Lie-admissible irreversible biological entanglements and their apparent initiation of hadronic medicine

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Abstract

In a preceding paper, two of us (R. M. S. and Th. V.) initiated the mathematical representation of life intended as the difference between organic and inorganic molecules, via the Lie-admissible hyperstructural branch of hadronic mechanics representing the size of biological entities, their contact interactions and their irreversibility over time. In this paper, we review the time reversal Lie-isotopic branch of hadronic mechanics, as well as its Lieadmissible covering, by showing that 20th century reversible Lie theories can be extended to an irreversible form by adding symmetric Jordan algebra brackets to antisymmetric Lie brackets. We then introduce, apparently for the first time: a Lie and Jordan admissible axiomatic formulation of the two directions of time; the Lie and Jordan admissible irreversible formulation of biological entanglements; their lack of visible use of energy; and a smooth connection between hadronic uncertainties at small distances and full determinism at classical distances. We then show that biological entanglements can provide a quantitative representation of the behavior by biological entities beyond our sensory perception, with expected diagnostic and curative values suggesting the apparent initiation of the novel hadronic medicine. Keywords: quantum mechanics, hadronic biology, entanglements.

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1 Introduction

Inspired by the historical 1935 argument by A. Einstein, B. Podolsky and N. Rosen (EPR) that "quantum mechanics is not a complete theory" [1] (Fig. 1), in the preceding paper [2] two of us (R. M. Santilli and Th. Vougiouklis) initiated the mathematical representation of life, intended as the difference between organic and inorganic molecules, via the Lie-admissible hyperstructural branch of hadronic mechanics for:

1.1. A representation of the *actual size* of biological systems and their constituents, such as a collection of cells in condition of mutual contract, or equivalently, of deep entanglement (Fig. 2) with ensuing interactions that generally are *non-linear* (in the wave functions) first studied by W. Heisenberg [3], *non-local* (because extended over surfaces or volumes) first studied by L. de Broglie and D. Bohm [4] and *non-potential* (because of contact, thus zero range character) first studied by R. M. Santilli [5] via the conditions of variational self- adjointness and technically called *variationally non-self-adjoint* (NSA);

1.2. A representation of the irreversibility over time of biological structures via *Santilli Lie-admissible branch of hadronic mechanics* initiated in the 1967 paper [6] (see the 1968 paper [7] for a review of Lie-admissible algebras, and subsequent works [8]-[12]) formulated via *Vougiouklis* H_v hyperstructures [13]-[15];

1.3. The apparent multi-valued character of biological structures which appears to be necessary for attempting a representation of the extreme complexities of life (Fig. 3).

By using a language accessible to the general educated audience, in this paper: we review Einstein's criticisms of quantum entanglements [1] by showing that particles entangled at large mutual distances can only be represented by quantum mechanics as being free; we then review the time reversible Lie-isotopic particle entanglements of hadronic mechanics [16] and show their actual entanglement at large mutual distance due to non-linear, non-local and non-potential/NSA interactions in the overlapping of wave packets of the constituents; we review the implications of Lie-isotopic entanglements for the explicit and concrete representation of Bohm's *hidden variables* [17] with ensuing by-passing of Bell's inequalities [18] and progressive recovering of *Einstein's determinism* [19]-[24].

We then introduce, apparently for the first time: 1) An axiomatic formulation of the two directions of time via an isobimodular structure; 3) A smooth connection between hadronic uncertainties at very small distances and full determinism at classical distances. 3) The *Lie-admissible irreversible biological entanglements* verifying conditions 1.1, 1.2, 1.3, also called hadronic entanglements, and show



Figure 1: In support of the EPR argument [1] according to which "quantum mechanics cannot be a complete theory," we present nuclear fusions, chemical combustion and the beginning of life that cannot be consistently represented via quantum mechanics due to its strict reversibility over time compared to the known irreversibility of biological systems.

their lack of visible energy.

We then indicate that biological entanglements can provide a physical representation of the behavior of biological entities beyond our sensory perception, with expected diagnostic and curative values and apparent initiation of a new branch of medicine here submitted for the first time under the name of *hadronic medicine*.

We have made an effort to render this paper minimally self-sufficient, with the understanding that a deeper knowledge of the proposed hadronic medicine can be solely reached via the study of the literature in the field (see the originating works [5]-[16]; adjoining studies on the mathematics underlying this study [25] [26]; paper [27] on the classification of hadronic mechanics; [28] for a recent collection of papers; and independent monographs [29]-[36]).

2 Lie-admissible and Jordan-admissible biological entanglements

2.1. Particle entanglements.

According to experimental evidence dating back to the early part of the past century, particles that were initially bounded together and then separated, can continuously and instantaneously influence each other at a distance, (see, e.g., [37] [38] and papers quoted therein), and at the classical level, e.g., for pairs of classical oscillators [39].

As an example, consider a pair of electrons that were initially bonded together into the valence bond and then separated. The entanglement here considered represents the experimental evidence, that when one electron is subjected to a 180°

spin flip, the second electron experiences the same spin flip instantaneously at arbitrary distances without any human intervention or any visible interaction, and the same entanglement appears to occur at the classical level under certain conditions.

2.2. Quantum entanglements.

The experimental evidence on particle entanglements is generally *assumed* to be represented by quantum mechanics for which reason particle entanglements are widely called *quantum entanglement* (Fig. 3). However, Albert Einstein strongly criticized such an assumption because it would imply superluminal communications that violate special relativity.

In support of Einstein's view, Santilli [16] pointed out that, at sufficiently big mutual distances, quantum mechanics can only represent entangled particles as being free.

Recall that quantum mechanics can only represent interactions derivable from a potential V which is *additive* to the kinetic energy K in the Hamiltonian H = K + V, while *no* quantum mechanical potential is conceivably possibly to represent continuous and instantaneous interactions at arbitrary distance.

