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Problematic aspects of generalized quantum theories and their apparent resolution via hadronic mechanics

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Abstract

We show that nonlinear and nonlocal completions of quantum mechanics on conventional spaces over conventional numeric fields are generally afflicted by a number of problematic aspects such as the general loss of observability, superposition principle and causality as well as the general inability to predict the same numeric values under the same conditions at different times. We show that said problematic aspects are resolved by hadronic mechanics in view of its embedding of all nonlinear and nonlocal interactions in the new operator $\hat{S} > 0$ of the axiom-preserving product $A \hat{\times} B = A\hat{S}B$ of the enveloping algebra, resulting in the representation of new potential-free interactions between extended particles, while reconstructing linearity, locality and potentiality on isospaces over numeric isofields. We prove, apparently for the first time, a theorem according to which hadronic mechanics admits as particular incomplete cases all possible nontrivial generalizations of quantum mechanics, by therefore unifying seemingly disparate experimental tests. In particular, we show that available experiments on the violation of Bell's inequalities, or on departures from quantum mechanical predictions, constitute direct experimental verifications of hadronic mechanics.

Keywords: quantum mechanics; nonunitary transforms; hadronic mechanics[1]

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1 Introduction

As it is well known, the 1935 historical argument by A. Einstein, B. Podolsky and N. Rosen that *"Quantum mechanics is not a complete theory"* [1] (see also related works [2]-[5]) has stimulated a large number of completions/generalizations of quantum mechanics as well as of its basic laws, such as: 1) Nonlinear theories [6]- [8]. 2) Nonlocal theories [9]-[10]. 3) Theories with nonpotential interactions [11] [12]. 4) Generalizations of Heisenberg's uncertainty principle [13]-[15]. 5) Generalizations of Bell's inequalities [16]-[18]. 6) q-deformations [19]-[21]. 7) Operator deformations [22]- [26]. 8) Squeezed states theories [27]. 9) String theories [28], and others.

Let us recall the majestic axiomatic structure of quantum mechanics for the conditions of its original conception and experimental verification (point-like particles in vacuum under electromagnetic interactions), such as the capability of predicting *the same numeric values under the same conditions at different times.* This important feature is due to the unitary character of the time evolution of a quantum mechanical observable A,

$$U(t)U^\dagger(t) = U^\dagger(t)U(t) = 1, \tag{1}$$

e.g. in the finite Heisenbergs representation

$$A(t) = e^{Hti}A(0)e^{-itH} = U(t)A(0)U^\dagger(t), \tag{2}$$

with ensuing verification of causality laws and preservation over time of the numeric value of the (multiplicative) unit of quantum mechanics

$$\hbar = 1 \ \rightarrow \ 1' = U1U^\dagger \equiv 1. \tag{3}$$

In turn, the preservation of the unit over time implies the preservation over time of the numeric field $\mathcal{R}$ in which measurements belong, and the preservation over time of Hermiticity/observability on a Hilbert space $\mathcal{H}$ with states ψ over the field of complex numbers $\mathcal{C}$

$$\langle\psi[U(t)HU^\dagger(t)]|\psi\rangle \equiv \langle\psi|[U(t)HU^\dagger(t)]^\dagger\psi\rangle \in \mathcal{R}. \tag{4}$$

resulting in unquestionable experimental verifications that have justly made quantum mechanics part of history.

In this paper we note that, nothwistanding their clear historical character, generalized theories 1-9 generally depart from the above axiomatic structure and, consequently, are afflicted by a number of problematic aspects, such as [29]-[36]:

1.1. Loss of unitary time evolution. A necessary and sufficient condition for a generalization of quantum mechanics to be nontrivial is to exit from the infinite class of unitary theories (1), which is possible if and only if the time evolution of the generalized theory is *nonunitary*

$$A(t) = W(t)A(0)W^\dagger(t), \quad W(t)W^\dagger(t) \neq 1, \tag{5}$$

with ensuing: i) lack of preservation over time of the basic unit

$$\hbar = 1 \ \rightarrow \ 1' = W(t)1W^\dagger(t) \neq 1, ; \tag{6}$$

ii) consequential loss over time of the numeric field $\mathcal{R}$; and iii) lack of consistent experimental verifications.

