Ratio Mathematica Volume 52, 2024

Lie-isotopic representation of stable nuclei I: Apparent insufficiencies of quantum mechanics in nuclear physics

Ruggero Maria Santilli*

Abstract

In this paper, we recall the majestic axiomatic consistency of quantum mechanics for *point-like particles and electromagnetic waves in vacuum.* By following the 1935 historical argument by A. Einstein, B. Podolsky and N. Rosen that *quantum mechanics is not a complete theory,* we identify a number of apparent insufficiencies of quantum mechanics in nuclear physics with particular reference to the lack of numerically exact representation in one century of nuclear data, the prohibition by Heisenberg's uncertainty principle to represent the neutron synthesis from the electron and the proton in the core of stars despite their extremely big Coulomb attraction and the ensuing inability to represent the nuclear stability. We then point out that the axiomatic origin of the indicated insufficiencies appears to be due to the representation of nuclear constituents as *dimensionless* particles, compared to the experimentally measured *extended* character of the charge distribution of protons and neutrons in conditions of partial mutual penetration within a nuclear structure, with consequential strong interactions of nonlinear, nonlocal and nonpotential. In the second paper, we attempt a resolution of the indicated insufficiencies with ensuing exact and invariant representation of the Deuteron data. In the third paper, we present a consequential representation of nuclear stability with ensuing new means of recycling nuclear waste by nuclear power plants and other advances.

Keywords: nuclear physics 81V35, EPR argument, hadronic mechanics, nuclear data. [1]

---

*The Institute for Basic Research, Palm Harbor, Florida USA; research@i-b-r.org

[1] Received on March 9, 2024. Accepted on May 30, 2024. Published on June 30, 2024. DOI:10.23755/rm.v52i0.1607. ISSN: 1592-7415. eISSN: 2282-8214.

1 Introduction

This author has expressed various times in his writings the view that *non-relativistic and relativistic quantum mechanics, thus including Galilean and special relativities, have a majestic axiomatic structure* and an impressive amount of experimental verifications within the conditions of their original conception, those of *dimensionless particles in vacuum under linear, local and potential interactions.*

Contrary to historical discoveries in atomic physics, quantum mechanics has been unable to achieve in one century under billions of dollars of research funds: exact representations of nuclear data; a structural (rather than kinematical) representation of the synthesis of the neutron from the hydrogen in the core of stars; the representation of nuclear stability despite the natural instability of the neutron as well as the strongly repulsive protonic Coulomb forces; industrially viable recycling of radioactive nuclear waste; sustainable and controllable nuclear fusions; and the lack of other advances of societal need.

Consequently, for societal as well as scientific accountability, we need to reinspect the *exact* validity of quantum mechanics in nuclear physics, with the understanding that its *approximate* validity in nuclear physics is beyond scientific doubts.

Among supporting views, we mention the widely forgotten (and often opposed) 1935 historical view by A. Einstein, B. Podolsky and N. Rosen (EPR) that *"quantum mechanics is not a complete theory"* or that *"the wave function of quantum mechanics dies not provide a complete description of physical reality"*, herein called the EPR argument [1], which was based on the experimental evidence on *particle entanglements,* namely, on the evidence that particles initially bonded together and then separated, can continuously and instantaneously influence each other at arbitrary mutual distances (see the recent paper [2] for entanglements at the classical level).

By noting that electromagnetic and gravitational interactions are ignorable at large mutual distances and superluminal interactions would violate special relativity, Ref. [1] argued that particle entanglements constitute experimental evidence on the lack of universal validity of quantum mechanics, thus suggesting the study of its completion.

On theoretical grounds, R. M. Santilli [3] has recently proved Einstein's rejection of the very name "quantum entanglement" on grounds that quantum mechanics can solely represent interactions derivable from a potential, and consequently, can solely represent entangled particle as being free due to the lack of any conceivable potential interactions at large mutual distances, by therefore suggesting the study of a suitable completion of quantum mechanics.

On experimental grounds, the lack of exact character of quantum mechanics in nature has been established by direct experiments [4] [5] [6] citing the EPR ar-

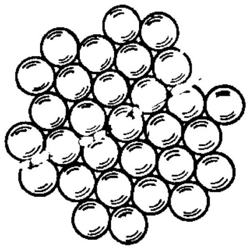

Figure 1: *In this figure, we illustrate experimental data on nuclear volumes compared to the volume of individual nucleons [23]-[27], according to which nuclei are composed by extended nucleons in conditions of partial mutual penetration of their dense charge distributions, with ensuing non-linear, non-local and non-potential interactions which have been technically studied via the conditions of variational self-adjointness in monograph [30].*

gument in their titles, as well as measurements in virtually all branches of physics, including: electrodynamics [7] [8] [9] [10]; large ion physics [11]; particle physics [12] [13]; Bose-Einstein correlation [14] [15]; propagation of light within physical media [16]; cosmology [17] [18]; and in other fields.

Following decades of studies in the field initiated in the late 1970's at Harvard University under support from the U. S. Department of Energy, the protracted general ignorance in nuclear physics of opposing experimental evidence [4]-[18] appears to be a primary reason for the failed achievement to date of an industrially viable controlled nuclear fusion despite the use of trillions of dollars of public funds, with known implications for our decaying environment.

In fact, the engineering realization of reactors for nuclear fusions may well be inadequate to achieve sustainability because: 1) Quantum mechanics has been experimentally established not to be exact for complex physical processes [4]-[18]. 2) Quantum mechanics has been unable to achieve an exact representation of nuclear data and stability. 3) The engineering components of nuclear reactors are based on the theoretical assumption of the validity for nuclear structures of Heisenberg's uncertainty principle, contrary to the recently achieved, progressive recovering of Einstein's determinism in the structure of hadrons, nuclei and stars, and its full recovering at the limit of gravitational collapse [19] [20] [21].

