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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

## THE NOTION OF NONRELATIVISTIC ISOPARTICLE

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## ABSTRACT

We introduce the notion of *nonrelativistic isoparticle* as a representation of a Galilei-isotopic symmetry studied in preceding works or, equivalently, as the generalization of the conventional notion of particle characterized by the isotopic liftings of the unit. We show that the lifting represents the transition from massive points moving in vacuum to extended-deformable particles moving within physical media. As explicit examples, we work out the cases of an extended-deformable particle: 1) in free conditions; 2) under external potential-selfadjoint interactions; and 3) under external potential-selfadjoint and nonhamiltonian-nonseladjoint interactions. The emerging methods are applied to a first classical and nonrelativistic treatment of Rauch's experiments on the spinorial symmetry of thermal neutrons under external (magnetic and) nuclear fields. The notion *nonrelativistic isoparticle* is submitted as a conceivable classical basis for future operator studies.

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As well known, the conventional notion of *classical nonrelativistic particle* is a representation of the Galilei group  $G(3,1) = [O_3 \otimes T_3^*(3)] \times [T_3^*(3) \times T_3^*(1)]$  on  $\mathfrak{R}_T^* \times T^*E(r, \delta, \mathfrak{R})$  and, as such, it is characterized by conventional units, the scalar unit 1 for the time field  $\mathfrak{R}_T$ , and the unit matrix 1 for the cotangent bundle  $T^*E$ .

By recalling that the Galilei symmetry holds only for interactions which are of local (differential) and potential (selfadjoint) type, the notion essentially characterizes the historical Galilei's concept of dimensionless particle moving in vacuum under action-at-a-distance interactions.

In particular, the intrinsic characteristics of the particle (mass, spin, charge, etc.) are immutable, classically and quantum mechanically, because points are immutable geometrical objects.

This perennial character of the intrinsic characteristics of particles was challenged in the first half of this century, particularly in nuclear physics. As an example, the total magnetic moments of nuclei have remained essentially unresolved despite over half a century of research. One can therefore read in ref. [1], p. 31, the possibility that "the *intrinsic magnetic moment of a nucleon is different when it is in close proximity to another nucleon*".

In different terms, the early (but not the contemporary) studies in nuclear physics admitted the possibility that the value of the magnetic moments of nucleons changes in the transition from the physical conditions under which they have been measured until now (long range electromagnetic interactions only), to the *different* physical conditions when members of a nuclear structure. Similar doubts were also expressed for the spin (see, e.g., ref. [1], p. 254).

A conjecture was then submitted by this author in ref. [2] according to which massive physical particles can experience an alteration of all their intrinsic characteristics, called *mutation*, when experiencing physical conditions broader than those permitted by the Galilean symmetry, such as:

- a) a composite particle with an extended charge distribution experiencing a deformation of its shape under sufficiently intense external, potential interactions; or
- b) an extended charge distribution experiencing a deformation of its shape due to contact, nonhamiltonian-nonseladjoint interactions (as conceivable in collisions); or
- c) a particle totally immersed in an external physical medium,

such as the core of a star.

In all these cases, the deformation of the shape of the particle constitutes the first physical origin for a necessary mutation of the intrinsic magnetic moment, as well established in classical and atomic physics. The mutation of all the remaining intrinsic characteristics of the particle, under sufficient physical conditions, can then be inferred from a number of arguments, e.g., of relativistic character, as we shall show in details in subsequent studies.

However, the primary physical origin of mutation was identified in ref. [2] as being due precisely to the (operator version of) the contact, nonlinear, nonlocal and nonhamiltonian interactions studied in these notes, as expected for a proton in the core of a star, or to a lesser extent, in a nuclear structure.

More generally, it was pointed out in ref. [2] that a necessary condition for the mutation of elementary particles is the existence of interactions which violate the Galilei (and Lorentz) symmetry. The deformation of the charge distribution is the simplest possible mechanism of breaking conventional space-time symmetries, because they are well known to be applicable only for rigid bodies. The contact, nonlinear, nonlocal and nonhamiltonian interactions under study in these notes characterize the most general possible violation of the Galilei (and Lorentz) symmetry at all their structural levels, e.g., inertial, local, canonical, etc. It is hoped that, in this way, the reader begins to see the implications of the interactions herein considered.

The above results were reached in ref. [2] (Sect. 4.19) via the addition of a (variationally) nonselfadjoint coupling to the conventional Dirac's equation. In fact, these interactions are notoriously velocity-dependent, as it must be the case for all drag forces, whether Newtonian or field theoretical. In turn, the addition of a velocity-dependent coupling to Dirac's equation implies the necessary alteration of the conventional gamma matrices. The mutation, in general, of the intrinsic magnetic momentum, spin and other characteristics is then a necessary consequence.

