of the uncertainty principle for point-like particles under electromagnetic interactions (as occurring in the atomic structure) is beyond doubt, its validity for nuclear constituents is a personal opinion by the individual physicist due to the lack of any direct experimental verification denounced beginning with the title of the 1978 Harvard memoir [27] while, by comparison, the synthesis of the neutron from the electron and the proton in the core of stars is a physical reality.

It is easy to see that the *isouncertainty principle* of hadronic mechanics (Sect. II-4.7) [1] [14] [15]

$$\Delta r \Delta p = \frac{1}{2}| < \hat{\psi}(\hat{r})| \star [\hat{r},\hat{p}] \star |\hat{\psi}(\hat{r}) > \frac{1}{2}\hbar\hat{T} \ \ll\ 1, \tag{22}$$

resolves the above impasse, because in view of very small value of $\hat{T}$, Eq. (II-15), the isotopic image of values (51)

$$\Delta r_e \leq R_n = 0.87 \times 10^{-13}\ cm,$$

$$\Delta v_e \ll 10^{10}\ m/s, \tag{23}$$

$$\Delta K_e = \frac{1}{2m_e} \times (\Delta p_e)^2 \ll m_n = 939.56\ MeV/c^2,$$

implies standard deviations of the coordinates of the order of the neutron radius and electron kinetic energy much smaller than the neutron mass.

3.3.2. Representation of the neutron spin. Assumption 3.3.A requires that the hadronic angular momentum of the isoelectron be equal to the spin of the proton, thus having value $\hat{L}_{3,\hat{e}} = 1/2$. The study of this assumption was initiated in the 1984 papers on the isotopies of the rotational symmetry [74] [75] and presented in the 1990 paper [52] via the isotrigonometric isofunctions (see also p. 304 on of Ref. [4]), as well as the use of the Lie-Santilli isoalgebra $\hat{so}(3)$. We here study, apparently for the first time, the nonrelativistic representation of the hadronic angular momentum $\hat{L}_3 = 1/2$ of the isoelectron in model (18) under the assumptions in first approximation that *the orbit of the isoelectron is contained in a plane and it is a perfect circle with radius* $R = 10^{-13}\ cm$. In fact, deviations from the above assumptions imply instabilities generally preventing a representation of the neutron mean life of $887\ s$. Under the above assumptions, the acting isosymmetry is the two-dimensional Lie-Santilli isogroup $\hat{O}(2)$ [74] [75] (see also Sect. 6.4, p. 233 on of [5]).

Consider the conventional $O(2)$ symmetry which is classically formulated on the two-dimensional Euclidean space $E(z, \delta, I)$, and quantum mechanically treated on a Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$. In accordance with the simple rule of Sect. II-3.9 (for the construction of isotopic images of quantum models to add the representation of extended nucleons and their non-Hamiltonian interactions), we map the entire classical and quantum mechanical formulation of $O(2)$ under the nonunitary transformation

$$\begin{gathered} UU^\dagger = \hat{I} = 1/\hat{T} == Diag.(n_1^2, n_2^2), = \\ = Diag.(b_1^{-2}, b_2^{-2}), \quad b_k = 1/n_k > 0, \ \ k = 1, 2, \end{gathered} \tag{24}$$

and introduce Bohm's hidden variable [8]

$$\frac{1}{n_1} = \frac{1}{n_2} = b_1 = b_2 = \lambda > 0. \tag{25}$$

Therefore, the isorepresentation occurs in the two-dimensional iso-Euclidean isospace $\hat{E}(\hat{r}, \hat{\Delta}, \hat{I})$ over the isofield $\hat{\mathcal{C}}$ with isocoordinates

$$\hat{r} = r\hat{I} = \{x, y\}\lambda^2 I_{2\times 2}, \tag{26}$$

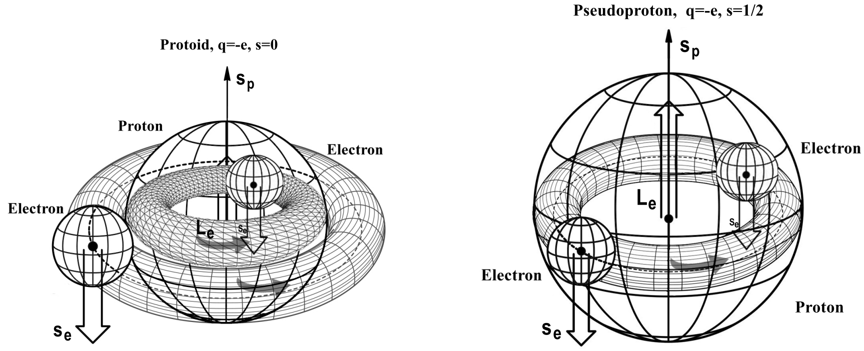

Figure 5: *In this figure, we illustrate the prediction by hadronic mechanics of the synthesis in the core of stars of negatively charged strongly interacting particles characterized by a hadronic singlet bond between an electron and a neutron, with null total spin, as well as the pseudoproton on the right with total spin* $1/2$*, and other hadronic states [101].*

isometric

$$\hat{\Delta} = (\hat{\delta})\hat{I}, \hat{\delta} = \hat{T}\delta = \lambda^2\delta, \tag{27}$$

isoinvariant

$$\hat{r^2} = \lambda^2 r^2, \tag{28}$$

and isotrigonometric representation (App. 5C, p. 300 on of Ref. [4])

$$\begin{gathered} x = r\lambda^{-1}cos\hat{\phi}, \quad y = r\lambda^{-1}sin\hat{\phi}, \\ \hat{\phi} = T_\phi\phi = n_1n_2\phi = \lambda^{-2}\phi \quad \hat{T}_\phi = b_1{-}1b_2^{-2} = \lambda^{-2}. \end{gathered} \tag{29}$$

The isounitary isoirreducible isorepresentations of $\hat{O}(2)$ is defined on a Hilbert-Myung-Santilli isospace $\mathcal{H}$ over the iso-complex isofield $\mathcal{C}$ with iso-states $|\hat{\psi}(\hat{r}) >$, isonormalization

$$\langle\hat{\psi}(\hat{r})|\hat{\times}|\hat{\psi}(\hat{r})\rangle = \hat{I}, \tag{30}$$

isoeigenvalues

$$\begin{gathered} \hat{R}(\hat{\phi})\hat{\times}|\hat{\psi}\rangle = \hat{e}^{iM\hat{\phi}}\hat{\times}|\hat{\psi}\rangle = (e^{i\hat{M}\hat{\phi}})\hat{I}_\psi\hat{\times}|\hat{\psi}\rangle = (e^{i\lambda^2M\hat{\phi}})|\hat{\psi}\rangle, \\ \hat{M} = b_1b_2M = \lambda^2M, \end{gathered} \tag{31}$$