In the author's view and experience, a primary reason for the century old opposition by mainstream physicists against the completion of relativistic quantum mechanics in nuclear physics, as well as for the oblivion of experimental deviations [4]-[18], is a widespread resilience against the consequential completion of *special relativity* for point-like particles in vacuum into a covering relatvity, known as *isorelativity,* for extended, therefore deformable and dense particles [22].

In this paper, we illustrate the validity of the EPR argument in nuclear physics via the following primary aspects:

1.1. The experimental evidence [23]-[27] that nuclei are composed by *extended* protons and neutrons (collectively called nucleons) in conditions of partial mutual penetration of their *dense* charge distribution, with ensuing expectation that, in addition to conventional linear, local and potential terms, nuclear forces contain additional terms that are *non-linear* (in the sense of depending on powers of the wave-function) as first studied by W. Heisenberg [28], *non-local* (in the sense of occurring in volumes) as first studied by L. de Broglie and D. Bohm [29], and *non-potential* (in the sense of being of contact, thus zero-range type not derivable from a potential), as first studied by R. M. Santilli in monographs [30] [31].

1.2. The consequential lack of exact representation in one century of nuclear data, such as nuclear spins, nuclear magnetic moments and nuclear stability.

1.3. The time reversal invariance of quantum mechanics (in view of the invariance of Heisenberg's equations under anti-Hermiticity), compared to the time irreversibility of all physical, chemical and biological energy releasing processes.

By using a language specifically intended for nuclear physicists, in Sect. 2 of this paper we identify a number of apparent insufficiencies of quantum mechanics in nuclear physics. In Sect. 3, we identify their apparent axiomatic origin as a necessary pre-requisite for their possible resolution.

In the second paper [32] (hereon indicated with the prefix II) we attempt a resolution of the insufficiencies indicated in this paper via the representation of the *extended* character of nucleons and their contact, thus *non-potential interactions,* with ensing exact and invariant representation of the Deuteron data.

In the third paper [33] (hereon referred with the prefix III), we use the methods of paper II for a quantitative representation of the synthesis of the neutron from an electron and a proton in the core of stars with a cinsequentiuak representation of nuclear stability despite the natural *instability* of the neutron and despite the strongly *repulsive* protonic Coulomb forces. We then apply the results to the recycling of radioactive nuclear waste and other problems.

2 Apparent insufficiencies of quantum mechanics in nuclear physics

It is conceivably possible that above insufficiencies 1.1, 1.2 and 1.3 may be at least partially responsible for the lack of achievement to date of controlled nuclear fusions, thus deserving their closer inspection.

2.1. Insufficient representation of strong nuclear forces

As it is well known, the achievement of a full representation of nuclear forces has remained elusive to this day despite the use in one century of a large number of potentials, including the Yukawa potential [34], the Woods-Saxon potential [35], the Reid potential [36], magnetic resonance technique [37], solutions of the Klein-Gordoin equatin [38], meson exchange formulations and other methods [39] (see Refs. [40]-[41] for historical accounts). This insufficiency suggests the study of possible additional terms in the nuclear force of non-linear, non-local and non-potential type which have been technically identified as being *variationally non-selfadjoint* (NSA) [30]. An explicit and concrete representation of strong nuclear forces is presented in Sect. II-3.4 and applied in Sect. II-5, III-3, III.4.

2.2. Insufficient representation of nuclear stability due to strongly repulsive protonic forces

Let us recall that protons repel each other with a Coulomb force, which at the mutual nuclear distance of $10^{-13}\ cm$, is extremely big for particle standards, since it is of the order of hundreds of Newtons. As an example, the two protons of the Helium experience the *repulsive* Coulomb force of 230 Newton,

$$\begin{aligned} F = +\tfrac{e^2}{r^2} = \\ = +(8.99 \times 10^9)\tfrac{(1.60\times 10^{-19})^2}{(10^{-15})^2} = +230N. \end{aligned} \tag{1}$$

It is possible that the Yukawa, Woods-Saxon, Reid and other attractive nuclear potentials may overcome such a big repulsive force, but it is unlikely whether there exists a residual attractive force sufficient for a quantitative representation of the stability of the Helium and other nuclei. A representation of nuclear stability despite the strongly repulsive protonic Coulomb forces is presented in Sect. III-4.2.

2.3. Inability to represent the synthesis of the neutron from the Hydrogen atom in the core of stars

Stars begin their lives as an aggregate of hydrogen atoms that increases in time via the accretion of hydrogen from intergalactic spaces. When the temperature (and pressure) in the core of stars reaches values estimated to be in the order of $10M\ F$, the electron is "compressed" inside the proton according to E. Rutherford [42] resulting in a new particle called the *neutron* $p^+ + e^- \ \rightarrow \ n.$

W. Pauli noted that Rutherford's synthesis does not conserve the spin, by therefore vioiating special relativity, and suggested for their preservation the emission of a hypothetical massless particle with spin $1/2$ called by E. Fermi the *neutrino* [40] [41]

$$p^+ + e^- \ \rightarrow \ n + \nu. \tag{2}$$

It appears that W. Pauli did succeed in maintaining the validity of special relativity for the neutron synthesis, but he jointly failed to maintain the validity of quantum mechanics. In fact, despite the existence of an extremely big *attractive* Coulomb force of 230 Newtons between the (negatively charged) electron and the (positively charged) proton, Eq. (1), quantum mechanics prohibits the synthesis of the neutron from the hydrogen for various reasons identified by R. M. Santilli in the 1978 Harvard University memoir [43].