According to these results, we can visualize a hierarchy of different physical conditions, of increasing complexity and methodological needs, such as [2]:

- A) The atomic structure, in which no mutation is possible because of the large mutual distances among the constituents;
- B) The nuclear structure, in which small mutations are

conceivable because available experimental data on the volumes of nuclei and of individual nucleons establish that nucleons, when members of a nuclear structure, are not only in contact, but actually in conditions of mutual penetration of about  $10^{-3}$  parts of their charge volume. In turn, such mutual penetrations is expected to characterize an additional (small) term in the nuclear force, precisely of the contact, nonlinear, nonlocal and nonhamiltonian type studied in these notes;

C) The hadronic structure, in which case we expect a proportionately higher mutation because, as indicated earlier in these notes, the size of all hadrons is approximately the same and coincides with the size of the wavepackets of all known massive particle. The hadronic constituents, to be massive physical particles, are therefore expected to be in conditions of total mutual penetration, resulting precisely in the contact nonlocal and nonhamiltonian interactions under consideration here. Still in turn, these interactions are expected to require a generalized notion of particle as hadronic constituent;

D) The core of stars, where a proportionately higher mutation is expected because, in addition to the hadronic conditions of total mutual penetration of the wavepackets, we have their compression; and

E) The gravitational collapse, where we expect the most extreme possible mutations because we have the most extreme conceivable physical conditions of particles, consisting not only of total mutual penetration of their wavepackets, and their compression, but also the condensation of an extremely large number of particles in an extremely small region of space.

The studies on the Lie-isotopic liftings of the Galilei relativity of ref. [3,4] were conducted for the purpose of permitting a quantitative study of the conjecture of mutation of the intrinsic characteristics of particles which is subjectable to experimental verification.

*DEFINITION 1: A nonrelativistic isoparticle is a representation of one of the infinitely possible Galilei-isotopic symmetries  $\hat{G}(3.1)$  on isospace  $\mathcal{H}_T \times T^*E(r, G, \mathcal{H})$  [4]*

$$\begin{aligned} \hat{G}(3.1): \quad a' &= \hat{g}(w) * a = \hat{g}(w) T_2 a \\ &= \{ \exp (w_K \omega^{\mu\sigma} \hat{I}_2^{\sigma\nu} (\partial_\nu X_K) (\partial_\mu) \hat{I}_2) * a, \end{aligned} \quad (1)$$

the comments in the preceding note regarding the possibility of representing the  $\pi^0$  as a "compressed positronium" and the neutron  $n$  along Rutherford's historical conception as a "compressed hydrogen atom", each case requiring a mutation of the constituents' characteristics for consistency).

Needless to say, a long chain of studies is needed for a quantitative, mathematical, theoretical and experimental appraisal of the above possibilities. The fundamental step is, and will remain, the primitive Newtonian setting which is the arena of our direct intuitions. In this note, we shall therefore present a few classical nonrelativistic examples of isoparticles. As we shall show in subsequent works, the operator formulation will be merely consequential, and will actually enhance the classical mutations of this note.

Let us begin by briefly outlining the underlying methodology for the reader's convenience. The fundamental classical discipline is the *Birkhoffian mechanics* [3] with general Pfaffian variational principle of the autonomous type for a system of  $N$  particles on  $\mathfrak{R}_T \times T^*E(r, \delta, \mathfrak{A})$

$$\delta \dot{A} = \delta \int_{t_1}^{t_2} dt [ R_\mu(a) \dot{a}^\mu - B(a) ] = 0, \quad (3a)$$

$$a = (a^\mu) = (r, p) = (r_{ia}, p_{ia}), \quad \mu = 1, 2, \dots, 6N, \quad i = 1, 2, 3, \quad a = 1, 2, \dots, N, \quad (3b)$$

where  $\dot{B}$  is the *Birkhoffian* (i.e., a quantity generally different than the total energy), which characterizes the covariant *Birkhoff's equations*

$$\Omega^{\mu\nu} \dot{a}^\nu = \partial_\mu B(a), \quad \partial_\mu = \partial / \partial a^\mu, \quad (4)$$

where  $\Omega^{\mu\nu} = \partial R_\mu / \partial p_\nu - \partial R_\nu / \partial p_\mu$  is the most general possible, exact symplectic tensor in local coordinates.

General systems (3) are then *restricted* to those characterized by the *symplectic-isotopic geometry* on  $\mathfrak{R}_T \times T^*E_1(r, g, \mathfrak{R})$  [6]

$$\delta \dot{A} = \delta \int dt [ p_{ia} g_{ij}(r, p) \dot{r}_{ja} - H(r, p) ] = 0, \quad (5a)$$

$$\dot{R}^\circ = (pg, 0) = (p_{ia} g_{ij}(r, p), 0_{ja}), \quad (5b)$$

*Equivalently, a nonrelativistic isoparticle can be defined as the generalization of the conventional notion of particle induced by the isotopic liftings of the unit*

$$1 \in T^*E(r, \delta, \mathfrak{A}) \Rightarrow \hat{1}_2 = \text{diag} (G^{-1}, G^{-1}) \in T^*E_2(r, G, \mathfrak{A}), \quad (2a)$$

$$\delta = \text{diag.} (1, 1, 1) \Rightarrow G = \text{diag.} (B_1^{-1}, B_2^{-2}, B_3^{-2}) > 0. \quad (2b)$$

Stated in different terms, once the actual dimensions of a particle are admitted, the existence of deformations becomes consequential. But then the same particle can have an infinite number of intrinsic characteristics depending on the infinitely possible local characteristics.