Consequently, the Schrödinger equation for two entangled particles with coordinates r_k , k = 1, 2 at a sufficiently big relative distance r on a Hilbert space \mathcal{H} over the field \mathcal{C} of complex numbers is given by (for $\hbar = 1$)

$$\Sigma_{k=1,2} \frac{1}{2m_k} p_k \times p_k \times |\psi(r_1) \times \psi(r_2)\rangle =$$

$$= [\Sigma_{k=1,2} \frac{1}{2m_k} (-i\frac{\partial}{\partial r_k}) \times (-i\frac{\partial}{\partial r_k})] |\psi(r_1) \times \psi(r_2)\rangle =$$
(1)
$$= E |\psi(r_1) \times \psi(r_2)\rangle$$

by, therefore, representing two free particles without any possible or otherwise visible interaction.

Note that, in the event quantum mechanical interactions are detectable at small and/or big distances, they cannot explain the instantaneous action at a distance, such as the indicated electron spin flip.

2.3. Lie-isotopic entanglements.

Recall that the wave packet of a particle fills up the entire universe because it is identically null only at infinity, even though it is measurable with current technologies only in the range of a few $fm = 10^{-13}$ cm. Consequently, Santilli [16] proposed that the entanglement of particles is due to the overlapping of their wave packets which is continuous, and instantaneous at arbitrary distance (Fig. 5), by therefore avoiding superluminal interactions.



Figure 2: We present examples of physical, chemical or biological systems illustrating their extended constituents in condition of partial mutual penetration, by therefore resulting in the most general known linear and non-linear, local and non-local and potential as well as non-potential interactions.

Additionally, the initial bond of the wave packets of particles causes their synchronicity whose conservation can explain the occurrence of actual physical events, such as a spin flip of a far away particle without any human intervention, without any visible interaction and without any visible use of energy (see Sect 2.5).

Furthermore, Santilli [16] recalled that the interaction caused by the overlapping of wave packets is non-linear, non-local and non-potential/NSA [5], and as such, can be quantitatively represented via the Lie-isotopic branch of hadronic mechanics according to the following main steps.

Recall that quantum mechanics is based on the universal enveloping associative algebra

$$U: \{X_i; X_j \times X_k; 1; i, j, k = 1, 2, ..., N; \}$$
(2)

of Hermitean operators X_i on a Hilbert space \mathcal{H} over a numeric field $F(n, \times, 1)$ with millenary associative product and related unit

$$X_i X_j = X_i \times X_j,$$

$$\times X_i = X_i \times 1 \equiv X_i \,\forall X_i \in U;$$
(3)

attached N-dimensional Lie algebra $L \equiv U^-$ with commutation rules

1

$$[X_i, X_j] = X_i \times X_j - X_J \times X_I = C_{ij}^k X_k;$$
(4)

Lie group of transformations with parameters $w \in F$ of an observable A(w)

$$A(w) = [\Pi_k e^{X_k w i}] A(0) [\Pi_k e^{-iwX_k}];$$
(5)

and related *Lie symmetries and conservation laws*, e.g., for the invariance of a Hamiltonian $X_1 = H(r, p)$ under time evolution with w = t that, for infinitesimal



Figure 3: We illustrate the complexity of biological structures by noting that the correlation of two atoms in a DNS, which is mathematically represented by the multiplication can produce an entire organ with billions of atoms, thus suggesting the use of multi-valued hyperstructures [13]-[15] to initiate their quantitative representation.

values of time, implies Heisenberg's equation for an operator A(t)

$$i\frac{dA}{dt} = [A, H] = A \times H - H \times A,$$
(6)

with ensuing conservation law of the energy

$$i\frac{dH}{dt} = [H, H] = H \times H - H \times H \equiv 0.$$
⁽⁷⁾

Lie-isptopic branch of hadronic mechanics, or *isomechanics* for short, is based on the *universal enveloping isoassociative algebra* of the same Hermitean generators X_i (because they are observable) first introduced in the 1983 monograph [8]

$$\hat{U}: \{X_i; X_j \times X_k; \ \hat{1}; \ i, j, k = 1, 2, ..., N; \}$$
(8)

characterized by the new product

$$X_i \hat{\times} X_j = X_i S X_j, \quad S > 0, \tag{9}$$

called *isoproduct* because verifying the axiom of associativity

$$X_i \hat{\times} (X_j \hat{\times} X_K) = (x_i \hat{\times} X_j) \hat{\times} X, \tag{10}$$

and related isounit

$$\hat{1} = 1/S,$$

$$\hat{1} \times X_i = X_i \times 1 \equiv X_i \ \forall X_i \in \hat{U},$$
(11)



Figure 4: We illustrate the experimental evidence clearly extendable to chemical structures and biological entities according to which two particles or wave packets that were initially bonded together and then separated, can continuously and instantaneously influence each other at arbitrary distances without any human intervention, without any visible energy source.

where X_iS and SX_j are conventional associative products and the quantity S, nowadays called the Santillians [24] [40] [23], is solely restricted by the conditions of being dimensionless and positive-definite while admitting an arbitrary dependence on time t, relative coordinates r, momenta p, density d, wave functions ψ , and any other needed local quantities $S = S(t, r, p, d, \psi, ...) > 0$, with realization for the non-relativistic entanglement of two particles in Euclidean space [16]

$$S = 1/\hat{1} = \Pi_{\alpha=1,2} Diag.(a_{1,\alpha}^2, b_{2,\alpha}^2, c_{3,\alpha}^2) \times d_{\alpha}^2 \times e^{-\Gamma},$$

(12)
$$a, b, c, d > 0, \ \Gamma > 0, , \ k = 1, 2, 3, \ \alpha = 1, 2,$$

where: the time-independent parameters $a_{k,\alpha}^2$, $b_{k,\alpha}^2$, $c_{k,\alpha}^2$ represent the two volumes of wave overlapping with a radius generally equal to the relative distance r; the time independent parameters d_{α}^2 represents the density of the individual wave packets; and Γ represents their non-linear, non- local and non-potential/NSA interactions.