The first insufficiency is caused by the fact that the validity of Heisenberg's uncertainty principle under strong interactions implies that the maximal value of the standard deviation Δr_e for the coordinate r_e of an electron inside a nucleus nay be bigger than the radius of the neutron, with consequential speed such that the kinetic energy of the electron K_e may be bigger than the mass m_n of the neutron,

$$\Delta r_e > R_n = 0.87 \times 10^{-13}\ cm,$$

$$\Delta v_e = \frac{\hbar}{\Delta r_e \times m_e} > 10^{10}\ m/s, \tag{3}$$

$$\Delta K_e = \frac{1}{2m_e} \times (\Delta p_e)^2 > m_n = 939.56\ MeV/c^2,$$

by therefore prohibiting any meaningful synthesis of the neutron, and suggesting the completion of Heisenberg's uncertainty principle for strong interactions presented in Ref. [21] (Sect. II-4.7), and applied to the resolution of insufficiencies (3) in Sect. III-3.3.1.

Additionally, Ref. [43] noted that the mass of the neutron is $0.782\ MeV$ *bigger* than the sum of the masses of the proton and of the electron

$$E_p = 938.272\ MeV,\ \ E_e = 0.511\ MeV,$$

$$E_n = 939.565\ MeV, \tag{4}$$

$$\Delta E = E_n - (E_p + E_e) = 0.782\ MeV > 0,$$

by therefore requiring a *positive binding energy* and resulting in a *mass excess* for which the Schrödinger equation *for a bound state with two constituents* admits no physically meaningful solution.

Ref. [43] finally noted that, while being fully valid for the representation for the Hydrogen atom, Dirac's equation fails to provide any representation of its "compression" into the neutron (for extensive theoretical studies in the neutron synthesis from the Hydrogen see the theretical [44], experimental studieds Ref. [45] and Sect. III-3).

It should be noted that the above insufficiency has been generally dismissed by 20th century nuclear physics on grounds that the missing energy of $0.782\ MeV$ is

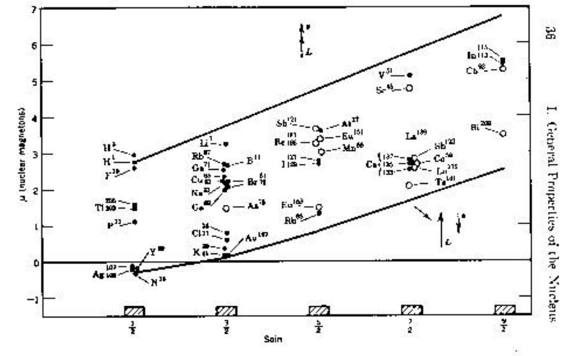

Figure 2: ***In this figure, we illustrate the well known Schmidt limits in nuclear physics (see, e.g., Ref. [41]) establishing deviations between the predictions of quantum mechanics for nuclear magnetic moments and their experimental values.***

carried by the neutrino of reaction (3). However, in the event that is the case, the synthesized neutron n in reaction (3) could not exist because of insufficient energy for its synthesis. Also, the missing energy of $0.782\ MeV$ cannot be provided by the $e-p$-relative kinetic energy because the $e-p$ cross section at $1M\ MeV$ is virtually null , thus preventing any synthesis. In the event said missing energy is provided by the $e-p$-relative energy, stars would never produce light because they synthesize about 10^{40} neutrons per second, by therefore requiring about $10^{40}\ MeV/s$. Additionally, scientific accountability prohibits the dismissal of a *generalized structure equation* for the $e-p$ bound state just because it does not verify quantum mechanics in view of its implications for controlled nuclear fusions and the recycling of radioactive nuclear waste [44] [45].

A review and upgrade of half a century of studies in the representation of *all* characteristics of the neutron in its synthesis from the Hydrogen in the coire of stars via hadronic mechanics is presented in Sect. III.3.

2.4. Insufficient representation of nuclear stability despite the instability of the neutron

As it is also well known, neutrons are naturally *unstable* (when isolated) with a mean life $\tau = 880\ s$ and spontaneous decay from Eq.(3) $n \rightarrow p^+ + e^- + \bar{\nu}$. Consequently, the stability of the neutron under strong nuclear forces deserves indeed a study. A representation of nuclear stability despite the natural instability of the neutron in presented in Sect. III-4.1.

Mainstream nuclear physicists generally dismiss the above problem (formulated in this paper apparently for the first time) on grounds that the evident nuclear stability dismisses the existence of the problem itself.

2.5. Insufficient representation of nuclear spins

According to quantum mechanical basic axioms, the only stable bound state of two particles with spin $1/2$ (such as the proton and the neutron) is the singlet coupling. Consequently, quantum mechanics predicts that the Deuteron D has the structure

$$D = (p_{\uparrow}, n_{\downarrow})_{qm}, \tag{5}$$

for which the total angular momentum is null, $J_D = 0$, contrary to the experimental value of the spin of the Deuteron $J_D = 1$.

As a result of this insufficiency, quantum mechanics represents the spin of the Deuteron via such a *collection of orbital contributions* with value $L_D = 1$ (see, e.g., Ref. [41]). However, the spin $J_D = 1$ has been measured for the Deuteron *in its true ground state,* i.e., the experimentally detected state for which $L_D \equiv 0$. Consequently, quantum mechanics does not allow a consistent representation of the spin of the Deuteron in its true ground state, with similar insufficiencies for heavier nuclei (Sect. III-4.1).

Mainstream nuclear physicists generally dismissed the above insufficiency on grounds that the spin of the Deuteron is fully recovered at the level of nuclear isospins. However, the nuclear isospin symmetry is a purely mathematical symmetry solely definable in a complex valued two-dimensional carrier space that, as such, cannot possibly have any sound connection with the spin of the Deuteron in a state that, by definition, has no excited orbital states. Other attempts at salvaging quantum mechanics via hypothetical symmetric and antisymmetric states of the Deuteron are afflicted by the same shortcoming.

Since spin is a fundamental nuclear characteristic with direct implications for nuclear fusions, recycling of nuclear waste and other unsolved problems of direct societal relevance, scientific accountability requires ay least the search for a consistent representation of the Deuteron spin *without* orbital contributions, which study is done in Sect. II-33.3.2.