1.2 . General loss of Hermiticity/observability. Even though condition (7) holds at time $t = t_0$ for generalized theories 1-9,

$$\langle\psi[W(t_0)HW^\dagger(t_0)]|\psi\rangle \equiv \langle\psi|[W(t_0)HW^\dagger(t_0)]^\dagger\psi\rangle \in \mathcal{R}. \tag{7}$$

the same property cannot be consistently defined at a later time with ensuing loss of the necessary conditions for the validity of essential self-adjointness. Also, the time evolution of a number of generalized theories has the structure

$$A(t) = W(t)A(0)W(t)^{-1} \tag{8}$$

in which case Hermiticity condition (7) no longer holds, with consequential loss of observability at all times (see [36] for a detailed study).

1.3. General loss of the superposition principle, i.e., the wave function Ψ of a bound state of two particles with relative coordinate r

$$[\Sigma_{k=1,2}\frac{1}{2m}p_k^2 + V(r)]\Psi(r) = E\Psi(r), \tag{9}$$

cannot be generally reduced to the sum of the wave functions of the individual particle constituents

$$\Psi(r) \ \neq \ \Sigma_{k=1,2}\psi_k(r), \tag{10}$$

with consequential inability to characterize the constituents of generalized bound states.

1.4. General violation of causality laws, e.g., the admission of possible solutions in which the effect precedes the cause (see the violation of causality for half- odd-integer eigenvalues of the angular momentum in Ref. [37]).

1.5. General loss of associative character of the enveloping algebras, with consequential, inapplicability of Lie's theory loss of all space-time symmetries and ensuing lack of preservation over time of numeric predictions (see Refs. [31] [34]).

In this paper, we show the apparent resolution of insufficiencies 1.1-1.5 for theories constructed via the mathematical and physical laws of *hadronic mechanics* which was proposed in 1978 by R. M. Santilli [11] [12] at Harvard University under DOE support and developed by several scholars (see [38] for a classification of hadronic mechanics, [39]-[41] for a review, monographs [42] [43] [44] for a comprehensive treatment and Refs. [45]-[54] for independent studies).

In this paper we assume a knowledge of the preceding papers of this Special Issue, with particular reference to: the insufficiencies of quantum mechanics in nuclear physics [55]; Sect. 2 (axiomatic formulation of quantum mechanics), Sect. 3 (outline of isomathematics) and Sect. 4 (outline of isomechanics) of [56]; Sect. 4 (representation of nuclear stability) and Sect. 6 (recycling of nuclear waste) of [57].

2 Apparent resolution of axiomatic insufficiencies

2.1. The fundamental assumptions of hadronic mechanics.
The first fundamental condition by R. M. Santilli [11] [12] for the completion of quantum mechanics into hadronic mechanics is the *preservation of the associative character of the quantum mechanical enveloping algebra,*

$$\begin{aligned} \xi = \{A, B, ...; AB = A \times B; 1\}, \\ A \times (B \times C) = (A \times B) \times C, \end{aligned} \tag{11}$$

as the only known way to avoid the mathematical and physical inconsistencies of nonassociative envelopes (see Ref. [34] for a technical treatement).

Santilli's [11] [12]second fundamental condition is the use of an enveloping algebra characterized by a generalized, yet axiom-preserving (thus isotopic) associative product, called isoproduct (First introduced in Charts 5.2, 5.3, 5.4, p. 154 on of [12] as part of the step-by-step isotopies of Lie's theory)

$$\begin{aligned} \hat{\xi} = \{A, B, ...; A\hat{\times}B; \hat{1}\}, \\ A\hat{\times}B = A\hat{S}B, \ \ \hat{S} > 0, \\ A\hat{\times}(B\hat{\times}C) = (A\hat{\times}B)\hat{\times}C, \end{aligned} \tag{12}$$

where $\hat{S}$, called the Santillian (see, e.g., [3]), with ensuing *necessary* generalization of the quantum mechanical unit $\hbar = 1$ into the *isounit* of hadronic mechanics

$$\hbar = 1 \ \rightarrow \ \ = \hat{1} = 1/\hat{S} > 0, \tag{13}$$

and resulting *nonunitary* character of he generalized theory (Sect. 4 of [56]).