(where $\hat{e}$ is, this time, the isoexponentiation in the ϕ-plane) with Lie-Santilli isogroup laws

$$\begin{gathered} \hat{R}(\hat{\phi})\hat{\times}\hat{R}(\hat{\phi}')\hat{\times}|\hat{\psi}\rangle = \hat{R}(\hat{\phi}')\hat{\times}\hat{R}(\hat{\phi})\hat{\times}|\hat{\psi}\rangle = \hat{R}(\hat{\phi}+\hat{\phi}')\hat{\times}|\hat{\psi}\rangle, \\ \hat{R}(\phi)\hat{\times}\hat{R}(-\hat{\phi})\hat{\times}|\hat{\psi}\rangle = \hat{R}(0)\hat{\times}|\hat{\psi}\rangle = |\hat{\psi}\rangle. \end{gathered} \tag{32}$$

The isoeigenvalue of the hadronic angular momentum $\hat{L}$ is given by

$$\hat{L}\hat{\times}|\hat{\psi}\rangle = \hat{M}|\hat{\psi}\rangle = \lambda^2 M|\hat{\psi}\rangle. \tag{33}$$

But isotopies preserve original numeric values. Therefore,

$$\hat{M} = \lambda^2 M = 0, 1, 2, 3, ... \tag{34}$$

Consequently, the angular momentum measured by the experimentalist is given by

$$M = \frac{\hat{M}}{\lambda^2}, \tag{35}$$

and can represent the *constrained* angular momentum of the isoelectron inside the proton for the value

$$M = \frac{\hat{M}}{\lambda^2} = \frac{1}{2}, \quad \lambda = \sqrt{2} = 1.4142. \tag{36}$$

The total spin of the neutron is then given by

$$S_n = s_p + s_e + L_e = \frac{1}{2} - \frac{1}{2} + \frac{1}{2}, \tag{37}$$

and therefore, *the spin of the neutron coincides with the spin of the proton* as expected.

For brevity, we leave to the interested reader the representation of the spin $1/2$ of the isoelectron via the isosymmetry $\hat{SO}(2)$, which can be derived from the above treatment with the $\hat{O}(2)$ symmetry with Santillian

$$\hat{T} = Diag.(\frac{1}{n_1}, \frac{1}{n_2}) = Diag.(b_1, b_2) = Diag.(\lambda, \lambda^{-1}), \quad Det.\hat{T} = I, \tag{38}$$

and the use of suitable isostates to separate positive and negative eigenvalues.

An important rule for the isorepresentation of the neutron synthesis from the Hydrogen atom is that *the total angular momentum of the isoelectron compressed inside the proton is identically null.* As we shall see, in case confirmed, the above rule appears to have basic implications for models of nuclear structures according to hadronic mechanics.

The spin of the neutroid according to Fig. 3 is characterized by the following value for the hadronic angular momentum of the isoelectron

$$M = \frac{\hat{M}}{\lambda} = 0, \quad \hat{M} = 0, \quad \lambda > 0. \tag{39}$$

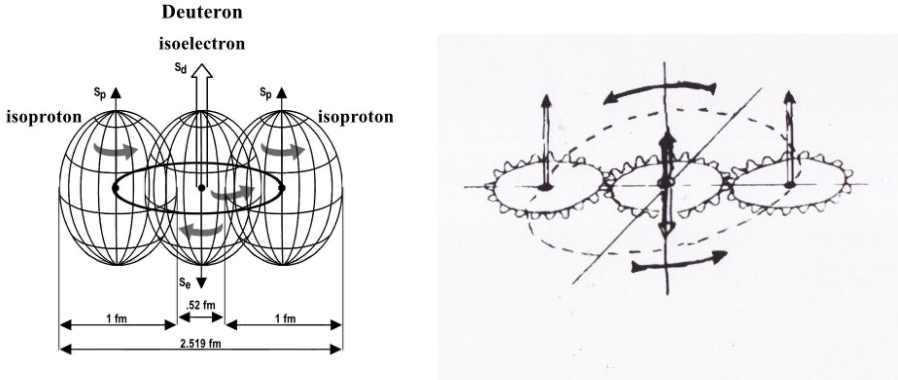

Figure 6: *In the left of this figure, we illustrate the first three-body structure model of the Deuteron permitted by the neutron synthesis from the Hydrogen [57]consisting of two protons in triplet coupling with an electron acting as their "gluon." Despite its stability illustrated in the right of this figure, this model had to be abandoned because of its inability to represent: 1) The spin $S_D = 1$ of the Deuteron as a bound state of a proton and a neutron in their ground state [2]; 2) The constrained angular momentum of the electron being equal to the proton spin with ensuing null total value $S_e^{tot} = 0$; and 3) The stability of the Deuteron under the natural instability of the neutron.*

Consequently, *the spin of the neutroid according to Figure 3 is predicted to be zero,* by therefore explaining the reason for their lack of detection via commercially available neutron detectors.

3.3.3. Representation of the neutron magnetic moment. The anomalous magnetic moment of the neutron according to model (18) has been first represented in the original proposal via the following *three* contributions [52]

$$\mu_n = \mu_p + \mu_e^{spin} + \mu_{\hat{e}}^{orb}. \tag{40}$$

The biggest difficulty of the above representation is that the magnetic moment of the electron ν_e^{spin} is too big for nuclear standard to prevent a quantum mechanical model of the neutron synthesis as well as to prevent that electrons can be members of nuclear structures. In fact, we have the value

$$\begin{gathered} \mu_e^{spin} = -9.284764 \times 10^{-24}\ J/T = \\ = -9.284764 \times 10^{-24} \times 1.979890761 0^{26}\ \mu_N = \\ = -928.4784 \times 1.979890 \mu_N = 1838.2851\ \mu_N, \end{gathered} \tag{41}$$

which is 961-times *bigger* than the neutron magnetic moment.

Under the assumption that the proton is a point-like particle, there is no known possibility to reduce magnetic value (41) down to the magnitude needed for the neutron magnetic moment. The above impasse is here resolved, apparently for the first time for value (41), via the magnetic moment of the *orbital* motion of the isoelectron $\mu_{\hat{e}}^{orb}$ which is *opposite* that of the isoelectron (Fig. 4) and its value is predicted to be [52]

$$\mu_{\hat{e}}^{orb} = 1833.580\,\mu_N. \tag{42}$$

By recalling the known values of the magnetic moments of the proton and the neutron [76] $\mu_p = 2.792\,\mu_N, \quad \mu_n = -1.913\,\mu_N$, we have the *numerically exact and time invariant representation of the anomalous magnetic moment of the neutron* [52]

$$\begin{gathered}\mu_n = \mu_p + \mu_e^{spin} + \mu_{\hat{e}}^{orb} = \\ = 2.792\,\mu_N - 1838.285\,\mu_N + 1833.5801\,\mu_N = -1.913\,\mu_N.\end{gathered} \tag{43}$$

It should be noted that the assumption of the above orbital contribution from the isoelectron not only allows a representation of the *numeric value* of the anomalous magnetic moment of the neutron, but also of its *negative value.*

3.3.4. Representation of the neutron rest energy, mean life and charge radius. In this section we shall identify, apparently for the first time, the numeric value of *neutron density* $n_4 = 1/b_4$ from isoequivalence principle (13) (the isoaxiom IV of special isorelativity, p . 139 of [33]) in remarkable agreement with the density of the proton-antiproton fireball of the Bose-Einstein correlation [77] [78].