2.6. Insufficient representation of nuclear magnetic moments

Under the use of the tabulated values of the magnetic moments of the proton and of the neutron in vacuum (see, e.g., Ref. [23])

$$\mu_p = +2.79285\, \mu_N, \quad \mu_n = -1.91304\, \mu_N, \tag{6}$$

(where μ_N represents the *nuclear magneton*) quantum mechanics (qm) predicts that the magnetic moment of the Deuteron is given by

$$\mu_D^{qm} = (2.79285 - 1.91304)\, \mu_N = 0.87981 \mu_N, \tag{7}$$

while the experimentally measured value is given by

$$\mu_D^{ex} = 0.85647\, \mu_N, \tag{8}$$

resulting in a deviation for about 3% of the quantum mechanical prediction μ_D^{qm}, Eq. (7), from the experimental value μ_D^{ex}, Eq. (8), with bigger deviations for heavier nuclei (Figs. 1, 2). A numerically exact and time invariant representation of the Deuteron magnetic moment in its true ground state (that without orbital contributions) is presented in Sect. II.3.3.

The above insufficiency of quantum mechanics is generally dismissed by mainstream nuclear physicists on grounds that an exact representation of the deuteron magnetic moment can be achieved via the angular contribution to its spin. The credibility of the dismissal is however in question due to the vast experimental evidence for which the *Deuteron spin* $S_D = 1$ *holds in the ground state,* that is, the experimentally detected state without orbital contributions by its very definition.

2.7. Insufficient representation of nuclear irreversible processes

As it is well known, quantum mechanics is a theory invariant under time reversal in view of the invariance of Heisenberg's time evolution of an observable A under anti-Hermiticity

$$i\frac{dA}{dt} - [A, H] \equiv -\{i\frac{dA}{dt} - [A, H]\}^\dagger. \tag{9}$$

Consequently, under the assumption that the total energy $H(r, p)$ is an observable (rather than a complex-valued, thus non-observable quantity), the same Schrödinger equation in the relative coordinate r has to be applied for time irreversible energy producing processes, such as the controlled fusion of two nuclei into a third, moving forward in time (fw), as well as for their image under motion backward in time (bw), such as the disintegration of the synthesized third nucleus,

$$\begin{gathered} H = H^\dagger, \\ H|\psi(r)\rangle = E^{fw}|\psi(r)\rangle, \quad \langle\psi(r)|H = \langle\psi(r)|E^{bw}, \\ |\psi(r)\rangle \equiv (\langle\psi(r)|)^\dagger, \quad E^{fw} \equiv E^{bw}, \end{gathered} \tag{10}$$

by therefore lacking an axiomatically consistent representation of the irreversibility of energy releasing processes at large and for controlled nuclear fusions in particular.

Additionally, quantum mechanics can solely represent the conservation of the energy due to the known antisymmetric character of Lie's brackets,

$$i\frac{dH}{dt} = [H, H] = HH - HH \equiv 0. \tag{11}$$

Hence, quantum mechanics additionally lacks an axiomatically consistent representation of the *energy production* of controlled nuclear fusions (see Ref. [46] for details).

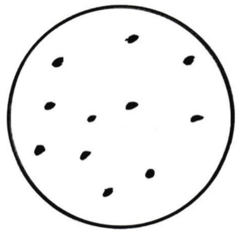

Figure 3: ***In this figure, we illustrate the conception of nuclei originating from the locality of quantum mechanics [1] as ideal spheres with isolated dimensionless particles in their interior. Such a view is disproved by actual value*** 0.87×10^{-13} cm ***of the charge distribution (rather than of the wave packet) of protons and neutron [27].***

For the apparent intent of maintaining quantum mechanics under physical conditions beyond those of its original conception, the limitation caused by time reversibility has been generally dismissed by mainstream nuclear physicusts on grounds that the direction of time is set by thermodynamical laws. The scientific character of the dismissal is however in question due to the notorious incompatibility of quantum mechanics with thermodynamics, as well as by known opposition against the EPR completion of quantum mechanics with non-potential interactions to achieve compatibility between mechanics and thermodynamics.

3 Apparent axiomatic origin of quantum mechanical insufficiencies

In this section, we attempt to identify the axiomatic origin of the insufficiencies of quantum mechanics studied in the preceding section as a necessary pre-requisite for their possible resolution studied in paper II [32], Sects, II-3 and II-4.

3.1. Is the linearity of quantum mechanics an axiomatic insufficiency?
As it is well known from the axiomatic structure of *linear* operators on a Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$ [41], *quantum mechanics is linear in the wave function,* while non-linear interactions are expected in nuclear structures. Various nuclear models with interactions non-linear in the wave functions have been studied (see the first study by W. Heisenberg [28]), but all of them have been formulated via the quantum mechanical *Hamiltonian* structure

$$H(r, p, \psi, ...)|\Psi(r)\rangle = E\psi(r)|\Psi(r)\rangle, \tag{12}$$

in which case: A) The Hamiltonian does not generally represent the total energy. B) There is the general violation of the superposition principle for the decomposition of a state $|\Psi(r)\rangle$ representing a nucleus with A nucleons into states $|\psi_k(r_k)\rangle$ representing the individual nucleons

$$|\Psi(r)\rangle \neq \Pi_{k=1,2,...,A}|\psi_k(r_k)\rangle, \tag{13}$$

with consequential inability to characterize individual constituents of a nucleus. C) Being additive to the kinetic energy in a Hamiltonian, non-linear interactions are generally interpreted as having a potential energy against their expected non-potential character (Fig. 1). A possible resolution of the above axiomatic insufficiency is presented in Sect. II-3.4 and applied in Sect. II-5, III-3, III.4.