Santilli's [42] [43] third fundamental condition is the generalization of the entire 20-th century applied mathematics, with no known exceptions, into *isomathematics* (Sect. 3 of [56]) which is based on isoproduct (12) and related isounit (13), resulting in the novel isonumbers, isofunctions, Lie-isotopic theory, isospaces, isogeometries, isosymmetries, isorelativities, etc. The Lie-isotopic branch of hadronic mechanics is the axiom-preserving completion of quantum mechanics based on isomathematics (see Sect. 3 of [56] for a review and references).

The above third condition is important for the resolution of the mathematical and physical inconsistencies of nonunitary generalizations of quantum mechanics treated in Sect. 1. This important point can best be seen as follows. Note that the Santillian is positive-definite $\hat{S} > 0$ like the unit $1 > 0$. Therefore, at the abstract, realization-free level, the isoproduct $A \hat{\times} B$ coincides with the conventional product $A \times B$ and the isounit $\hat{I}$ coincides with the conventional unit 1. It then follows that the Lie-isotopic branch of hadronic mechanics [38] coincides at the abstract level with quantum mechanics to such an extent that they can be expressed with the same symbols, only subject to different realizations.

This occurrence guarantees that *hadronic mechanics preserves all properties of quantum mechanics* to such an extent that, to the authors best knowledge, any possible preservation by hadronic mechanics of inconsistencies 1.1-1.5 is due to insufficient knowledge or use of isomathematics.

The reader should keep in mind that Santilli's proposed "hadronic" mechanics is intended for the *time invariant* (hence the need for isosymmetries) representation of the numeric value of the generally ignored actual dimension, shape and density of nucleons (spheroids with $0.78\ fm$ radius), as well as for the representation of nuclei as generalized bound states of extended, thus deformable nucleons in conditions of partial mutual penetration [56] with conventional, action-at-a-distance, variationally selfadjoint (SA) interactions represented with the Hamiltonian H plus contact, zero-range, variationally nonselfadjoint (NSA) interactions represented with the Santillian $\hat{S}$ [11] [12], under the condition of recovering quantum mechanics identically, uniquely and unambiguously for all distances sufficiently bigger than the range of the strong interactions (about $1\ fm$) [42] [43].

Recall also that the Santillian is solely subjected to the condition of being positive-definite but otherwise possesses an unrestricted functional dependence for the representation of the indicated, generally ignored, nuclear structure, e.g., a dependence on: time t, relative coordinates r, relative momentum p, acceleration a, energy E, charge radius R, nucleon density d, pressure π, temperature τ, wave function $\psi(r)$, their derivatives $\partial_r \psi(r)$ and any other needed local quantity,

$$\hat{S} = 1/\hat{I} = \hat{S}(t, r, p, a, E, R, d, \pi, \tau, \psi(r), \partial_r \psi(r), ...) > 0, \tag{14}$$

with explicit realization for a hadronic two-body system given by Eq. (15) of Ref. [56], i.e.,

$$\hat{S} = \hat{S}_{4\times4} = \Pi_{\alpha=1,2,3,4} \begin{pmatrix} \frac{1}{n^2_{1,\alpha}} & 0 & 0 & 0 \\ 0 & \frac{1}{n^2_{2,\alpha}} & 0 & 0 \\ 0 & 0 & \frac{1}{n^2_{3,\alpha}} & 0 \\ 0 & 0 & 0 & \frac{1}{n^2_{4,\alpha}} \end{pmatrix} \times$$

$$\times e^{-\Gamma(r,p,a,R,d,\psi,\hat{\psi}(r),\partial_r\psi(r)...)} \ > \ 0, \tag{15}$$

$$n_{\mu,\alpha} > 0, \ \ \Gamma > 0,$$

where the exponential structure (which is crucial for the results of this paper) originates from the axiom-preserving restriction on the isodifferential (for spherical nucleons)[58]

$$\hat{d}\hat{r} = \hat{S}d(r\hat{1}) = \hat{S}d[r\hat{I}(r,...)] = dr - rd\Gamma(r,...). \tag{16}$$