Despite their simplicity, generalized basic assumptions (8)-(11) of isomechanics imply a corresponding generalization of the totality of 20th century applied mathematics into the new *isomathematics* [9] [30] [33] [34] [36] that cannot possibly be reviewed here. We merely limit ourself to recall that isoenvelope (8) is formulated on a Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ [41] with isostate $|\hat{\psi}(\hat{r})\rangle$ over the isofield of isocomplex numbers $\hat{\mathcal{C}}$, $\hat{c} = c\hat{1}$, $c \in C$ [42] with isocoordinates $\hat{r} = r\hat{1}$ and isonormalization

$$\langle \hat{\psi}(\hat{r}) | \hat{\times} | \hat{\psi} \rangle = \langle \hat{\psi}(\hat{r}) | \hat{\times} | \hat{\psi} \rangle = S.$$
(13)

Under the correct treatment via isomathematics, isoenvelope (8) uniquely and

unambiguously characterize: 1) The Lie-Santilli isoalgebra [8]

$$[X_i, X_j] = X_i \hat{\times} X_j - X_J \hat{\times} X_I = \hat{C}_{ij}^k \hat{\times} X_k;$$
(14)

2) The Lie-Santilli group of isotransformations with isoparameters $\hat{w} \in \hat{C}$ of an observable A(w)

$$A(w) = [\Pi_k \hat{e}^{X_k wi}] \hat{\times} A(0) \hat{\times} [\Pi_k \hat{e}^{-iwX_k}] =$$

=
$$[\Pi_k e^{X_k Swi}] A(0) [\Pi_k e^{-iwSX_k}],$$
 (15)

where we have used the isoexponent $\hat{e}^X = [e^{XS}|\hat{1} = \hat{1}[e^{SX}]$ [9]; 3) Lie-Santilli isosymmetries and conserved quantities generally given by the N generators X_k [8] [30]; 4) The Heisenberg-Santilli isoevolution of an observable A(t)

$$i\frac{dA}{dt} = [A,\hat{H}] = A\hat{\times}H - H\hat{\times}A,$$
(16)

with related conservation of the energy for $X_1 = H$, $i\frac{dH}{dt} = [H, H] \equiv 0$; The Schrödinger- Santilli isoeigenvalue equations

$$H\hat{|}\hat{\psi}\rangle = HS\hat{|}\hat{\psi}\rangle = \hat{E}\hat{\times}\hat{|}\hat{\psi}\rangle = E\hat{|}\hat{\psi}\rangle; \tag{17}$$

and the remaining features of the Lie-isotopic branch of hadronic mechanics [10].

By recalling the expression of the *iso-linear iso-momentum* characterized by the completion of the local Newton-Leibnitz differential calculus into the non-local *iso-differential calculus* [43] (see also [36] for independent mathematical studies)

$$\hat{p} \times \hat{\psi}(\hat{r}) = \hat{p} \hat{S}(\hat{r}, ...) \psi(\hat{r}) = -i \frac{\hat{\partial}}{\hat{\partial}_{\hat{r}}} \hat{\psi}(\hat{r}) = -i \hat{1}(\hat{r}, ...) \frac{\partial}{\partial_{\hat{r}}} \hat{\psi}(\hat{r}),$$
(18)

the non-relativistic version of particle entanglements is characterized by the Schrödinger-Santilli isoequation

$$\begin{aligned} H\hat{|}\hat{\psi}(\hat{r})\rangle &= HS|\hat{\psi}(\hat{r})\rangle = (\Sigma_{k=1,2}\frac{1}{2m_k}\hat{p}_k\hat{\times}\hat{p}_k\hat{\times}|\hat{\psi}(\hat{r})\rangle = \\ &= [\Sigma_{k=1,2}\frac{1}{2m_k}(-i\hat{1}\frac{\partial}{\partial\hat{r}_k})(-i\hat{1}\frac{\partial}{\partial\hat{r}_k})]|\hat{\psi}(\hat{r})\rangle = \\ &= \{\Sigma_{k=1,2}[(-\frac{\hat{1}^2}{2m_k}(\frac{\partial}{\partial\hat{r}_k})(\frac{\partial}{\partial\hat{r}_k}) - \frac{\hat{1}}{2m_k}(\frac{\partial\hat{1}}{\partial\hat{r}_k})(\frac{\partial}{\partial\hat{r}_k})]\}||\hat{\psi}(\hat{r})\rangle = \\ &= \{\Sigma_{k=1,2}[(-\frac{\hat{1}^2}{2m_k}(\frac{\partial}{\partial\hat{r}_k})(\frac{\partial}{\partial\hat{r}_k}) - \frac{\Gamma}{2m_k}(\frac{\partial\Gamma}{\partial\hat{r}_k})(\frac{\partial}{\partial\hat{r}_k})]\}|\hat{\psi}(\hat{r})\rangle = \\ &= \hat{E}\hat{\times}|\hat{\psi}(\hat{r})\rangle = E|\hat{\psi}(\hat{r})\rangle = E|\hat{\psi}(\hat{r}_1)\rangle\hat{\times}|\hat{\psi}(\hat{r}_2)\rangle, \end{aligned}$$



Figure 5: We illustrate the central notion of this paper called "hadronic entanglement" [16] according to which the entanglement of particles of Fig. 4 is due to the overlapping of their wave packets, thus being a continuous and instantaneous physical event at arbitrary distances equally holding for physical, chemical and biological structures.

which identifies the interaction term $-\frac{\Gamma}{2m_k}(\frac{\partial\Gamma}{\partial\hat{r}_k})(\frac{\partial}{\partial\hat{r}_k})$ additive to the kinetic term for two isoentangled particles, which term is completely absent for quantum entanglement (1).

Recall that the primary aim of this paper is to indicate the apparent existence of the entanglement in biological structures which evidently requires a smooth transition from operator settings at particle distances to full determinism at classical distances. This transition has been studied via the following steps:

2.3-I: Bohm's "hidden variables". In 1952, D. Bohm [17] introduced the hypothesis that a number of variables λ may be hidden in the formalism of quantum mechanics in such a way to recover Einstein's determinism [1]. Hadronic mechanics has been constructed for, and has indeed achieved an explicit and concrete realization of Bohm's hidden variables via their identification with the Santillian as being hidden in associativity axiom (10) with new realization

$$X_i \hat{\times} X_j = X_i S X_j = X_i \lambda X_j,$$

$$X_i \lambda (X_j \lambda X_k) = (X_i \lambda X_j) \lambda X_k,$$
(20)

by therefore offering a realistic possibility of weakening Heisenberg's uncertainty principle in the transition from its original conception for isolated point-like atomic electrons in vacuum to complex physical, chemical and biological systems with extended constituents under action-at-a-distance potential as well as contact non-potential interactions.