3.2. Is the locality of quantum mechanics an axiomatic insufficiency?
This axiomatic insufficiency is due to the fact that, as voiced by A. Einstein, B. Podolsky and N. Rosen [1], the wave function $\psi(r)$, the potential $V(r)$, the Laplacian Δ_r and the Newton-Leibnitz differential calculus of the Schrödinger equation

$$H(r,p)\psi(r) = [-\frac{\hbar^2}{2m}\Delta_r + V(r)]\psi(r) = E\psi(r), \tag{14}$$

can only be defined at a finite number of isolated points r, resulting in an excessively simple conception of nuclei as ideal spheres with isolated massive points in their interior (Fig. 3). In reality, nuclei are composed by extended, therefore deformable and dense nucleons with a charge radius of about $0.87 \times 10^{-13}\ cm$ [27] in conditions of partial mutual penetration, with ensuing non-linear, non-local and non-potential interactions (Sect. 1). It appears evident that these complex conditions cannot be represented in a final form by the linear, local and potential structure of quantum mechanics.

It should be noted that, for the apparent intent of maintaining quantum mechanics under non-local conditions beyond its representational capability, 20th century nuclear physics dismissed the insufficiency on the dimensionless character of nuclear constituents on grounds that the dimension of protons and neutrons are represented by their wave packets. In reality, such a dismissal does not appear to have solid physical grounds because:

1) Wave packets fill up the entire universe, while protons and neutrons have the measured dimension of $0.87 \times 10^{-13}\ cm$.

2) Assuming that wave packets can be used for the representation of the size of particles, their interactions are non-local (because occurring in volumes not reducible to points), thus being outside the representational capabilities of quantum mechanics.

3) The dismissal of dimensionless constituents is in disrespect of A. Einstein, B. Podolsky and N. Rosen because their concluding statement is that ... *the wavepackets of quantum mechanics cannot represent the entire physical reality* [1].

For the apparent intent of opposing the representation of the actual, measured, dimension of nucleons achieved in Paper II, mainstream nuclear physicists generally dismiss the extended character of nucleons on grounds that their size is represented by the standard deviations Δr of Heisenberg's uncertainty principle. This additional dismissal does not appear to have solid physical grounds because:

i) The validity of Heisenberg's uncertainty for nuclear constituents is a personal opinion by individual physicists because said principle has never been directly tested under strong nuclear interactions.

ii) To have physical value, the dismissal should prove in refereed publications that, while being a constituent of a nucleus such as the Zirconium, a proton has a standard deviations $\Delta r, \ \Delta p$ *exactly equal* to that when the same proton is the nucleus of the hydrogen atom.

iii) To remain within the boundaries of science, the indicated dismissal should first disprove the *iso-uncertainty principle under strong interactions* of Refs. [19] [20] [21].

A possible resolution of the above axiomatic insufficiency is presented in Sect. I-3.4 and applied in Sect. I-5, II-3, II.4.

3.3. Is the potentiality of quantum mechanics an axiomatic insufficiency?

Let us recall the following technical treatment of potential forces by R. M. Santilli in the 1978 Springer-Verlag monograph [30]:

DEFINITION 3.1: A force is said to be 'variationally self-adjoint' (SA) when it verifies all necessary and sufficient conditions for its derivation as an additive potential in a Hamiltonian.

However, as recalled in Sect. 1.1 and Fig. 1, experimental nuclear data [23]-[27] suggest the presence of new terms in nuclear force that can be technically identified via the following:

DEFINITION 3.2 [30]: A force is said to be 'variationally non-self-adjoint' (NSA) when it violates the necessary and sufficient conditions for its derivation from an additive potential in a Hamiltonian.

NSA forces are known to exist since Newton's time, they are represented by the external terms in Lagrange's and Hamilton's equations, and their operator counterpart cannot be ignored in view of the *No-Reduction Theorem* studied by R. M. Santilli in the 1983 Springer-Verlag monograph [31] whose understanding requires a knowledge of the progressive recovering of Einstein's determinism studied in Refs. [19] [20] [21].

THEOREM 3.1: A macroscopic system with SA and NSA internal forces cannot be consistently decomposed into a finite number of quantum mechanical par-

ticles all with sole SA forces, and vice versa, a quantum mechanical system with sole SA internal forces cannot represent a classical system with SA and NSA forces under the correspondence or other principles.

As an example, the resistive force experienced by a spaceship during re-entry in our atmosphere is a NSA force, that analytically, can be solely represented via the external terms in Lagrange's and Hamilton's equations. Theorem 3.1 establishes that said spaceship cannot be consistently decomposed into a collection of quantum mechanical particles thus suggesting the EPR completion of quantum mechanics via the inclusion of NSA forces.

In any case, following decades of studies by various scholars along the EPR argument, quantum mechanics has been completed into the covering *hadronic mechanics* [47] [48] [49] [50] which has achieved a numerically exact and time invariant representation of nuclear data [52] thanks to the characterization of strong interactions as the most general conceivable superposition of SA and NSA forces [53] by therefore establishing that *irreversibility originates at the most elementary possible level in nature,* e.g., in meson's spontaneous decays. A possible resolution of the above axiomatic insufficiency is presented in Sect. I-3.4 and applied in Sect. I-5, II-3, II.4.

3.4.Is the modularity of quantum mechanics an axiomatic insufficiency?

We are here referring to the conventional associative action (called modular) of a Hamiltonian on a state $|\psi\rangle$ of the Hilbert space verifying conditions (10), which modular action is evidently responsible for possible causality violation by quantum mechanical treatments of controlled nuclear fusions. Axiomatic insufficiency 3.4 implies the expectation that quantum mechanics cannot represent consistently time irreversible systems, such as nuclear fusions as well as high energy particle collisions. We can therefore say that the causal treatment of controlled nuclear fusions suggests the construction of a suitable completion of the modular structure of quantum mechanics to be studied in subsequent papers.