We see in this way the emergence of the new term $-rd\Gamma$ responsible for the numerically exact and time invariant representation of nuclear data [55]-[57], which term is completely absent in quantum mechanics.

2.2. Isounitarity.

Recall that the *projection* of hadronic mechanics on conventional spaces over conventional fields can be easily constructed via a nonunitary transformation representing the size, shape, density of nucleons and their interactions

$$UU^\dagger = U1U^\dagger = \hat{1}, \tag{17}$$

under the condition of subjecting the *totality* of 20th century applied mathematics to said transformation, such as:

$$A \times B \ \rightarrow \ A'\hat{\times}B' = U(AB)U^\dagger =$$

$$= (UAB)(UU^\dagger)^{-1}(UBU^\dagger) = A'\hat{S}B', \tag{18}$$

$$\hat{S} = (UU^\dagger)^{-1} > 0.$$

To prevent a rather frequent misrepresentation, it should be stressed that Eqs. (17)-(18) *do not* belong to hadronic mechanics because they are not formulated according to isomathematics, as one can seen from the fact that all products are conventional. In fact, said equations belong to the class of *nonunitary generalizations of quantum mechanics* with ensuing physical inconsistencies identified in Sect. 1.

Most insidious for physical applications, is the lack of invariance of the Santillian under an additional nonunitary transformation $WW^\dagger \neq 1$,

$$W(A'\hat{S}B')W^\dagger = A\text{''}\hat{\times}\hat{S}'\hat{\times}B\text{''},$$
$$\hat{S}' = W\hat{S}W^\dagger \neq \hat{S}, \tag{19}$$

with consequential lack of invariance over time of nonpotential/NSA interactions.

The above insufficiency was resolved in the 1998 paper [59] with the reformulation of all possible nonunitary transformations in terms of the *isounitary transformations*

$$U = \hat{U}\hat{S}^{1/2},$$
$$UU^\dagger = \hat{U}\hat{\times}\hat{U}^\dagger = \hat{U}^\dagger\hat{\times}\hat{U} = \hat{1}, \tag{20}$$

under which we have indeed the isoinvariance of the isounit and of the Santillian, provided that all quantities are formulated on isospaces over isofields (Sect. 3.12 of [56])

$$\hat{I} \to \hat{U}\hat{\times}\hat{I}\hat{\times}\hat{U}^\dagger = \hat{I}' \equiv \hat{I},$$
$$\hat{A}\hat{\times}\hat{B} \to \hat{U}\hat{\times}(\hat{A}\hat{\times}\hat{B})\hat{\times}\hat{U}^\dagger = \hat{A}'\hat{S}'\hat{B}', \tag{21}$$
$$\hat{S}' \equiv \hat{S}.$$

The resolution of the mathematical and physical inconsistencies indicated in Sect. 1 crucially depends on the above isotopic reformulation of nonunitary theories.

The notion of isounitarity can be best expressed via the *iso-Heisenberg equation* (first achieved in p.153 of [12] and developed in Chap. 4, p. 127 of [43]) also called the *Heisenberg-Santilli isoequation,* here presented for the particular case with $\hat{t} = t,\ I_t = 1$ in the infinitesimal form

$$i\frac{d\hat{A}}{dt} = [\hat{A}\hat{,}\hat{H}] = \hat{A}\hat{\times}\hat{H} - \hat{H}\hat{\times}\hat{A} =$$
$$= \hat{A}\hat{S}(e, p, \psi, ...)\hat{H} - \hat{H}\hat{S}(r, p, \psi, ...)\hat{A}; \tag{22}$$