By assuming the iso-Galilean invariant hadronic structure model of Sect. II-5.7, first presented in Eqs. (5.1.14), p. 835 of Ref. [27] (see the review in Sect. 2.5.4 of Ref. [79]) we have the following *hadronic structure equations of the neutron*

$$\begin{gathered}\left[\tfrac{1}{r^2}\left(\tfrac{d}{dr}r^2\tfrac{d}{dr}\right) + \bar{m}\left(E + e^{\frac{2}{r}} + N\tfrac{e^{-br}}{1-e^{-br}}\right)\right] = 0, \\ E_n^{tot} = E_p + E_{\hat{e}} - E = 939.565\,MeV, \\ \tau^{-1} = 2\pi\lambda^2|\hat{\psi}(0)|^2\tfrac{\alpha^2 E_e}{\hbar} = 877\,s, \\ R = b^{-1} = 10^{-13}\,cm = 1\,fm,\end{gathered} \tag{44}$$

which can be reduced to the *algebraic* equations in the parameters $k_1,\ k_2$ (Eqs. (5.1.32a) and (5.1.32b), p. 840 of Ref. [27])

$$\begin{gathered}E = k_1[1-(k_2-1)2]\tfrac{2\bar{h}c}{b} = 939.565\,MeV, \\ \tau = \tfrac{48\times(137)^2}{4\pi bc}\tfrac{k_1}{(k_2-1)^3} = 877\,s, \quad \xi = \tilde{n},\end{gathered} \tag{45}$$

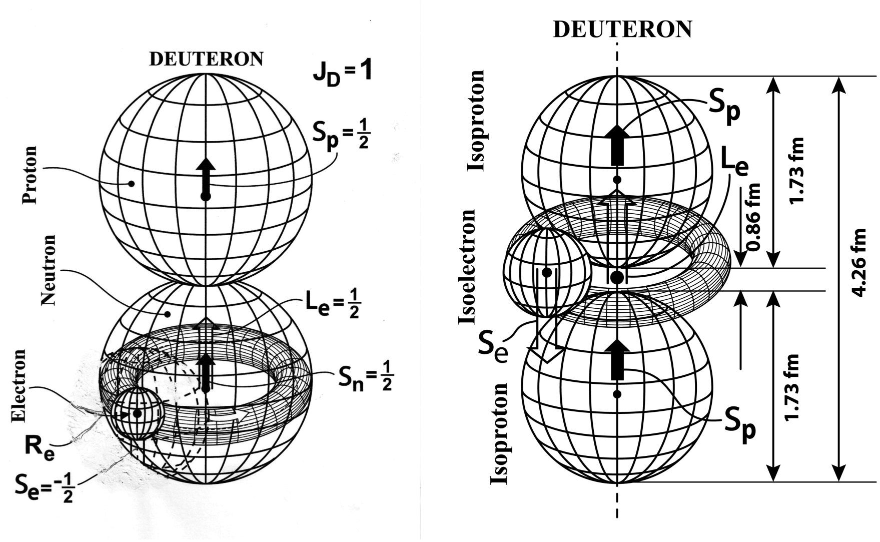

Figure 7: *In the left of this figure, we illustrate the structure of the Deuteron as an axial triplet coupling (Fig. II-8) of a proton and a neutron which permits the representation of the stability of the Deuteron (illustrated in the right of this figure) via the decoupling of the neutron into its permanently stable constituents, the proton and the electron. An important point for the consistency of the representation is that the electron in the indicated axial triplet configuration preserves its null total angular momentum due to the persistence of its total immersion within the proton which is not possible for the planar singlet coupling (Fig. II-8) of Fig. 6.*

energy spectrum

$$E = -\frac{1}{4K_h k_2}\left(\frac{k_2}{N} - N\right)^2 = 0, \quad k_2 = K_h \frac{\tilde{E}_{\tilde{e}}}{\hbar^2 b^2} = 1, \tag{46}$$

and numeric solutions[52] (see also Refs. [63])

$$k_1 = 2.6, \quad k_2 = 1, \tag{47}$$

which numeric values should be compared with those for the meson and baryon octets of Refs.[27] [79].

The primary difference between the above equations and Eqs. (II-118) is the absence of a potential in the latter while the former exhibits indeed a potential, evidently given by the Coulomb potential between the electron and the proton.

We now compute the rest energy of the neutron via the isoequivalence principle (13) under the approximation that all particles are perfectly spherical, thus having $n_d = 1/b_d = 1, \ d = 1, 2, 3$ (see next section for the spheroid case). We shall then compare the results with other experimental particle data. By recalling from Sect. II-5.7 that *the Hulten potential does not generate a binding energy,* the total energy of the neutron in model (18) is given by

$$E_n = \tilde{E}_{\tilde{e}} + E_p - E, \tag{48}$$

where, from principle (13) for the assumed values $n_d = 1/b_d = 1,\ d = 1, 2, 3,$

$$\tilde{E}_{\tilde{e}} = \frac{E_e}{n_4^2} = \frac{0.511}{n_4^2}. \tag{49}$$

By keeping in mind that we cannot use quantum mechanical methods for the computation of the binding energy of neutron due to its *excess* energy of $0.782\ MeV$, we can assume in first approximation that the binding energy E of the neutron according to structure model (18) is of the same order of magnitude as that of the positronium in the $1s$ state because the charge structures and radii of the neutron and the positronium are essentially the same, [80]

$$E = -0.948\ eV. \approx 0. \tag{50}$$

Hence, we can assume in first approximation that

$$\tilde{E}_{\hat{e}} = E_n - E_p = 0.511 + 0.762\ MeV = 1.293\ MeV \tag{51}$$

resulting in the *best available estimate of the density of the proton* based on the isoequivalence law (13)

$$\begin{aligned} \tilde{E}_{\tilde{e}} &= \tfrac{E_e}{n_4^2} = 1.293\ MeV/c^2, \\ n_4^2 &= \tfrac{0.511}{1.293} = 0.395, \\ n_4 &= 0.628, \end{aligned} \tag{52}$$

resulting in the value of the neutron density $n_4 = 1/b_4 = 0.610$ which is in remarkable agreement with the numeric value of $n_4^2 = 0.429,\ \ n_4 = 0.654$ for the proton-antiproton fireball of the Bose-Einstein correlation, Eqs. (10.28b), p. 127 of [77] (see also Ref. [78]). Therefore, from Eqs. (44), (46) and (50), the rest energy of the neutron is given by the rest energy of the proton plus the isorenormalized rest energy of the isoelectron, $E_n = E_p + \tilde{E}_{\hat{e}}$.