3.5. Is the single-valudness of quantum mechanics an axiomatic insufficiency?

We are here referring to the quantum mechanical property for which the product of two quantities (such as numbers, matrices, operators, etc.) yields one single result. By contrast, the correlation of two atoms in a DNA may yield a full organ, such as a liver, with about 10^{30} atoms. The mathematical representation of the correlation is given by the multiplication. Hence, axiomatic limitation 3.5 suggests the need for an additional completion of quantum mechanics of *multi-valued character* in which the product of two quantities can yield an ordered, but otherwise arbitrary number of results in which case, there appears to be hope to initiate a quantitative representation of life [54].

4 Concluding remarks

In preparation of our search for new radiation-free controlled nuclear fusions of light elements preliminarily indicated in report [46], in this paper we recalled the majestic axiomatic consistency of non-relativistic and relativistic quantum mechanics with corresponding Galilean and special relativities, as well as their impressive record of experimental verifications for the conditions of their original conception, those of *point-like particles in vacuum under electromagnetic interactions.*

As an illustration of the 1935 historical argument by A. Einstein, B. Podolsky and N. Rosen that *quantum mechanics is not a complete theory* [1], we have identified a number of apparent insufficiencies of quantum mechanics for *extended protons and neutrons under strong interactions,* with particular reference to the inability of achieving in one century under billions of dollars of research funds: exact representations of nuclear data; a structural (rather than kinematical) representation of the synthesis of the neutron from the hydrogen in the core of stars; the representation of nuclear stability despite the natural instability of the neutron as well as the strongly repulsive protonic Coulomb forces; the recycling of radioactive nuclear waste; sustainable and controllable nuclear fusions; and the lack of other advances of societal needs.

In the author's view, the insufficiency at the foundation of the preceding ones is the impossibility by quantum mechanics to achieve a representation via a structure equation (rather than "ad hoc" kinematic arguments) of the synthesis of the neutron from an electron and a proton in the core of stars despite their extremely big Coulomb attraction (1), because Heisenberg's uncertainty principle implies a maximal standard deviation for the electron coordinate bigger than the size of the neutron and independently, a maximal standard deviation of the momentum yielding a kinetic energy of the electron bigger than the mass of the neutron (3), by therefore prohibiting any possible neutron synthesis.

This rather serious insufficiency confirms the need for a suitable EPR completion of quantum mechanics including, most importantly, a completion for strong interactions of Heisenberg uncertainty principle studied in Sect. II-4.7, which has been assumed for one century to be valid in nuclear physics without any direct or indirect experimental verification.

We have then identified the apparent axiomatic origin of the indicated insufficiencies in the *point-like* approximation of nuclear constituents, compared to the experimentally established *extended* character of nucleons, with consequential, expected new terms in the nuclear force of *nonlinear* type initiated by W. Heisenberg [28], *nonlocal* type initiated by L. de Broglie and D. Bohm [29] and *nonpotential* type initiated by R. M. Santilli [30] [31].

In paper II [32], we present in a language specifically intended for nuclear physicists an apparent resolution of the indicated insufficiencies and apply the results to achieve exact and invariant representations of the Deuteron data.

In paper III [33], we apply the same methods to achieve an exact and invariant representation of *all* characteristics of the neutron in its synthesis from the Hydrogen in the core of stars thanks to the completion of Heisenberg's uncertainty principle into the *isouncertainty principle for strong interactions* [21] (Sect. II-4-7), and apply the results for new means of recycling radioactive nuclear waste by nuclear power plants and other applications.

The methods underlying our studies are given by the *axiom-preserving EPR completion*— of quantum mechanics into *hadronic mechanics* [47] [48] [49] [50] according to the following branches with increasing complexity for the representation of progressively more complex nuclear conditions (see Ref. [51] for a summary classification of hadronic mechanics):

4.1. Lie-isotopic branch of hadronic mechanics [31] [48] comprising a time reversal invariant completion of quantum mechanics for the representation of stable nuclei as a collection of extended and dense nucleons in conditions of partial mutual penetration with linear and non-linear, local and non-local and potential, as well as non-potential internal forces (Fig. 1).

4.2. Lie-admissible branch of hadronic mechanics [48] [46] comprising a time irreversible completion of the Lie-isotopic branch for the causal representation of the controlled nuclear fusion, recycling of radioactive nuclear waste and other problems of societal relevance.

4.3. Hyperstructural branch of hadronic mechanics [48] [54] comprising a multi-valued completion of the irreversible Lie-admissible branch in the hope of initiating a quantitative representation of life intended as the difference between organic and inorganic molecules.

Acknowledgements

The author would like to express sincere thanks for penetrating critical comments received from the participants of the *2020 International Teleconference on the EPR argument,* the *2021 International Conference on Applied Category Theory and Graph-Operad-Logic dedicated to the memory of Prof. Zbigniew Oziewicz,* the *Seminars on Fundamental Problems in Physics,* and the *2023 Sustainable Industrial Processing Summit and Exhibition*. Additional thanks are due to various colleagues for technical controls and to Mrs. Sherri Stone for linguistic control of the manuscript. However, the author is solely responsible for the content of this paper due to several revisions in its final form.

References

[1] Einstein, A. Podolsky, B. and Rosen, N.: Can quantum-mechanical description of physical reality be considered complete?, *Phys. Rev.* **47**, 777-780 (1935),
http://www.eprdebates.org/docs/epr-argument.pdf

[2] Berkowitz, R.: Macroscopic systems can be controllably entangled and limitlessly measured, *Physics Today* July issue, 16-18 (2021).