Note that Bohm's hidden variables are represented under the full validity of the abstract axioms of quantum mechanics, only subjected to a broader realization. In fact, at the abstract, realization-free level, quantum and hadronic mechanics coincide to such an extent that they can be expressed via the same equations only subjected to a broader realization of the associative product.

2.3-II. Bell's inequalities. In 1964, J. S. Bell [18] introduced a rigorously proved mathematical theorem according to which a system of quantum mechanical particles with spin 1/2 does not admit a classical counterpart, by therefore prohibiting the admission of Bohm's hidden variables and any possible reconciliation with Einstein's determinism. The theorem was proved by showing that a certain quantity D^{Bell} representing all possible conditions of the system of spin 1/2 particles is always *smaller* than the corresponding classical value D^{Clas} ,

$$D^{Bell} < D^{Clas}.$$
 (21)

In 1998, R. M. Santilli [44] proved a theorem according to which the derivation of Bell's inequality (21) via hadronic mechanics (hm) for *extended* particles with spin 1/2 in deep mutual entanglement, hence under both Hamiltonian and Santillian interactions, implies the re-definition of Bell's quantity (Eq. (5.8), p. 189 of [44])

$$D^{HM} = \frac{1}{2} (\lambda_1 \lambda_2^{-1} + \lambda_1^{-1} \lambda_2) D^{Bell},$$
(22)

which can be equal to the corresponding classical quantity D^{Cl} , by therefore confirming Einstein's view on the possible recovering of classical determinism at small distances.

2.3-III. Einstein's isodeterminism. In 1981, R. M. Santilli [19] submitted the hypothesis that protons and neutrons under the extremely attractive strong nuclear forces should have standard deviations Δr and Δp smaller than those of atomic electrons in vacuum (Fig. 6). In 1994, R. M. Santilli [20] proved that said standard deviations are identically null, with ensuing classical determinism, for particles in the interior of black holes.

Following the achievement of maturity of the novel isomathematics, R. M. Santilli [21] (see the review in [22]) finally introduced in 2019 the *Lie-isotopic completion of Heisenberg's uncertainty principle for strong interactions*, nowa-days fully applicable to entangled wave packets (Eq. (35), p. 14 of [21])

$$\Delta r \Delta p \approx \frac{1}{2} |\langle \hat{\psi}(\hat{r}) | \hat{\times}[\hat{r},\hat{p}] \hat{\times} | \hat{\psi}(\hat{r}) \rangle | \ll \frac{1}{2} S = \frac{\lambda}{2} \ll 1.$$
(23)

By recalling that the energy, and therefore the density d_{α} of wave packets decreases in a way inversely proportional to the square of the distance, the above principle establishes that the standard deviations Δr and Δp progressively recover Einstein's classical determinism [1] with the increase of the distance, i.e, with the decrease of the densities d_{α} of the wave packets, and said deviations are null, $\Delta r = m\Delta p = 0$ for null densities $d_{\alpha} = 0$ in Eqs. (12) and (23). We reach in this way the needed smooth reconciliation of microscopic hadronic entanglements at very small mutual distances with macroscopic deterministic biological



Figure 6: We here illustrate the main argument of the 1981 paper [19] under DOE support according to which protons and neutrons under extremely attractive strong nuclear forces cannot possibly have the same Heisenberg's uncertainties of atomic electrons in vacuum under long range electromagnetic interactions.

entanglements at classical distances (see Ref. [39] and papers quoted therein for experimental evidence of classical, deterministic, mechanical entanglements).

2.4. Lie and Jordan-admissible entanglements.

The main axiomatic assumption of Lie-isotopic entanglements is the systematic use, for all possible products, of the isoproduct $X_i \times X_j = X_i S X_j$ which is *invertible* in the sense that the isoproduct to the right of X_i to X_j , here denoted $X_i > X_j = X_i S X_j$ is identical to the isoproduct to the left of X_j to X_i , here denoted $X_i < X_j = X_i S X_j$ resulting in the axiom of product invertibility $X_i < X_j \equiv X_i > X_j$.

Upon due development of the resulting new isomathematics and related methods, the Lie-isotopic entanglement allowed: the first known numerically exact and time invariant representation of; the experimental data for stable nuclei [45]-[47]; the first known attraction between identical electrons in a valence bond despite their extremely big Coulomb repulsion [11]; the first known exact representation of the experimental data of the Hydrogen [48] and water [49] molecule; and other exact representations that had been impossible to achieve via quantum mechanics in one century of research.

Despite the indicated advances, Lie-isotopic entanglements and related methods are *inapplicable* to biological structures because of the invariance of Lieisotopic formulations under time reversal (as shown, e.g., by the invariance of Heisenberg-Santilli isoequation (16) under anti-Hermiticity), compared to the strict time irreversibility of biological entities.

The first axiomatic formulation of irreversibility over time was reached in 1981 by R. M. Santilli [50], then at the Department of Mathematics of Harvard University, via:

1) The ordering of all isoproducts to the right $X_i > X_j$ in representation of motion forward in time;

2) The ordering of all isoproducts to the left $X_i < X_j = X_i R X_j$ in representation of motion backward in time;

3) The assumption that the isoproducts are characterized by two different, time dependent, forward and backward Santillians to assure irreversibility $R \neq S$.

By noting that the ordered isoproducts $X_i > X_j$ and $X_i < X_j$ are individually associative, the above axioms imply the decomposition of the universal enveloping isoassociatve algebra (8) of Lie-isotopic formulations into the Kronecker product of the forward and backward isoenvelopes over the corresponding numeric genofields ${}^{<}C \times C^{>}$ [42]

$$\hat{U} = {}^{<} U \times U^{>},$$

$$U^{>} : \{X_{i}; X_{i} > X_{j} = X_{i}SX_{j}; 1^{>}; i, j, k = 1, 2, ..., N\}$$

$${}^{<}U : \{X_{i}; X_{i} < X_{j} = X_{i}RX_{j}; 1^{>}; i, j, k = 1, 2, ..., N\}$$

$$1^{>} = 1/S \neq {}^{<} 1 = 1/R,$$
(24)

resulting in the *Lie-admissible and Jordan-admissible mathematics*, also called *genomathematics* [9] [7] [51] [52].