Note that values (52) imply the following *superluminal tangential speed of the electron within the proton structure*

$$C = \frac{c}{n_4} = 1.592\ c, \tag{53}$$

which is fully possible for hadronic mechanics because contact nonpotential interactions carry no energy [13] [81].

Intriguingly, values (52) confirm the nonlocal effects at the foundation of the isoequivalence principle (13), the second digit difference between the value $n_4 = 0.628$ of Eqs. (48) and the value $n_4 = 0.654$ of Ref. [77] being due to the assumption of perfect spheridicity of the particle in isoprinciple (13) compared to the spheroid shape of the particles in Ref. [77].

3.4. Relativistic representation of the neutron synthesis. Recall that the Dirac equation has acquired a justly historical stature because of its exact and time invariant relativistic representation of the electron under the Hamiltonian interaction of the *point-like* proton in the structure of the Hydrogen atom.

The Dirac-Santilli isoequation (I-95) has been constructed to attempt the exact and time invariant representation of the isoelectron under the Hamiltonian and non-Hamiltonian interactions of the *extended* proton in the structure of the neutron according to Fig. 3, thus requiring its characterization via the isotopies of the spinorial covering of the Poincaré symmetry $\hat{\mathcal{P}}(3.1) = \hat{SL}(2.\hat{C})\hat{\times}\hat{\mathcal{T}}(3.1)$, Eq. (1) first introduced in the 1995 paper [26] jointly with the Dirac-Santilli isoequation and the first relativistic representation of the neutron synthesis.

For consistency, model (18) requires that the absolute value of the hadronic angular momentum of the isoelectron $\hat{L}_3$ is equal to its spin $\hat{S}_3$, thus requiring that

$$\hat{L}_3 = \hat{S}_3, \quad \hat{L}^{\hat{2}} = \hat{S}^{\hat{2}}. \tag{54}$$

From Eqs. (8) and (10) of the isospinorial isosymmetry $\hat{\mathcal{P}}(3.1)$, we therefore obtain the conditions on the characteristic quantities of the isotopic element (2)

$$\begin{aligned} b_1^{-1}b_2^{-1} &= \tfrac{1}{2}b_1 b_2, \\ b_1^{-2}b_2^{-2} + b_2^{-2}b_3^{-2} + b_3^{-2}b_1^{-2} &= \tfrac{1}{4}(b_1^2 b_2^{-2} + b_2^2 b_3^{-2} + b_3^2 b_1^{-2}), \end{aligned} \tag{55}$$

with numeric value for the case of spherical symmetry [26]

$$b_1^2 = b_2^2 = \frac{1}{n_1^2} = \frac{1}{n_2^2} = \sqrt{2} = 1.415, \quad b_1 = b_2 = \frac{1}{n_1} = \frac{1}{n_2} = 1.189. \tag{56}$$

The value for the semi-axis $b_3^2 = 1/n_3^2$ of the isoelectron can be found by assuming the volume preserving normalization $n_1^2 + n_2^2 + n_3^2 = 3$ used for neutron, Eq. (I-89), resulting in the value [26]

$$n_1^2 = n_2^2 = 0.707, \quad n_3^2 = 1.587, \tag{57}$$

by therefore suggesting that the wave packet of the isoelectron is *oblate* (because $n_3^2 < n_1^2 = n_2^2$).

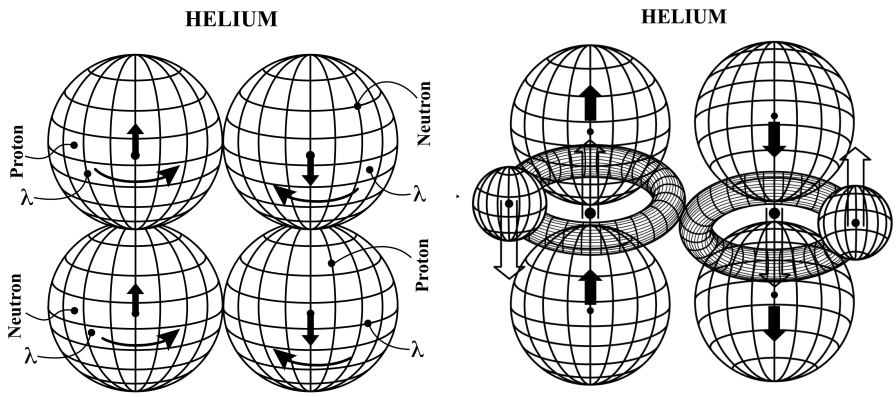

Figure 8: *In this figure we compare the structure of the Helium according to ref. [2] (left side) and that according to the analysis of paper (right view) to illustrate the achievement of stability as well as the identification of an internal attractive force in the transition from the former to the latter.*

To compute the rest energy of the neutron, we now assume the full form of the isoequivalence principle (13) from which, instead of Eq. (48), we have the following expression for the energy $\tilde{E}_{\hat{e}}$ of the isoelectron from the value (52) of the shape of the electron wave packet

$$\tilde{E}_{\hat{n}} = \frac{n_3^2}{n_4^2} E_e = 1.37\ MeV/c^2,$$
$$n_4^2 = \frac{1.587}{1.37} 0.511\ MeV/c^2 = 0.592, \tag{58}$$
$$n_4 = 0.789,$$

which remains close to the value $n_4 = 0.6$ for the density of the fireball of the Bose-Einstein correlation of [77] [78].

The relativistic representations of neutron magnetic moment is essentially the same as that of the preceding non-relativistic analysis (see Eq. (7.4), p. 182 of [26] for an alternative derivation). As it is well known, the radial equation of Dirac's equation for the Hydrogen atom is essentially the same as the corresponding nonrelativistic equation, and the same holds for the Dirac-Santilli isoequation. Consequently, the relativistic representation of the mean life and charge radius of the neutron per model (19) is essentially the same as that of Sect. 3.3.3. of Ref. [26] and, as such, it is omitted for brevity.

4 Representation of nuclear stability

4.1. Representation of nuclear stability despite the neutron instability. We shall now search for a mechanism according to which the naturally unstable neutron (when isolated) becomes permanently stable under strong nuclear forces.

Let us recall our initial structure model of the Deuteron as a three-body hadronic bound state of two protons in plane triplet alignment with spins $1/2$ and one isoelectron with total angular momentum $J = 0$ (Fig. 6) [52] [57]. This model had to be abandoned despite the representation of the Deuteron spin $S_D = 1$ and its great stability (illustrated with the 'gear model' in the right of Fig. 6), because of the impossibility to represent proton-neutron exchange forces (due to the inability that the isoelectron could be consistently transferred from the neutron to the proton), the inability to represent the stability of the Deuteron despite the neutron natural instability, and for other reasons.