with integrated form showing its clean isounitary structure

$$\hat{A}(t) = e^{H\hat{S}ti}\hat{A}(0)e^{-it\hat{S}H} = \hat{e}^{Hti}\hat{\times}\hat{A}(t)\hat{\times}\hat{e}^{-itH} =$$
$$= \hat{U}(t)\hat{\times}\hat{A}(0)\hat{\times}\hat{U}^\dagger(t), \quad \hat{U}\hat{\times}\hat{U}^\dagger = \hat{U}^\dagger\hat{\times}\hat{U} = \hat{1}, \tag{23}$$

where we have used the isoexponential of an operator X characterized by the Poincaré-Birkhoff-Witt-Santilli isotheorem of the isoenvelope $\hat{\xi}$ (Chart 5.2, p. 154 on of [12])

$$\hat{e}^X = (e^{X\hat{S}})\hat{1} = \hat{1}(e^{\hat{S}X}). \tag{24}$$

Note that hadronic mechanics requires *two* operators for the representation of systems: the Hamiltonian $\hat{H}$ for the representation of Hamiltonian interactions and the Santillian $\hat{S}$ for the representation of non-Hamiltonian interactions;

2.3. Isolinearity.

Consider the Schrödinger-Santilli isoequation for a two-body hadronic bound state on the iso-Hilbert space $\hat{\mathcal{H}}$ with isostates $|\hat{\Psi}(\hat{r})\rangle$ over the isofield $\hat{\mathcal{C}}$ with relative Isocoordinates $\hat{r} = UrU^\dagger = r\hat{I}$ (first introduced in Eq. (15), p. 259 of [12]), developed in Chapter 5, p. 182 of [43] and reviewed in Sect. 4 of [56])

$$\begin{gathered} \hat{H}\hat{\times}|\hat{\Psi}(\hat{r})\rangle = \hat{H}\hat{S}(\hat{r},\hat{p},\hat{\psi},...)|\hat{\Psi}(\hat{r})\rangle = \\ = [\Sigma_{k=1,2}\frac{\hat{1}}{2m_k}\hat{\times}\hat{p}_k\hat{\times}\hat{p}_k + \hat{V}(\hat{r})]\hat{\times}|\hat{\Psi}(\hat{r})\rangle = \\ = \hat{E}\hat{\times}|\hat{\Psi}(\hat{r})\rangle = E|\hat{\Psi}(\hat{r})\rangle. \end{gathered} \tag{25}$$

Since all nonlinear terms are embedded in the Santillian $\hat{S}$ (or, equivalently, in the isounit $\hat{1}$), the above equation is clearly *linear* in the isowave function when formulated on isospaces over isofields, a property called *isolinearity* (Append. 4,C.1 and Definition 4.C.1, p. 166 of [43]), although highly nonlinear when *projected* on conventional spaces over conventional fields.

In a language accessible to nuclear physicists, we introduce the following important property:

THEOREM 2.3.1 [43]: Any regular, algebraic or partial differential equation which is nonlinear on the three-dimensional Euclidean space $E(r,\delta,1)$ over $\mathcal{R}$ always admits an identical isolinear reformulation on the isospace $\hat{E}(\hat{r},\hat{\delta},\hat{1})$ over $\hat{\mathcal{R}}$

In fact, for any given nonlinear equation, there always exists at least one Santillian $\hat{S}$ for the representation of all nonlinear terms with ensuing isotopy of the product (12) and/or or of the differential calculus (16) [58].

Th above setting has stimulated the following problem that has remained open to this day:

QUESTION 2.3.1 [43]: Is an elementary solution of an isolinear equation on an isospace over an isofield a solution of the nonlinear projection of said equation on a conventional space over a conventional field?

In the event the answer to the above question is positive, isotopies can provide new means to identify at least one solution of nonlinear equations.