The jointly Lie and Jordan admissible group of transformations of an observable A(w) is given by

$$A(w) = [\Pi_k \hat{e}^{X_k w i,>}] > A(0) < [\Pi_k e^{-iwX_k,<}] =$$

=
$$[\Pi_k e^{X_k R w i}] A(0) [\Pi_k e^{-iwSX_k}],$$
 (25)

where we have used the genoexponeniations $e^{X,>} = [e^{XR}]1^>$, $e^{X,<} = 1[e^{SX}]$ [9], resulting, for small values of the parameter, in the *Heisenberg-Santilli Lie-admissible and Jordan-admissible time evolution* for an observable A(t) (first introduced in the 1967 paper [6], see the 1968 paper [7] for the definition of a jointly Lie-admissible and Jordan-admissible algebra and monographs [8] [10] [12] for subsequent works)

$$i\frac{dA}{dt} = (A, H) = A < H - H > A = ARH - HSA =$$

= (ATH - HTA) + (AJH + HJA) = [A,H] + {A,H}, (26)
$$R = T + J, \quad S = T - H,$$

where the totally antisymmetric brackets [A,H] verify the Lie algebra axioms while the totally symmetric brackets $\{A,H\}$ verify the axioms of Jordan algebras, by therefore illustrating the joint Lie and Jordan-admissible character of the main time evolution of hadronic mechanics [6] [7] [8] [10].

In fact, time evolution (26) shows on algebraic grounds that reversible 20th century Lie theories can be extended to represent irreversibility by adding Jordan brackets to Lie brackets in a way reminiscent of supersymmetric theories [61] [62].

On analytic grounds, Santilli's Lie-admissible time evolution (26) shows that irreversibility originates from the forgotten external terms F of Lagrange's and Hamilton's analytic equations, as illustrated by the realization generally used in applications

$$R = 1, \quad S = 1 - F/H,$$

$$i\frac{dA}{dt} = (A, H) = ARH - HSA = [A, H] + AF,$$
(27)

which is the operator counterpart, first proposed in paper [55], of the classical, Lieadmissible time evolution of an observable for Hamilton's equations with external terms, whose study was initiated in the 1978 Harvard's memoir [51] and studied in details via Birkhoff-admissible equations in monograph [8] and subsequent works on irreversible processes.

Alternatively, we can say that Jordan algebras provide an operator representation of Lagrange's and Hamilton's *non–conservative* external terms, as illustrated by the *time rate of variation of the energy* of Eqs. (26) for R = 1, S = 1 - F/H

$$i\frac{dH}{dt} = (H,H) = H(R-S)H = HF,$$
(28)

by therefore fulfilling Jordan's dream that his symmetric algebras may, one day, find applications in physics.

We finally mention the Schrödinger- Santilli genoequations

$$\langle {}^{<}\psi| < H = \langle {}^{<}\psi|RH = \langle {}^{<}\psi|{}^{<}E,$$

$$H > |\psi^{>}\rangle = HS|\psi^{>}\rangle = E^{>}|\psi^{>}\rangle,$$
(29)

in which irreversibility is assured for ${}^{<}E \neq E^{>}$.

All remaining aspects of Lie-isotopic formulations, including the representation of the non-potential origin of entanglements (19), are valid for Lie- and Jordan-admissible entanglements when all their isoproducts are restricted to the right. We shall therefore define our central notion of *organic entanglements*, also called *hadronic entanglements*, as being the extension of Lie-isotopic entanglements (19) with all products being ordered to the right and the forward Santillian being explicitly dependent on time.

Let us note that, since the abstract algebra B with elements a, b, ... and product $a \star b = aRb - bSa$ is Lie-admissible, i.e., its attached antisymmetric algebra B^-

with product $[a, b] = a \star b - b \star a$ is Lie, it seems natural to assume the algebra B as the universal enveloping *non-associative* algebra of irreversible entanglements.

It is important to note that the 1981 paper [50] selected instead for the axiomatization of time irreversibility the universal enveloping algebra ${}^{<}U \times U^{>}$ in which each branch is isoassociative. This is due to the fact that non-associative enveloping algebras have serious mathematical and physical shortcomings, such as the impossibility to formulate the Poincaré-Birkoff-Witt theorem for a nonassociative envelope with consequential impossibility of defining a unique and unambiguous transition from algebras to groups, consequential loss of symmetries and relativities and other shortcomings [63] [64].

2.5. No visible energy in entanglements.

The most mysterious, and therefore, most intriguing aspect of physical, chemical or biological entanglements is that their action occurs without any visible use of energy. We are referring to the case of two electrons of Sect. 2.1 that were initially bonded together and then separated. When the first electron is subjected to a 180 degree spin flit (following the supply of the needed energy), the second far away electron experiences an instantaneous 180 degree spin flip without any visible use of energy.

This intriguing feature was first identified in Sect. 5, p. 819 on of the 1978 memoir [65], studied in detail specifically for valence electrons in Chap. 4, p. 168 on of monograph [11], upgraded in the 2020 paper [66] and can be outlined as follows.

The electron pair valence bond with relative coordinate r can be represented in quantum mechanics by the Schrödinger equation (for $\hbar = 1$)

$$\left[\frac{1}{m_e}p \times p + \frac{e^2}{r}\right]|\psi(r)\rangle = E|\psi(r)\rangle, \tag{30}$$

where the positive sign + of the Coulomb potential $V = +\frac{e^2}{r}$ represents *repulsion*, thus being in gross disagreement with the visual, let alone experimental evidence on the *attraction* in valence pair bonds.

Hadronic mechanics has achieved the numerically exact and time invariant representation of *all* characteristics of valence electron bonds, resulting in a hadronic state called *isoelectronium* [11], via the following simple Santillian

$$S = e^{-V_h/V_c},\tag{31}$$

exhibiting the Hulten potential (because strongly attractive at short distances)

$$V_h = W \frac{e^{-br}}{(1 - e^{-br})},$$
(32)

where $b^{-1} = 10^{-11} cm$ represents the charge radius of the isoelectronium.