In the hope of resolving the above insufficiencies, we have then assumed model (I-81) on the Deuteron structure as a two-body hadronic bound state of a isoproton and an isoneutron in axial triplet couplings with spins $1/2$. We here submit the hypothesis that, when it is a member of the Deuteron structure, *the isoelectron of model (18) decouples from the neutron to acquire a position intermediate between the two protons* (right of Fig. 7).

$$D = \begin{pmatrix} \hat{p}^{\uparrow} \\ \hat{n}^{\uparrow} \end{pmatrix} = \begin{pmatrix} p^{\uparrow} \\ \hat{e} \\ p^{\uparrow} \end{pmatrix}, \tag{59}$$

in which the isoelectron remains immersed within dense hadronic matter as a necessary condition to activate hadronic mechanics with ensuing restricted angular momentum equal to the proton spin and null total angular momentum (Sect. 3.3.1 and Fig. 4)

$$J_{\hat{e}}^{tot} = S^{spin} + L^{orb} = \frac{1}{2} - \frac{1}{2} = 0. \tag{60}$$

Consequently, the decoupled isoelectron assumes an intermediate position between the two protons by occupying the distance between their charge distributions of $0.3745 \times 10^{-13}\ cm$ indicated in Figure II-9 [82].

Note that the above hypothesis is supported by null values (46) and (50) of the neutron binding energies fior the SA and NSA components of strong interactions, according to which the isoelectron is *quasi-free,* and indeed decouples from the neutron under the strong Coulomb attraction by the second proton.

It appears that, unlike the case for the planar configuration (Fig. 6), the axial triplet coupling of the isoproton and the isoneutron in the Deuteron structure (left of Fig. 7) allows the representation of: all experimental data of the Deuteron in its true ground state; the representation of proton-neutron exchange forces via the mere exchange of the isoelectron in between the two protons; and the representation of the deuteron stability despite the natural instability of the neutron.

Additionally, the decoupling of the isoelectron from the neutron indicates the apparent existence of a strongly *attractive* Coulomb force $e - p$ in the Deuteron structure which is absent in quantum mechanical treatments, by therefore contributing to the stability of the Deuteron.

4.2. Representation of nuclear stability despite repulsive protonic forces. Let us recall that, according to quantum mechanics, protons *repel* each other (in view of their equal charge) with a Coulomb force of such an extreme value not to be counter balanced by potential nuclear forces.

We here indicate, apparently for the first time, that hadronic mechanics can represent this additional problem of nuclear stability via a mechanism similar to the achievement by *hadronic chemistry* of a strongly *attractive* force between the *identical* electrons of valence couplings [83] [84] [85].

The proposed isotopic mechanism is essentially that of counter-balancing repulsive nuclear force via the completion of a conventional repulsive quantum mechanical model with non-Hamiltonian forces generated by a suitably selected nonunitary transform (Sect. II-3.11).

Various measurements [86] have established that the Helium $He(2,4,0)$ [87] has a charge radius of $1.678 \times 10^{-13}\ cm$, against the radius of two protons and two neutrons each having the value of $0.841 \times 10^{-13}\ cm$[20] with a total radius of $1.682 \times 10^{-13}\ cm$.

The above measurements confirm the primary assumption of hadronic mechanics, according to which nuclei are composed by extended nucleons in conditions of partial mutual penetration of their dense charge distributions, with ensuing non-Hamiltonian interactions. Consequently, the indicated measurements justify the presence of non-Hamiltonian interactions in the structure of the Helium.

Let us assume in first approximation that the Helium is a quantum mechanical (qm) bound state of two Deuterons with anti-parallel spins (Fig. 8)

$$He(2,4,0) = [D(1,2,1_\uparrow), D(1,2,1_\downarrow)]_{qm}, \tag{61}$$

under the sole action of the Coulomb potential between the two protons,

$$V_c(r) = +\frac{e^2}{r}, \tag{62}$$

where, as usual, the positive sign denotes *repulsion,* with the understanding that conventional, attractive, nuclear potentials will be added later on.

In order to add non-Hamiltonian interactions in model (61), we subject its

Schrödinger equation to the nonunitary transformation (Sect. II-3.7)

$$\begin{aligned} UU^\dagger = \hat{I} = 1/\hat{T} = e^{\frac{V_h(r)}{V_c(r)}} = e^{\frac{K\frac{e^{-br}}{1-e^{-br}}}{\frac{e^2}{r}}}, \\ \hat{T} =\approx 1 - \frac{V_h(r)}{V_c(r)} = 1 - \frac{K\frac{e^{-br}}{1-e^{-br}}}{\frac{e^2}{r}}, \end{aligned} \tag{63}$$

where N is a normalization constant, resulting in the iso-Schrödinger equation (Sect. II-5.7 and Sect. 3 of Ref. [79])

$$\begin{aligned} U[\tfrac{1}{2m}\delta^{ij}p_ip_j + V_c(r)] \mid \psi(r) \rangle U^\dagger = \\ = U(\tfrac{1}{2m}\delta^{ij}p_ip_j)U^\dagger \hat{T} \mid \hat{\psi}(r)\rangle + UV_c(r)U^\dagger \hat{T} \mid \hat{\psi}(r)\rangle \approx \\ \approx \{-\tfrac{1}{m}\hat{\partial}_r\hat{\partial}_r + V_c(\hat{r})[1 - \tfrac{V_h(r)}{V_c(r)}]\} \mid \hat{\psi}(r)\rangle = \\ = [-\tfrac{1}{m}\Delta_r + \tfrac{e^2}{r} - K\tfrac{e^{br}}{1-e^{br}}] \mid \hat{\psi}(r)\rangle, \end{aligned} \tag{64}$$

where K is a normalization constant. But the Hulten potential behaves like the Coulomb potential at short distances (Eq. (5.1.6), p. 833, of the 1978 memoir [27]), by therefore *absorbing* the Coulomb potential via a mere redefinition of its normalization constant from K to K'

$$V_h(r) \approx V_c(r), \quad V_c + V_h = +\frac{e^2}{r} - K\frac{e^{br}}{1-e^{br}} \approx -K'\frac{e^{br}}{1-e^{br}}, \tag{65}$$

resulting in the radial equation without any presence of repulsive forces

$$\left[\frac{1}{r^2}(\frac{d}{dr} + m(E + K'\frac{e^{-br}}{1-e^{-br}})\right] |\hat{\psi}\rangle = 0. \tag{66}$$

Consequently, it appears that *nonlinear, nonlocal and nonpotential nuclear interactions due to the mutual penetration of nucleons can be so strongly attractive to overcome repulsive Coulomb force between protons.*

For brevity, we leave to the interested reader the completion of the above isotopic model with some of the attractive nuclear potentials such as the *Yukawa potential* [88], the *Woods-Saxon potential* [89] and other potentials or their combination [90].

4.3. Open problems in nuclear stability. It should be indicated that the representation of the nuclear stability of Sect. 4.1 via the decoupling of the neutron into its original, permanently stable constituents appears to be solved on both mathematical and physical grounds.