[3] Santilli, R. M.: A quantitative representation of particle entanglements via Bohm's hidden variables according to hadronic mechanics, *Progress in Physics* **18**, 131-137 (2022),
www.santilli-foundation.org/docs/pip-entanglement-2022.pdf

[4] Fadel, M. et al.: Spatial entanglement patterns and Einstein-Podolsky-Rosen steering in Bose-Einstein condensates. *Science* **360**, 409–415 (2018),
www.santilli-foundation.org/Basel-paper.pdf

[5] Colciaghi, P. et al.: Einstein-Podolsky-Rosen Experiment with Two Bose-Einstein Condensates, *Phys. Rev. X* **13**, 021031-1/021031-10 (2023),
https://journals.aps.org/prx/pdf/10.1103/PhysRevX.13.021031

[6] Aspect, A. et al.: Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities, *Phys. Rev. Lett.* **49**, 91-94 (1982),
https://ui.adsabs.harvard.edu/abs/1982PhRvL..49...91A

[7] Miller, J. P. et al.: Muon (g2): experiment and theory. *Rep. Prog. Phys.* **70**, 795–881 (2007),
news.fnal.gov/2021/04/first-results-from-fermilabs-muon-g-2-experiment-strengthen-evidence-of-new-physics

[8] Santilli, R. M.: Apparent Unsettled Value of the Recently Measured Muon Magnetic Moment, *Progress in Physics* **18**, 15–18 (2022),
http://www.santilli-foundation.org/docs/muon-meanlife-2022.pdf

[9] Santilli, R. M.: Representation of the anomalous magnetic moment of the muons via the Einstein-Podolsky-Rosen completion of quantum into hadronic mechanics, *Progress in Physics* **17**, 210–215 (2021),
https://www.santilli-foundation.org/muon-anomaly-pp.pdf

[10] Santilli, R. M.: Representation of the anomalous magnetic moment of the muons via the novel Einstein-Podolsky-Rosen entanglement in: H. M. C.

Garcia, J. J. C. Guzman, L. H. Kauffman and H. Makaruk, Editors,*Scientific Legacy of Professor Zbigniew Oziewicz,* World Scientific (2024), https://www.santilli-foundation.org/docs/santilli-546-(1).pdf

[11] Schukraft, K.: Heavy-ion physics with the ALICE experiment at the CERN Large Hadron Collider. *Trans. R. Soc.* **A370**, 917–932 (2012), royalsocietypublishing.org/doi/10.1098/rsta.2011.0469

[12] Cardone, F., Mignani, R. and Santilli, R. M.: On a possible energy-dependence of the K^0 lifetime. Part I *J. Phys. G: Part. Phys.* **18**, L141-L144 (1992),
www.santilli-foundation.org/docs/Santilli-32.pdf

[13] Cardone, F., Mignani, R. and Santilli, R. M.: On a possible energy-dependence of the K^0 lifetime. Part II *J. Phys. G: Part. Phys.*, **18**, L61-L65 (1992),
www.santilli-foundation.org/docs/Santilli-32.pdf

[14] Santilli, R. M.: Nonlocal formulation of the Bose-Einstein correlation within the context of hadronic mechanics, *Hadronic J.* **5**, 1–50 and **15**, 81–133 (1992),
www.santilli-foundation.org/docs/Santilli-116.pdf

[15] Cardone, F. and Mignani, R.: Nonlocal approach to the Bose-Einstein correlation, *JETP* **83**, 435 (1996),
www.santilli-foundation.org/docs/Santilli-130.pdf

[16] Ahmar, H. et al.: Additional experimental confirmations of Santilli's IsoRed-Shift and the consequential lack of expansion of the universe, *Journal of Computational Methods in Sciences and Engineering* **13**, 321-375 (2013),
www.santilli-foundation.org/docs/IRS-confirmations-212.pdf

[17] Mignani, R.: Quasars redshift in isominkowski space, *Physics Essay* **5**, 531 (1992),
http://www.santilli-foundation.org/docs/Santilli-31.pdf

[18] Santilli, R. M.: Experimental Verifications of IsoRedShift with Possible Absence of Universe Expansion, Big Bang, Dark Matter and Dark Energy, *The Open Astronomy Journal*, **3**, 124-132 (2010),
www.santilli-foundation.org/docs/Isoredshift-Letter.pdf

[19] Santilli, R. M. Generalization of Heisenberg's uncertainty principle for strong interactions, *Hadronic J.* **4**, 642-663 (1981),
www.santilli-foundation.org/docs/generalized-uncertainties-1981.pdf

[20] Santilli, R. M. Isorepresentation of the Lie-isotopic $SU(2)$ Algebra with Application to Nuclear Physics and Local Realism,*Acta Applicandae Mathematicae* **50**, 177-190 (1998), www.santilli-foundation.org/docs/Santilli-27.pdf

[21] Santilli, R. M.: Studies on the classical determinism predicted by A. Einstein, B. Podolsky and N. Rosen, *Ratio Mathematica* **37**, 5-23 (2019), www.eprdebates.org/docs/epr-paper-ii.pdf

[22] Santilli, R. M.: Lie-isotopic Lifting of Special Relativity for Extended Deformable Particles, *Lettere Nuovo Cimento* **37**, 545-555 (1983), www.santilli-foundation.org/docs/Santilli-50.pdf

[23] IAEA, Nuclear data services, website: https://www.iaea.org/about/organizational-structure/department-of-nuclear-sciences-and-applications/division-of-physical-and-chemical-sciences/nuclear-data-section

[24] Vonsovsk, S.: *Magnetism of Elementary Particles,* Mir Publishers (1975).