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Under value (31), the Schrödinger-Santilli isoequation of the isoelectronium is given by

$$\begin{aligned} &\left[\frac{1}{m_e}\hat{p}\hat{\times}\hat{p} + V_c\right](1 - \frac{V_h}{V_c})|\hat{\psi}(\hat{r})\rangle = \\ &= \left[\frac{1}{m_e}\hat{p}\hat{\times}\hat{p} + V_c - V_h\right]|\hat{\psi}(\hat{r})\rangle = E|\hat{\psi}(\hat{r})\rangle. \end{aligned}$$
(33)

Recall that the Hulten potential behaves like the Coulomb potential at small distances, by therefore absorbing the latter irrespective of whether it is attractive or repulsive due to its strength. Therefore, Eq. (33) is well approximated by the reduced form

$$\left[\frac{1}{m_e}\hat{p}\hat{\times}\hat{p} - V_h\right]|\hat{\psi}(\hat{r})\rangle = E|\hat{\psi}(\hat{r})\rangle.$$
(34)

whose energy spectrum was derived in details in Refs. [?] [11] [66], resulting in the sole value for n = 1 (see, e.g., Eqs. (4.15) and (4.17), p. 171 of [11])

$$E = K(\frac{1}{n} - n)^2, = 0, \quad n = 1,$$
(35)

(where K is a positive isorenormalization factor) confirming that, contact nonpotential/NSA interactions caused by the overlapping of wave packets provide no contribution to binding energies of entangled particles, as expected from their very definition [5], and all binding energies are entirely characterized by conventional potential interactions.

This important property has been confirmed by all applications of hadronic mechanics to bound states of extended particles with potential and non-potential interactions, such as electron valence bonds [11], stable nuclei [45]-[47], structure models of elementary particles with detectable constituents [66] and other models. In all cases, the deep entanglement of the wave packet of particles in singled (triplet) coupling resulted to be strongly attractive (repulsive).

In Sect. 2.3 we indicated that a possible mechanism of particle entanglements may be the tendency by synchronized frequencies of wave packets overlapping to remain synchronized, with the understanding that said mechanism and the origin of the related energy are basically unknown at this writing.

Despite that, the lack of visible energy by entanglements may have significant, yet unexplored applications, such as the preference of the *EPR computers* based on entanglements [16] over quantum computers that do not represent entanglements (Sect.2.2), a possible explanation of extremely long bird flights without supplementary food, and other possibilities here left to the interested colleague.

We should indicate that a deeper understanding of the transition from a conventional Coulomb repulsive behavior of electron pairs at large relative distances to their strongly attractive behavior under condition of essentially total entanglement of their wave packets, may imply a generalization of the 20th century notion of particle (as a unitary irreducible representation of the spinorial covering of the Poincaré symmetry) to the *isoparticles* of hadronic mechanics to characterize an expected mutation of the very structure of the elementary charge [67].

3 Apparent examples of hadronic entanglements

3.1. Foreword.

In this section we review applications in biology of quantum entanglements and reexamine them in terms of hadronic entanglements in view of the insufficiencies of the former reviewed in Sect. 2.2. We then present apparent new application in biology of hadronic entanglements.

3.2. Entanglement in quantum biology.

The ability to sense magnetic fields is common to many animals. This ability can be very useful in nature, especially in migratory species. During research on the perception of magnetic field by robin (Erithacus rubecola), the Wiltscho couple discovered that these birds do not perceive the direction of the magnetic field but are aware of its inclination [68]. This opened up several questions about the mechanisms of their sense for magnetic fields, since it behaved in a clearly different way from a normal compass.

Entanglement is currently considered to play a crucial role in the functioning of the robin's sense. According to the hypothesis put forward by Thorsten Ritz and Klaus Schulten, the detection of the magnetic field is carried out thanks to the cryptochromes, proteins present in the eye.

The cryptochrome contains a molecule of FAD with a pair of electrons in entangled state. The hypothesis predicts that when the cryptochrome is hit by photons, loses an entangled electron, and generates a pair of radicals in the state of triplet or singlet. The probability of generating the triplet or singlet has a value dependent on the inclination of the magnetic field. The quantities of radicals in the two different states would then influence the successive reactions, allowing according to mechanisms still unknown, the detection of the inclination of the magnetic field by the robin [69].

3.3. Mother-child information transfer.

Quantum correlation unites all the molecules of a living organism besides them being distant components of a living being since all organisms come from a single cell in the first stage of their formation.

Extending the reasoning, entanglement should relate not only all parts of a living organism but also its descendants.

Since there are cases of transmission of information from mother to child

through unknown pathways, we can consider the possibility that entanglement may play a role in the exchange of information between biological organisms.

The case of a mother who has had a blind child is highlighted. Even if the child cannot see, an experiment has shown that he can acquire visual information from the mother.

The experiment consisted simply in showing letters of the English alphabet or numbers to the two subjects, in two laboratories about 10 km away to isolate the two individuals from any form of communication. The son obviously could not see them and if he tried to guess he could rarely give the right answer, in line with what you would expect from statistical calculations (about 1 out of 26). However, the boy could guess the symbols that were shown to his mother with a greater precision than one would expect from pure chance.

The boy hit 20 numbers out of 58 on the first try and 19 out of 58 on the second. Statistically very significant values, having the boy guessed 34.3% and 32.7% of the times against a theoretical probability of 10%.

During the latter test, the result was even more astonishing: on two attempts with 45 letters, the boy guessed 17 and 12 times (37.8% and 26.2% respectively). A similar result obtained randomly occurs only 1 time out of 10^6 [70].

This case suggests that in a situation of acute need, together with the presence of emotional bond and desire for mutual help, there may be transmission of information, albeit not 100% accurate, from mother to child by apparently telepathic means. Rather than invoking supernatural causes, we suggest entanglement as a possible explanation.

3.4. Correlation between mothers and children during the lactation period.

Another phenomenon that suggests a correlation between mothers and children is lactation. Kary Barber's research suggests that breastfeeding stimulation is influenced by information transfer between mother and child, although again communication is not 100% accurate.

The study involved 19 women who were responsible for keeping a diary of the times when milk was lost, especially during periods when they were away from their baby.

At the same time, those who took care of the child wrote in a second diary the periods when the infant showed signs of discomfort, hunger, or crying.

The results of 10 women were excluded from the experiment because they did not report the data correctly or they never left the baby long enough.