By contrast, the representation of nuclear stability under repulsive proton-proton Coulomb forces of Sect. 4.2 is purely *mathematical,* by therefore creating the fundamental open problem on the *physical* origin of the attraction.

In the author's view, the above open problem can be best studied via the completion of the quantum mechanical notion of *point-like charge* into the nonlocal notion of *isocharge* of isopartices (Definition 2.1). In turn a nonlocal formulation of the Coulomb law requires a *structure model of the charge,* e.g., along the author's undergraduate thesis [91] [92] [93] (see also the review in Sects. 3.2 and 3.3 of [35]).

5 Implications of the neutron synthesis

5.1. Stimulated neutron decay. In the author's view, an intriguing implications of the synthesis of the neutron from the Hydrogen is the possible existence of mechanisms suitable to trigger the neutron decay, of course, under the strict validity of all nuclear conservation laws.

As it is well known, according to quantum mechanics the electron is not a constituent of the neutron, which therefore with consequential prohibition to stimulate the neutron decay. Nevertheless, the representation by hadronic mechanics of *all* characteristics of the neutron at the *nonrelativistic and relativistic levels* as a hadronic bound state of an isoproton and an isoelectron admits indeed the stimulated neutron decay. In fact, the decoupling of the neutron for nuclear stability predicts a possible stimulated decay of the neutron via the irradiation with resonating photons γ_{res} with the frequency [57]

$$E_{res} = E_{\hat{e}} = 1,293\ MeV. \tag{67}$$

Note that the stimulated neutron decay is generally *not* admitted by stable nuclei, except for double beta decays of the type

$$\begin{gathered}\gamma_{res}(0,0,1) + Mo(100,42,0) \rightarrow Tc(100,43,1) + \beta^-(0,-1,0),\\ Tc(100,43,1) \rightarrow Ru(100,44,0) + \beta^-(0,-1,1),\end{gathered} \tag{68}$$

which transmutation has been tentatively verified by Santilli [6] [67]) and by the experimental team [94] (see Fig. 9 and review [67]) via the following simple and inexpensive experimental set up:

5.1.1) Purchase a small sample of the commercially available radioisotope Europe-152 (emitting photons with E_r) and of the pure isotope $Mo(100,42,0)$.

5.1.2) Place the Europe-152 sample in contact with the sample of $Mo(100,42,0)$.

5.1.3) The emission of easily detectable electrons from the $Mo(100,42,0)$ sample, or the possible detection of traces of $Ru(100,44,0)$ in the originally pure sample of $Mo(100, 42, 0)$, would establish the existence of the stimulated double beta decays (75) beyond scientific doubt.

5.2. The etherino hypothesis. It should be indicated that, despite the best possible efforts conducted over decades, *the author has been unable to identify any possibility of maintaining the Pauli-Fermi neutrino hypothesis in a structural (rather than kinematical) representation of the neutron synthesis in a star.*

This impasse has left no other option than that, in line with Rutherford's original conception (18), of assuming that *the neutron is a generalized bound state of the permanently stable electron and proton without any additional constituent.* Different models are encouraged, provided that they interpret the totality of the characteristics of the neutron during its synthesis from the electron and the proton as occurring in nature.

To begin, recall that bound states of two particles were assumed throughout the 20-th century to have *negative* binding energies, resulting in a *mass defect* which is at the foundation of nuclear fusions. By comparison, the neutron synthesis misses $0.872\ MeV$ which can be provided, quite simply, by the kinetic energy of the electron (Sect. 3.3.4). However, the author has located no plausible source of the additional energy needed to accomodate the neutrino.

Similarly, the only known structural representation of the neutron synthesis is Rutherford's compression of the electron inside the dense proton in the core of stars. In turn, such a compression mandates that, to prevent extreme resistive forces, the angular momentum of the internal electron must be equal to the proton spin, thus implying Rutherford's original conception (18) rather than Pauli- Fermi conception (15). Despite years of attempts, the author has found no possibility of accommodating the neutrino spin $1/2$ in the neutron synthesis.

To illustrate the intriguing character of the problem, let us also indicate that the studies reviewed so far on the neutron synthesis have left open the fundamental problem of the *origin* of the missing energy of $0.782\ MeV$ (Sect. I-1). Contrary to a rather general view, said missing energy cannot be provided by the relative kinetic energy between the electron and the proton in the core of stars, because at that energy, the electron-proton cross section is essentially null, thus prohibiting any synthesis.

Similarly, said missing energy cannot be provided by the Sun, because the Sun synthesizes about 10^{38} neutrinos *per second* [95], that would require about

$$\Delta E = 10^{38}\ MeV\ per\ second, \tag{69}$$

by therefore implying such an energy loss for which the Sun would cool down and never produce light.

For these and other reasons, Santilli proposed in 2007 the hypothesis that *the missing energy in the neutron synthesis is provided by the ether conceived as a* *universal substratum with extremely big energy density,* and that the energy of $0.782\ MeV$ is transferred from the ether to the neutron by a massless, chargeless and spinless *longitudinal impulse* called *etherino* (denoted with the letter a from the Latin *aether*) [96] according to the synthesis

$$\hat{e}^- + a + \hat{p}^+ \ \rightarrow \ n. \tag{70}$$

Note that, contrary to the case of the neutrino in the Pauli-Fermi synthesis (15), the etherino is on the *left* of synthesis (14) as a condition to supply the missing energy. It should also be recalled that (see the original 1956 study [91] and Chapter 3 of the review [35]):

5.2.1) The ether as a universal substratum is necessary for a coherent conception of electromagnetic *waves.*

5.2.2) The ether as a universal substratum is not in conflict with special relativity because of the inability by us to identify a reference frame at rest with the ether due to inertia.

5.2.3) The *aethereal wind,* that has been historically used to maintain the validity of special relativity, is eliminated by admitting that, as it is the case for electromagnetic waves, the electron and other truly elementary particles are oscillations of points of the ether.

It should be also indicated that Santilli proposed the etherino hypothesis to *assure the gravitational stability of the Sun,* in view of the fact that the Sun releases into light $2.3 \times 10^{38}\ MeV/s$ [97] (see also [98]), corresponding to about $4.3 \times 10^{6}\ t/s$. Since in a Gregorian year there are 10^7 seconds, the loss of mass by the Sun ΔM_S per year due to light emission is given by

$$\Delta M_S = 10^{23}\ \text{metric tons per year}. \tag{71}$$

The above loss of mass is of such a magnitude to cause a change of planetary orbits detectable in astrophysical laboratories, which change is contrary to centuries of measurements on the stability of the Sun gravitational field. However, the energy needed for the star to synthesize neutrons, Eq. (50), is essentially equal to the energy loss into light, by therefore allowing a representation of the gravitational stability of the star, with the understanding that it implies a *return to cosmology based on the continuous creation of matter in the universe* [99], with intriguing implications, e.g., for a realistic representation of the energy released in supernova explosions, since it is of such a dimension and of such instantaneous character not to be realistically representable via the sole use of nuclear fusions.