2.4. Isolocality

In view of the abstract identity between quantum and hadronic formulations (beginning with the abstract identity between coordinates r and isocoordinates $\hat{r}$), it is evident that iso-Schrödinger equation (25) is *local* on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$ yet it is *nonlocal* when projected on $\mathcal{H}$ over $\mathcal{C}$, resulting in a property called *isolocality* (Append. 4.C.1 and Definition 4.C.1 p. 166 on of [43]). In fact, the isocoordinate $\hat{r}$ of an isolated nucleon is a *point* on an isospace over an isofield, yet its projection in an ordinary space over an ordinary numeric field represents the *volume* occupied by the charge distribution of the considered nucleon

$$\hat{r} = r\hat{1} = rDiag.(n_1^2, n_2^2, n_3^2, n_4^2). \tag{26}$$

Consequently, Santilli's isodifferential calculus [58] (see S. Georgiev's extended studies [54]) is a generalization of the Newton-Leibnitz differential calculus from points to volumes, as needed to represent *extended* particles.

When we have two nucleons under strong nuclear interactions, the Santillian (15) represents the experimentally established *overlapping* of the charge distribution of the two nucleons, resulting in the contact, zero range NSA interactions that are at the foundation of the exact representations of nuclear data [56] and stability [57].

Since the wave packet of a particle is identically null only at infinity, the volume characterized by Santillian (15) for the EPR entanglement of particles is generally infinite.

2.5. Isouncertainties.

As indicated earlier, a necessary condition for a generalization of quantum mechanics to be nontrivial is that its time evolution must be *nonunitary,* Eqs. (23). In turn, a nonunitary time evolution implies a necessary generalization of Heisenberg's uncertainty principle first identified by R. M. Santilli [60] in 1981 for strong interactions, extended in 1994 to black holes [61] and finalized in 2019 [62] (see also studies [3] [40] and the possible experimental verification [62]) resulting in the *isouncertainty principle* of hadronic mechanics, also called *Einstein's isode-*

terminism, which can be written for spherical particles

$$\begin{aligned}
\Delta r \Delta p &\approx \tfrac{1}{2}|U\{\langle\psi r|\,[r,p]\,|\psi(r)\rangle\}U^{\dagger}| = \\
&= \tfrac{1}{2}|\langle\hat{\psi}(\hat{r})|\hat{\times}\,[\hat{r,}\hat{p}]\,\hat{\times}|\hat{\psi}(\hat{r})\rangle| = \\
&= \tfrac{1}{2}|\langle\hat{\psi}(\hat{r})|\hat{S}\,[\hat{r,}\hat{p}]\,\hat{S}|\hat{\psi}(\hat{r})\rangle| = \\
&= \tfrac{1}{2}\hbar e^{-\Gamma(t,r,p,a,E,R,d,\pi,\tau,\psi(r),\partial_r\psi(r),...)} \approx \\
&\approx \tfrac{1}{2}\hbar[1-\Gamma(t,r,p,a,E,R,d,\pi,\tau,\psi(r),\partial_r\psi(r),...) \ll \tfrac{1}{2}\hbar,
\end{aligned} \tag{27}$$

As one can see, the above principle includes as particular cases all axiomatically incomplete generalizations of the uncertainty principle known to the authors (see Refs. [13]-[15] and papers quoted therein). The significance of reformulation (27) is given by the resolution of the inconsistencies identified in Sect. 1 plus the isoinvariance for the prediction of the same numeric values under the same conditions at different times.

As an illustration, Heisenberg's uncertainty principle prohibits a quantitative representation of the synthesis of the neutron from the proton and the electron in the core of stars, because the standard deviation Δr_e for the coordinate of the electron is much bigger than the size of the neutron and the standard deviation Δp_e of the momentum implies a kinetic energy of the electron bigger than the rest energy of the neutron [55],

$$\begin{aligned}
&\Delta r_e > R_n = 0.87 \times 10^{-13}\, cm, \\
&\Delta v_e > \tfrac{\hbar}{\Delta r_e \times m_e} > 10^{10}\, m/s, \\
&\Delta K_e = \tfrac{1}{2m_e} \times (\Delta p_e)^2 > m_n = 939.56\, MeV/c^2.
\end{aligned} \tag{28}$$