In the remaining nine participants, a correlation was found between child discomfort symptoms and milk loss, which manifested at very close intervals despite the distance that prevented mothers from becoming aware of their child's status through known sensory channels. Of a total of 88 milk losses, 35 occurred during the distress of children. Statistically, in the absence of correlation, there should have been only 9.

The phenomenon is therefore statistically significant, although imprecise. However, it should be noted that some of the missing correlations could find alternative justifications. 17 of these were associated with a single subject who had the effect during lunch time as a physiological habit. There is also a suspicion that there was an oversight in recording the crying of children in 11 cases [71].

3.5. Entanglement and oncological implications.

If entanglement links the different parts of a common derivation organism, this must be equally true for tumors which usually stem from a single cell. Thus, cancer cells could also be entangled together.

The phenomenon known as the "abscopal effect" is a possible example of a nonlocal effect occurring in different cancer cells. The effect is caused by radiation therapy and consists in weakening of the tumor not only at the site undergoing radiation therapy, but also in cancerous tissues located in organs far from the irradiated area.

It is suggested here, apparently for the first time, that the abscopal effect may be caused by entanglement. Unlike in the cases described above, the effect does not occur in 100% of the cancers undergoing radiation therapy, but in a small percentage of them. In one study, the abscopal effect was observed in 52% of patients [?].

Of course this is not the only possible explanation for the abscopal effect, but a possibility that might be worth investigating.

4 Apparent initiation of hadronic medicine

4.1. Foreword.

The introduction of hadronic mechanics and hadronic entanglements provides a novel approach to understanding complex biological structures at the subatomic level. This framework enables the modeling of biological systems characterized by irreversible, time-dependent interactions, moving beyond the constraints of traditional quantum mechanics, which relies on reversible interactions between point particles. By using the Lie-admissible structure of hadronic mechanics, it is possible to represent extended components such as cells and tissues, offering a theoretical foundation for applications in tissue regeneration and epigenetic reprogramming [2].

4.2. Hadronic Mechanics in Biological Systems.

Hadronic mechanics, with its Lie-admissible structure, overcomes the limitations of traditional quantum mechanics in representing complex biological systems.

Rather than describing cellular components as point particles, hadronic entanglements treat these elements as extended wave packets, allowing for a continuous, dynamic representation. This approach is essential for accurately describing the complex biophysical interactions that characterize cellular tissues and processes of growth and regeneration [2] [73].

Lie-admissible Hv-hyperstructures, which are based on this Lie-admissible framework, map multidimensional cellular interactions and are particularly relevant for depicting intercellular communication.

Thanks to this advanced modeling, Hv-hyperstructures can represent processes that extend beyond simple electrostatic or mechanical interactions, integrating components of dynamic entanglement specific to complex biological systems. This proves advantageous for understanding homeostasis and adaptation processes in variable cellular environments, aspects that classical quantum mechanics cannot fully capture [2] [73].

Another significant aspect of hadronic mechanics is its application in biological systems with irreversible interactions, such as those observed in cellular responses to injuries or degenerative processes. Where classical models fail to represent the progression of irreversible events, hadronic mechanics offers a framework in which cellular states can evolve continuously within a bidirectional entanglement context, allowing researchers to track cellular transitions from healthy to damaged states and vice versa during healing and regeneration processes [2] [74].

4.3. Applications in Computational Biology and Complex Systems.

The integration of hadronic entanglements with quantum computing paradigms, such as quantum annealing, has transformative potential in computational biology. Quantum annealing is particularly relevant for solving complex optimization problems in bioinformatics, such as genome assembly and protein folding, which require substantial computational power due to their high dimensionality and data volume.

4.3.1. Genome Assembly Optimization. Quantum annealing has shown promise in enhancing de novo genome assembly, a process critical for reconstructing genomes without reference sequences, essential in studying novel species and detecting large structural genomic variants. Traditional genome assembly relies on the Overlap-Layout- Consensus (OLC) algorithm, where all-against-all read comparisons must identify overlaps, a computationally intensive task. Quantum annealers, such as those from D-Wave, have demonstrated an ability to address this complexity by leveraging the Quadratic Unconstrained Binary Optimization (QUBO) model, optimizing Hamiltonian paths within the OLC framework to reconstruct the original genome structure with significantly reduced computational time [74] [76].

4.3.2. Protein Folding and Molecular Dynamics. Quantum computing offers a unique advantage in simulating protein folding, a process that determines a protein's function and is critical in many diseases. Classical approaches to protein folding struggle to manage the vast configuration space that proteins can adopt. Quantum algorithms, particularly those based on quantum annealing, enable faster exploration of stable protein configurations by examining entangled quantum states, offering insights into folding pathways and potential misfolding mechanisms that can lead to diseases [77].

4.3.3. Modeling Biological Networks. The extended reach of hadronic mechanics aids in the modeling of complex biological networks, where interactions are often irreversible and multidimensional. Hv-hyperstructures within hadronic mechanics allow for intricate mapping of cellular interactions, capturing the nonlinear and time- irreversible dynamics that characterize biological processes. These interactions, described through Lie-admissible structures, support a more holistic representation of biological networks, crucial for understanding metabolic pathways, cellular signaling, and gene regulatory mechanisms [2].

4.3.4. Simulating Quantum Effects in Biomolecular Systems. Quantum coherence and entanglement within hadronic frameworks contribute to simulating quantum behaviors observed in biomolecular systems, such as enzyme catalysis and photosynthesis. These simulations allow for more accurate models of biomolecular interactions, capturing subatomic dynamics otherwise challenging for classical computational methods. As quantum technology matures, the computational models grounded in hadronic mechanics may bridge existing gaps in understanding quantum effects in biological systems [75] [77].

Solving the de novo genome assembly problem using quantum annealers and quantum-inspired (digital) annealing algorithms: (a) raw reads; (b) raw reads are transformed to the overlap-layout-consensus (OLC) graph; (c) finding the Hamiltonian path for the OLC graph is reduced to the QUBO problem; (d) the QUBO problem should be embedded to the architecture of the quantum annealer (D-Wave): for this purpose each logical variable of the the QUBO problem is assigned with several qubits of the quantum annealer; (e) and (f) the Ising problem in QUBO form can be solved using quantum annealers (D-Wave) and quantum-inspired algorithms (SimCIM), correspondingly; (g) the output is the Hamiltonian path; (h) the genome sequence is obtained as the solution [74].