By keeping in mind that (as it is the case for quarks) neutrinos cannot be directly detected, we should finally indicate for interested readers the existence of unpublished indications that available experiments on the *predictions* of the neutrino hypothesis [100] appear to be numerically representable via the corresponding *predictions* of the etherino hypothesis [96].

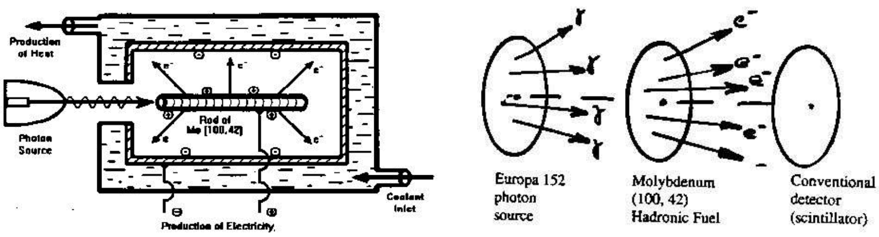

Figure 9: *In the left of this figure, we reproduce the original drawing of paper [105] showing (from the left) a beam of resonating photons irradiating a cylinder of* $Mo(100, 42, 0)$ *which emits electrons easily trapped by a metal casing with the production of a clean DC electric current of nuclear origin (for which reason the set up is called a 'nuclear battery'), plus clean heat utilized by a heat exchanger (because the metal casing heats up while trapping electrons with* $0.782\ MeV$ *and the transmutation is esoenergetic). In the right of this figure, we reproduce the original figure of paper [105] illustrating the simplicity as well as the low cost of experimental verifications [6] [94].*

5.3. The pseudoproton hypothesis. Following the synthesis of the neutron via the compression of an electron within the dense proton, hadronic reactors (Fig. 1) progressively synthesizes, in statistical smaller amounts, *various negatively charged, strongly interacting particles* [101] such as:

5.3.1) The *protoid* $\tilde{p}_1^-$) with spin 0, mass essentially that of the neutron and mean-life predicted to be of about $7\ s$ which is predicted to be a planar singlet coupling of an isoelectron and a neutron (left of Fig. 5).

5.3.2) The *pseudo-proton* $\tilde{p}_2^-$ with spin $1/2$, mass equal to that of the neutron and mean-life of the order of $5\ s$, which is predicted from the compression of the isoelectron, this time, inside the hyperdense neutron (right of Fig. 5).

5.3.3) Additional states with decreasing mean life [101] (see also Ref. [108]). The protoid and pseudoproton are representable with the unified notation

$$\tilde{p}^- = (\hat{e}^-, n)_{hm}, \tag{72}$$

where, as it is the case for the neutron synthesis, the isoelectron has total angular momentum $J^{tot} = S^{spin} + L^{orb} = 1/2$ for the exterior orbital configuration and $J^{tot} = S^{spin} + L^{orb} = 0$ for the constrained interior configuration of Fig. 5.

The existence of the above hadronic states is predicted from the fact that *non-Hamiltonian interactions caused by deep EPR entanglement of particles are insensitive to charges,* thus being somewhat reminiscent of the charge independence of strong interactions.

A reason currently stimulating mathematical, theoretical, experimental and industrial interest is that being negatively charged and strongly interacting, *pseudo-protons are attracted by nuclei* here represented in standard notation $N(Z, A, J, u)$

[87] with resulting unavoidable transmutation into *new nuclei called nucloids* [101]

$$\tilde{p}_s^- + N(Z, A, J) \ \rightarrow \tilde{N}(Z-1, A+1, J+s, u+m_{\tilde{p}}), \tag{73}$$

whose possible significance for the recycling nuclear waste, medical treatments and other fields is indicated in Sect. 6.

It should be finally noted that the existence of the pseudo-proton, let alone its long mean life for particle standards, is impossible without the EPR completion of quantum into hadronic mechanics.

6 Recycling radioactive nuclear waste

6.1. The societal problem of recycling radioactive nuclear waste. As it is well known [102], in view of the popular opposition against the transportation of radioactive nuclear waste through populated areas to reach an assigned general depository, the radioactive nuclear waste generated by nuclear power plants has been stored on their site since start up, by reaching current storage levels such to constitute a serious danger for local residents as well as to the environment.

Amidst a social problem of such a dimension, it appears recommendable to study *all* possible forms of recycling o nuclear waste irrespective of whether they are aligned with preferred theories. The problem is compounded by the failure to date of all forms of recycling based on 20-th century theories which have been attempted under billions of dollars of public funds.

The problem is compounded by the fact that radioactive nuclear waste can solely be handled by governmental or para-governmental conduits, by therefore preventing the open participation by the industry at large. Societal accountability then suggests the continuation of the ongoing research based on the assumption that quantum mechanics is exactly valid for nuclear waste, as well as the initiation of *governmental* research based on new vistas, such as the Einstein-Podolsky-Rosen argument that suggesting a suitable completion of quantum mechanics for nuclear structures as a recommendable premise for the successful recycling of nuclear waste or the development of radiation-free controlled nuclear fusions.

With the understanding that the standard model has produced a valid *classification* of particles, the recycling of nuclear waste is additionally precluded by the assumption that the hypothetical quarks are the actual physical constituents of protons and neutrons without the dismissal in refereed publications of related inconsistencies, such as the belief that the permanently stable electron and proton *disappear* from the universe at the time of the neutron synthesis to be replaced by the hypothetical quarks ,and at the time of the neutron decay, the hypothetical quarks disappear to be replaced by the original electron and proton [103]. For brevity, we refer the interested reader to the inconsistencies of quark conjectures in Sect. 8.2.2 of [33] and the *return to sanity* advocated by Sir Karl Popper [104] for the structure model of unstable particles with physical constituents produced free in the spontaneous decays as it is the case for the neutron and all unstable particles, of course, according to hadronic rather than quantum mechanics [27] [79]. It is hoped that governmental agencies do not expose themselves to a judgment by posterity for keeping the recycling of nuclear waste hostage to hypothetical quarks that are incompatible with the physical reality of the neutron synthesis, admit a rigorously proved finite probability of tunnel effect and have not been detected free despite hadron collisions at extremely high energies.

By keeping in mind the prohibition of transporting radioactive waste through populated areas, R. M. Santilli has proposed the *recycling of radioactive nuclear waste by the nuclear plants themselves via their stimulated decay reducing their very long mean lives down to minutes or seconds* [105]-[107] (see also [108][33] [109]. The main issue is that the stimulated decays predicted by quantum mechanics are insufficient for the indicated reduction of mean lives, while hadronic mechanics does indeed predict a number of effective stimulated decay deserving due scientific process for proper societal accountability.