By comparison, studies of the neutron synthesis via hadronic mechanics under isouncertainty principle (27), imply standard isodeviations for which Eqs. (28) become [56] [62]

$$\begin{aligned}
&\hat{\Delta} r_e = \hat{S}\Delta r_e \le R_n = 0.87 \times 10^{-13}\, cm, \\
&\hat{\Delta} v = \hat{S}\Delta v_e \ll 10^{10}\, m/s, \\
&\hat{\Delta} K_e = \hat{S}\Delta K_e \ll m_n = 939.56\, MeV/c^2,
\end{aligned} \tag{29}$$

by therefore allowing advances that cannot be formulated via quantum mechanics, let alone treated, such as:

2.5.1) Numerically exact and time invariant representation of *all* Deuteron data in its true ground state, namely, that without orbital contributions [56].

2.5.2) Numerically exact and time in variant representations of *all* characteristics of the neutron in its synthesis from the hydrogen at the nonrelativistic and relativistic levels [64] [57] (see [65] for experimental verifications).

2.5.3) Apparent elimination of the Coulomb barrier opposing nuclear fusions via the synthesis of *negatively charged* pseudonuclei [66] .

2.5.4) The possible return to the continuous creation of matter in the universe to explain the $0.782\ MeV$ missing in the neutron synthesis [65].

2.5.5) Apparent confirmation of the original conception of all matter in the universe as being composed by protons and electrons [64] [57].

2.6. Isosuperposition

It is evident that the regaining of linearity (Sect. 2.3) implies the recovering of the superposition principle on isospaces over isofields. This property can be illustrated via the simple nonunitary transformation of linear equations (9)

$$\begin{gathered} U\{\Sigma_{k=1,2}\tfrac{1}{2m}p_k^2 + V(r)]\Psi(r)\}U^\dagger = \\ = U[E_1|\psi_1(r)\rangle + E_2|\psi_2(r)\rangle]U^\dagger = E_1|\hat{\psi}_1(\hat{r})\rangle + E_2|\hat{\psi}_2(\hat{r})\rangle, \end{gathered} \tag{30}$$

where one should note the preservation of the eigenvalues under isotopies since nonpotential/NSA forces carry no energy [56].

Isosuperposition principle (30) has permitted the characterization of the individual nuclear constituents under nonlinear internal interactions [56] [57].

3 Experimental verification of hadronic mechanics via the violations of Bell's inequality.

We are now sufficiently equipped to state, prove and illustrate the following:

THEOREM 3.1: Particle experiments measuring deviations from numerical predictions of quantum mechanics constitute experimental verifications of hadronic mechanics.

PROOF: The sole possibility of achieving theoretical predictions numerically different than those predicted by quantum mechanics is via nonunitary transformations of quantum predictions that, as such, are within the class of isounitary equivalence of hadronic mechanics. Q.E.D.

Let us illustrate the above theorem with the following representative examples:

3.1) Quantum electrodynamics predicts the following value for the giromagnetic factor (g-2) of the muon [67] [68]

$$g^{QED} = 2.00233183620, \tag{31}$$

which value is invariant under the infinite class of unitary equivalence of quantum mechanics (Sect. 1). Consequently, the anomalous experimental value of the g-2 factor [69]

$$g^{EXP} = 2.00233184122, \tag{32}$$

can solely be achieved by subjecting the derivation of the theoretical value g^{QED} to a step-by-step nonunitary transformation that, as such, constitutes experimental verification of hadronic mechanics [72] [71]. The expectation of additional deviation caused by the reversibility of quantum mechanics versus the irreversibility of spontaneous decays should be noted [70].