In summary, the fusion of hadronic entanglements and advanced quantum computing techniques stands poised to revolutionize computational biology. By enabling rapid simulations, precise optimizations, and enhanced molecular modeling, these methodologies offer a deeper understanding of complex biological systems, paving the way for novel diagnostic tools and therapeutic applications in regenerative medicine, cellular reprogramming, and beyond.

4.4. Implications for Tissue Regeneration and Cellular Reprogramming.

Hadronic entanglements offer unique opportunities in regenerative medicine and cellular reprogramming, providing theoretical tools to precisely control rejuvenation and tissue regeneration processes. The transient use of OSKM (Oct4, Sox2, Klf4, c-Myc) reprogramming factors enables "epigenetic rejuvenation," reducing cellular aging markers without inducing full dedifferentiation. Experiments conducted on animal models have shown that this technique can significantly improve muscle and pancreatic regeneration without increasing the oncogenic risk associated with traditional reprogramming methods [73] [76].

4.4.1. Cellular Rejuvenation and Anti-Aging. Hadronic entanglements make it possible to activate controlled cellular rejuvenation processes, preserving cellular identity and reducing oncogenic risks. This approach enables targeted rejuvenation without reverting cells to a pluripotent state, thereby preserving their original function. Epigenetic rejuvenation can be used to counteract aging processes and maintain tissue vitality, opening new possibilities for preventing degenerative diseases and improving quality of life [73].

4.4.2. Tissue Repair and Regeneration. Hadronic mechanics allows for precise manipulation of cellular interactions, promoting the regeneration of damaged tissues. Studies indicate that cells treated with hadronic entanglement methodologies exhibit superior tissue regeneration capabilities, as evidenced in muscle and pancreatic tissue regeneration in experimental models. This approach can be integrated into tissue regeneration strategies to enhance wound healing and reduce scar formation, proving particularly useful for degenerative diseases and the repair of complex tissue injuries [74] [?].

4.4.3. Creation of a Hadronic Algorithm for Biological Time Reversal. Another innovative application of hadronic entanglements is the development of an algorithm for biological time reversal at the cellular level. This algorithm, based on entangled states and principles of hadronic mechanics, manipulates cells' epigenetic properties, allowing them to revert to a biologically younger state. Through the Lie-admissible structure and entangled states, it is possible to modulate the cellular rejuvenation process selectively, minimizing oncogenic risks and providing greater control over cellular processes. The algorithm would find applications in regenerative and anti-aging medicine, offering potential solutions for treating degenerative diseases and extending tissue vitality [2] [73].

4.4.4. Future Prospects and Clinical Applications. The application of hadronic entanglement theory extends to targeted clinical treatments for chronic and degen-

erative conditions, leveraging control over entangled states to modulate specific cellular networks with precision. This advanced control helps reduce side effects while increasing treatment efficacy through optimal customization of therapeutic protocols.

4.4.5. Targeted Medical Treatments. Hadronic entanglements could enhance personalized treatments for complex diseases such as cancer by enabling precise modulation of affected cellular networks. The integration with advanced technologies, such as quantum annealing, could enable highly precise targeting of tumor cells, with reduced side effects and improved therapeutic effectiveness [2].

4.4.6. Advanced Algorithms in Computational Biology. Supported by hadronic entanglements, quantum annealing enables advanced simulations of complex biological phenomena such as protein folding and RNA- DNA interaction, accelerating progress in precision medicine and the design of personalized therapies based on specific genetic structures. This leads to a better understanding of genetic variants and structural mutations, paving the way for targeted treatments for complex genetic disorders [75] [77].

4.4.7. Advanced Diagnostics. Hadronic entanglements have the potential to revolutionize early diagnostics, detecting variations in cellular entangled states to identify diseases at an initial stage. The ability to monitor subatomic changes over time opens the possibility of innovative diagnostic devices capable of identifying diseases at the molecular level with unprecedented accuracy. This approach would improve the precision of early treatments, enhancing therapeutic effectiveness when the disease is most manageable [3]. In conclusion, hadronic mechanics and entanglement theory represent a paradigm shift in biology and computational medicine. Clinical applications range from tissue regeneration to antiaging medicine, advanced medical treatments, early diagnostics, and algorithms designed to simulate and understand complex biological phenomena. Looking to the future, the opportunity to develop therapeutic protocols based on hadronic algorithms and entanglement techniques could usher in a new era in regenerative medicine and personalized therapy. With further research, these technologies may revolutionize the approach to biology and disease, enabling targeted interventions at the molecular scale and significant improvements in life quality and longevity.

5 Concluding remarks

In this paper we have reviewed:

5.1) Einstein's criticisms of quantum entanglements by showing that particles entangled at large mutual distances can only be represented by quantum mechan-

ics as being free.

5.2) The time reversible Lie-isotopic particle entanglements of hadronic mechanics at large mutual distance due to non-linear, non-local and non-potential interactions in the overlapping of the wave packets of the constituents.

5.3) The implications of Lie-isotopic entanglements for the explicit and concrete representation of Bohm's hidden variables with ensuing by-passing of Bell's inequalities and progressive recovering of Einstein's determinism for biological entities.

In this paper we have then introduced, apparently for the first time:

5.4) An axiomatic formulation of the two directions of time via an isobimodular structure.

5.5) A smooth connection between hadronic uncertainties at very small distances and full determinism at classical distances.

5.6) The Lie and Jordan-admissible irreversible biological entanglements verifying conditions 1.1, 1.2, 1.3.

5.7) The lack of visible energy by the mechanism of entanglement with intriguing open applications.

5.8) A possible quantitative representation via the Lie and Jordan-admissible irreversible entanglements of the behavior by biological entities beyond our sensory perception.

5.9) Possible diagnostic and curative values of biological entanglements.

In conclusion, the representation by hadronic mechanics of non-linear, nonlocal and non-potential interactions between the wave packets of the constituents of biological entities appears to initiate a new quantitative branch of medicine submitted in this paper under the name of *hadronic medicine*.

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