6.2. Recycling radioactive nuclear waste based on the neutron synthesis. In the author's view, the most important societal implication of the studies on the synthesis of the neutron from the proton and the electron [52]-[71] is the prediction of innovative means for the recycling of radioactive nuclear waste via nuclear transmutations.

The first possibility is offered by the representation of nuclear stability via the decoupling of the isoelectron $\hat{e}$ from the neutron (Sect. 4) since it implies that *neutrons can be stimulated to decay via irradiation with photons γ_r with the resonating frequency or energy $E_r = E_{\hat{e}}$* which, for the case of the Deuteron, is given by $E_{\hat{e}} = 1.293\ MeV$, according to the reaction first proposed in the 1994 paper [105]

$$\gamma_r^\uparrow + n^\downarrow \ \rightarrow +p^\downarrow + e^\uparrow + \nu, \tag{74}$$

with possible application to the transmutation of $Mo(100, 42, 0)$ into the stable $Ru(100, 44, 0)$ preliminarily verified by Santilli [6] as well as by the experimental team [94] (see also Refs. [66] [71])

$$\begin{gathered}\gamma_r(0,0,1) + Mo(100,42,0) \rightarrow Tc(100,43,1) + \beta^-(0,-1,0), \\ Tc(100,43,1) \rightarrow Ru(100,44,0) + \beta^-(0,-1,1).\end{gathered} \tag{75}$$

Note that, in the event confirmed, the above transmutation would provide a dual source of nuclear energy without the emission of harmful radiation or the release of radioactive waste (Fig. 9), which are given by the production of a DC current (since one resonating gamma would produce two easily trappable electrons) as well as of heat since the transmutation is isoenergetic with $\Delta E = 1.828\ MeV$,

$$\Delta E = E(100, 42, 0) - E(100, 44, 0) - E(\gamma) - 2E(e) = \\ = (3.034 - 0.184 - 1.022)MeV = 1.828MeV. \tag{76}$$

A second possible recycling of radioactive nuclear waste via their stimulated decay is given by the irradiation of waste pellets with a beam of low energy neutrons produced by the *Directional Neutron Source* of Hadronic Technologies Corporation (see Sect. 3.3 and Fig. 1). By remembering that radioactive nuclear waste can solely be handled by governmental agencies, the possibility can be illustrated via the following neutron irradiation of the unstable $42 - Mo - 100$ with half life of 10^{19} *years* into the isotope $42 - Mo - 102$ with half life of one *minute,*

$$2n + Mo(42, 100, 0) \rightarrow Tc(42, 102, 1). \tag{77}$$

For brevity, we leave to interested colleagues the extension of the above results to radioactive nuclear waste.

6.3. Recycling radioactive nuclear waste via pseudoproton irradiation. It should be mentioned that the confirmation and separation of the pseudoproton and/or the protoid (Sect. 3.6 and Ref. [101]) would imply the possible recycling of radioactive nuclear waste via their stimulated decay, as illustrated by the predicted transmutations of the type

$$2\bar{p}^-(-1, 1, 0) + Mo(42, 100, 0) \rightarrow Zr(40, 102, 0), \tag{78}$$

in which case the 10^{19} years main-life of $42 - Mo - 100$ is reduced to the of 2.9 seconds mean-life of $40 - Zr - 102$.

It should be noted that, being negatively charged and having low energy (estimated up to $100\ eV$), all protoids or pseudoprotons are expected to be absorbed by nuclei of an irradiated waste pellet. It should also be noted that protoids and pseudoprotons are evidently *unstable.* Nevertheless, their half life is predicted to be of the order of seconds (about 10% the half life of the neutron), thus being suitable for industrial applications.

In the author's view, the above application of protoids and pseudoprotons should not be dismissed on grounds of its incompatibility with the *antiprotons* allegedly produced at various particle laboratories, in view of their unsettled character as true antiparticles [101]. It is hoped that particle physics laboratories may develop reactors producing a beam of antiprotons/pseudoprotons to test possible recycling of radioactive nuclear waste.

In view of these and other unsettled aspects, Santilli has also proposed the test of the gravity of true antiparticles such as the positrons in a horizontal vacuum tube [110] (see also A . P. Mills [111], V. de Haan [112] and the Springer Nature monograph [113]), and test the gravity of the apparent anti-hydrogen atom only following the verification that its nucleus is indeed a true antiparticle.

7 Concluding remarks

In the hope that the studies presented in these papers can be applied to strong interactions at large, in the preceding [2] we have presented a systematic and updated study, specialized to nuclear physics of the axiom-preserving time reversal invariant *Lie-isotopic branch of hadronic mechanics,* whose knowledge is recommendable prior to the study of the time irreversible *Lie-admissible branch of hadronic mechanics* for the axiomatically consistent treatment of controlled nuclear fusions [114]- [116].

We have then presented numerically exact and time invariant representations of the experimental data of the Deuteron conceived as a bound state of a proton and a neutron via simple *quantum mechanical* nonunitary transformations of available models (Sect. I-3-11) under the condition of being reformulated in terms of isomathmatics and isomechanics (Sect. I–3.12) to avoid known inconsistencies of quantum nonunitary theories.

In this paper, we have presented apparently for the first time:

7.1) The relativistic completion of the quantum mechanical notion of *particle* into that of *isoparticles* via special isorelativity and related Lorentz-Poincaré-Santilli isosymmetry for extended nucleons under potential and nonpotential interactions.

7.2) The representation of *all* characteristics of the neutron at the nonrelativistic and relativistic levels in its synthesis from the isoelectron and the isoproton in the core of stars under their big *attractive* Coulomb force.

7.3) The identification of the numerical value of the density of the neutron in a way compatible with other experimental data.

7.4) The representation of nuclear stability despite the natural instability of the neutron via the *decoupling* of the electron from the neutron in the Deuteron due to the big Coulomb attraction from the second proton.

7.5) The representation of nuclear stability despite the big Coulomb *repulsion* between protons thanks to the admission of nonpotential forces between extended nucleons.

The main implications of the above advances are:

7.A) The prediction of means for the recycling of radioactive nuclear waste by the nuclear power plants themselves via new stimulated decays permitted by hadronic mechanics which apparently reduce very large mean lives down to seconds (Sect. 6).

7.B) The possible return to the historical cosmology with continuous creation of matter in the universe because the sole consistent possibility of explaining the missing $0.782 MeV$ in the neutron synthesis is that it originates from the ether as a universal substratum (Sect. 3).

7.C) The return to the first conception of the neutron and of nuclei at large as being bound states of the original constituents of stars, the proton and the electron. Said conception was prohibited by the theoretical assumption that Heisenberg's indeterminacy principle for point-like particles in vacuum is also valid inside nuclei. However, the conception of the neutron, and therefore, of all matter in the universe as being bound states of protons and electrons is fully valid under Einstein's isodeterminism (Sects. 3 and 4.7) [1] [14] [15] for extended particles within hadronic matter.

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