3.2) Under the use of the tabulated values of the magnetic moments of the proton and of the neutron in vacuum [67] [68]

$$\mu_p = +2.79285\,\mu_N, \quad \mu_n = -1.91304\,\mu_N, \tag{33}$$

(where μ_N represents the *nuclear magneton*) quantum mechanics (qm) predicts that the magnetic moment of the Deuteron is given by

$$\mu_D^{qm} = (2.79285 - 1.91304)\,\mu_N = 0.87981\mu_N, \tag{34}$$

which is about 3% off the experimentally measured value [67] [68]

$$\mu_D^{exp} = 0.85647\,\mu_N. \tag{35}$$

By remembering the invariance of quantum mechanical predictions under unitary transformations, the experimental value μ_D^{exp} can be solely achieved via nonunitary transformations of the quantum mechanical treatment that, as such, belong to hadronic mechanics (Sect. 2), as confirmed in Refs. [73] [74].

3,3) For various other cases, see the analysis of Refs. [44] [56] [57] and independent studies [45].

In 1964, J. S. Bell [75] published a theorem essentially stating that, under the assumption of quantum mechanics per axiomatic structure (12), (13), including the representation of the spin $1/2$ of particles via Pauli's matrices, and other assumptions, *a system of two point-like particles with spin* $1/2$ *does not admit a classical counterpart.* The theorem was proved by showing that a certain expression D^{Bell} (whose explicit value depends on the assumed conditions of the two particles) is always *smaller* than the corresponding classical value D^{Clas},

$$D^{Bell} < D^{Clas}, \tag{36}$$

for all possible values of D^{Bell}. Consequently, Bell's theorem confirms the impossibility for quantum mechanics to recover Einstein determinism [1].

In 1998, R. M. Santilli [76] published the value $\hat{D}^{Bell}$ for *two extended nucleons under strong interactions according to hadronic mechanics* via the *Pauli-Santilli iso-matrices* on an isospace $\mathcal{H}$ over the isofield $\mathcal{C}$ [40], resulting in the value (Eq. (5.8), p. 189 of [76])

$$\hat{D}^{Bell} = \frac{1}{2}(\lambda_1\lambda_2^{-1} + \lambda_1^{-1}\lambda_2)D^{Bell}, \tag{37}$$

where λ_1 and λ_2 are Bohm's hidden variables [77].

Experiments [78] [79] on the polarization of entangled photons have measured numeric values of a generalized Bell's inequality which violates inequality (36). Since the latter deviations can be solely achieved via nonunitary transformations of Bell's derivation, experiments [78] [79] constitute direct experimental verifications of hadronic mechanics, and the same holds for the additional experiments [80] [81] [82] whose papers quote the EPR argument in their titles (see Sect. 1 of [55] for a comprehensive list of experimental verifications of hadronic mechanics).

4 Concludig remarks

As shown in the preceding sections, all generalizations of quantum mechanics, thus including theories 1 to 9 of Sect. 1, must have a *nonunitary time evolution* as a necessary condition to be nontrivial. Consequently, all generalized theories imply isotopies (12) with the addition of nonlinear, nonlocal and nonpotential/NSA interactions represented by the Santillian $\hat{S}$. It then follows that all generalized theories 1 to 9 are incomplete particular cases of the infinite class of isounitary equivalence of hadronic mechanics.

The universality of hadronic mechanics can be more formally expressed by the following:

THEOREM 4.1: Hadronic mechanics is universal for the representation of all infinitely possible, regular, linear and nonlinear, local and nonlocal and potential as well as nonpotential interactions (universality) directly in the frame of the experimentalist (direct universality).

PROOF: Under the assumption that the Hamiltonian is conserved (closed systems, e.g., stable nuclei), the Hamilltonian H of the iso-Schrödinger equation (25) represents all infinitely possible linear, local and potential/SA interactions, while the Santillian $\hat{S}$ represents all infinitely possible nonlinear, nonlocal and nonpotential/NSA interactions. When the Hamiltoinian is not conserved (open *systems, e.g., nuclear fusions), the bimodular structure of Eqs. (25) is ordered to the right (left) to represent motion forward (backward) in time for the additional representation of irreversibility over time [83] [84]. Q.E.D.*

It is hoped that the unification of seemingly different generalized theories presented in this paper may remove the need of a number of experimental verifications for different generalized theories and their restriction to only one test, as it is the case for generalized uncertainties.

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