[25] Rau, S., et al.: Penning trap measurements of the deuteron and the HD^+ molecular ion, *Nature* **585**, 43-47 (2020), doi.org/10.1038/s41586-020-2628-7

[26] ScienceDirect, Helium nucleus, website, //www.sciencedirect.com/topics/mathematics/helium-nucleus

[27] Pohl, R.: Antognini, A. and Kottmann, F.: The size of the proton, *Nature* **466**, 213-216 (2010), www.nature.com/articles/nature09250

[28] Heisenberg, W.: *Nachr. Akad. Wiss. Gottingen* **IIa**, 111 (1953), https://link.springer.com/chapter/10.1007/978-3-642-70079-8_23

[29] Stanford Encyclopedia of Philosophy, Bohmian (de Broglie-Bohm) Mechanics (2021), https://plato.stanford.edu/entries/qm-bohm/

[30] Santilli, R. M.: *Foundation of Theoretical Mechanics,* Springer-Verlag, Heidelberg, Germany, Vol. I (1978) *The Inverse Problem in Newtonian Mechanics,* http://www.santilli-foundation.org/docs/Santilli-209.pdf

[31] Santilli, R. M.: *Foundation of Theoretical Mechanics,* Springer-Verlag, Heidelberg, Germany, Vol. II (1983) *Birkhoffian Generalization of Hamiltonian Mechanics,*
http://www.santilli-foundation.org/docs/santilli-69.pdf

[32] Santilli, R. M.:Elements of nuclear physics according to hadronic mechanics, II: Exact Lie-isotopic representation of the Deuteron data, submitted to *Ratio Mathematica*.

[33] Santilli, R. M.: Elements of nuclear physics according to hadronic mechanics, III: Exact Lie-isotopic representation of the nuclear stability, submitetd to *Ratio Mathematica*.

[34] Yukawa, H.: On the interaction of elementary particles, *Proc. Phys. Math. Soc. Jpn.* **17**, 48-57 (1935).

[35] Woods, R. D. and Saxon, D. S.: Diffuse Surface Optical Model for Nucleon-Nuclei Scattering, *Phys. Rev.* **95**, 577-578 (1954).

[36] Reid, R. V.: Local phenomenological nucleon–nucleon potentials, *Annals of Physics* **50**, 411–448 (1968).

[37] Rabi, I. I.: *Science: The Center of Culture,* World Publishing Co., New York. NY (1970).

[38] Ahmadov, A. I. et al.: Approximate bound state solutions of the Klein-Gordon equation with the linear combination of Hulthén and Yukawa potentials, *Physics Letters A* **383 (24)**, 3010-3017 (2019),
https://www.sciencedirect.com/science/article/abs/pii/S0375960119305791

[39] Christman, J. R.: *The Strong Interaction,* Project PHYSNET Physics Bldg. Michigan State University East Lansing (2001),
http://www.physnet.org/modules/pdf_modules/m280.pdf

[40] Fermi, E.: *Nuclear Physics,* University of Chicago Press (1949).

[41] Blatt, J. M. and Weisskopf, V. F.: *Theoretical Nuclear Physics,* Wiley and Sons (1952).

[42] Rutherford, H.: Bakerian Lecture: Nuclear Constitution of Atoms, *Proc. Roy. Soc. A,* **97**, 374 (1920),
royalsocietypublishing.org/doi/10.1098/rspa.1920.0040

[43] Santilli, R. M.: Need of subjecting to an experimental verification the validity within a hadron of Einstein special relativity and Pauli exclusion principle, *Hadronic J.* **1**, 574-901 (1978), http://www.santilli-foundation.org/docs/santilli-73.pdf

[44] Santilli, R. M.: Reduction of Matter in the Universe to Protons and Electrons via the Lie-isotopic Branch of Hadronic Mechanics, Progress in Physics, **19**, 73-99 (2023), https://www.ptep-online.com/2023/PP-65-09.PDF

[45] Norman, R. et al.: Experimental Confirmation of the Synthesis of Neutrons and Neutroids from a Hydrogen Gas, *American Journal of Modern Physics* **6**, 85-104 (2017), www.santilli-foundation.org/docs/confirmation-neutron-synthesis-2017.pdf

[46] Santilli, R. M.: Apparent Resolution of the Coulomb Barrier for Nuclear Fusions Via the Irreversible Lie-admissible Branch of Hadronic Mechanics, Progress in Physics, **18**, 138-163 (2022), http://www.santilli-foundation.org/hyperfusion-2022.pdf

[47] Santilli, R. M.: *Elements of Hadronic Mechanics*, Ukraine Academy of Sciences, Kiev (1995), Vol. I, *Mathematical Foundations,* http://www.santilli-foundation.org/docs/Santilli-300.pdf

[48] Santilli, R. M.: *Elements of Hadronic Mechanics*, Ukraine Academy of Sciences, Kiev (1995), Vol. II, *Theoretical Foundations,* www.santilli-foundation.org/docs/Santilli-301.pdf

[49] Santilli, R. M.: *Elements of Hadronic Mechanics*, Ukraine Academy of Sciences, Kiev (2016), Vol. III, *Experimental verifications,* www.santilli-foundation.org/docs/elements-hadronic-mechanics-iii.compressed.pdf

[50] Anderson, R.: *Collected OA Papers on Hadronic Mathematics, Mechanics and Chemistry,* https://www.santilli-foundation.org/docs/HMMC.pdf

[51] Anderson, R.: Outline of Hadronic Mathematics, Mechanics and Chemistry as Conceived by R. M. Santilli. *American Journal of Modern Physics*, 2016, v. 6, 1–16, www.santilli-foundation.org/docs/HMMC-2017.pdf

[52] Santilli, R. M. and Sobczyk, G.: Representation of nuclear magnetic moments via a Clifford algebra formulation of Bohm's hidden variables, *Scientific Reports* **12**, 1-10 (2022), www.santilli-foundation.org/Santilli-Sobczyk.pdf

[53] Santilli, R. M.: Lie-admissible invariant representation of irreversibility for matter and antimatter at the classical and operator levels, *Nuovo Cimento* **B121**, 443 - 485 (2006),
www.santilli-foundation.org/docs//Lie-admiss-NCB-I.pdf

[54] Santilli, R. M. and Vougiouklis, T.: A New Conception of Living Organisms and its Representation via Lie-Admissible H_v-Hyperstructures, *Algebras, Groups and Geometries* **37**, 741-764 (2020),
www.santilli-foundation.org/docs/Santilli-Vougiouklis-2020-epr.pdf