The infinite family of Galilei-isotopic relativities have been conceived precisely to represent these infinitely possible local conditions of each given extended-deformable particle.

On operator grounds, the ideal isoparticle is evidently a quark. In fact, all massive particles have a wavepacket of the order of  $1F$ . To be physical particles, quarks are therefore expected to have an extended wavepacket of the size of all hadrons.

This would evidently imply that a quark moves within the medium composed by all the remaining constituents, called *hadronic medium* [2], which results precisely in the nonlinear, nonlocal and nonhamiltonian forces under consideration in this series of papers.

We can therefore introduce the following

*DEFINITION 2: A nonrelativistic isoquark is a representation of one of the infinite family of Lie-isotopic symmetries  $\hat{G}(3,1) \times SU(3)$ , where  $\hat{G}(3,1)$  represents the space-time structure, and  $SU(3)$  represents the isotopic-unitary lifting of  $SU(3)$  first studied by Mignani [5]*

The above definition is introduced in the hope of resolving at least some of the now vexing problems of contemporary hadron physics, with particular reference to the possible identification of the hadronic constituents with mutated forms of physical particles which are produced free in the spontaneous decays, thus reacquiring their conventional intrinsic characteristics as currently measured under electromagnetic interactions only. [See

operator formulation of isotopies.

Nevertheless, we believe it is important to identify here the primitive Newtonian foundations of the expected mutation of the charge distribution and of the intrinsic magnetic moments for thermal neutrons in the conditions of experiments [11]. Operator formulations can then follow.

It is intriguing to note that our Galilei-isotopic relativities can provide, not only a conceptual, but also a first quantitative interpretation of data [11] already at this classical nonrelativistic level.

In fact, the addition of a sufficiently intense, external, potential field to the extended-deformable free particle characterized by isometric (9), implies a necessary mutation of shape which can be characterized via the "isotopy of an isotopy"

$$g = (b_1^2, b_2^2, b_3^2), \quad V = 0 \Rightarrow g' = (b_1'^2, b_2'^2, b_3'^2), \quad V \neq 0, \quad (14)$$

where we again assume the preservation of the volume, i.e.,

$$b_1^2 b_2^2 b_3^2 = b_1'^2 b_2'^2 b_3'^2, \quad (15)$$

Now, Rauch's experiments [11] were done on the  $4\pi$  spinorial symmetry of thermal neutrons in interactions with the external fields of Mu-metal nuclei via neutron interferometric techniques. In essence, a magnetic field was applied to the neutron beams passing through the electromagnet gap of such a value (7,496 G) which should produce two complete spin flips around the third axis ( $\theta_3 = 720^\circ$ ) under the assumption that the intrinsic magnetic moment of the neutron has the conventional value  $(-1.91304211 \text{ eh}/2\text{mpc})$ . The nuclear interactions occurred because the experimenters filled up the electromagnet gap with Mu-metal sheets to reduce stray fields.

In this way, the neutrons experience a first conventional interaction caused by the external magnetic field, plus an additional interaction with the external fields of the Mu-metal nuclei. Even though still preliminary and inconclusive at this writing, the best available measures are [11]

$$\theta_3 = 715.87^\circ \pm 3.8^\circ, \quad (16)$$

which, as such, do not contain the angle  $720^\circ$  needed for the exact symmetry in the minimal and maximal values, and show a violation of the conventional rotational symmetry of about 1%.

Again, if these data are confirmed by future experiments, they imply nothing mysterious but the mere alteration (mutation in our language) of the intrinsic magnetic moment of the neutrons under the conditions considered. Also, since the experimental angle is consistently lower than the angle  $720^\circ$  needed for the exact symmetry, we expect a decrease of the (absolute value of the) intrinsic magnetic moment of the neutron (this is called the "angle slow-down effect" [9]). Finally, from a decrease of the magnetic moment we can expect an increase of the oblate character of type (14) under condition (15) for the preservation of volume.

Since we have at best a small deviation, it is reasonable to assume that the mutation of shape is also small. In first approximation, we have from data (16) that the deviation is of the order of

$$716^\circ/720^\circ \approx 0.9944, \quad (17)$$

which can be assumed to be of the order of magnitude of the oblateness caused by the external nuclear fields. Then, our purely classical nonrelativistic treatment implies that mutation (14) for values (16) of Rauch's experiment [11] under condition (15) of preservation of the neutron's volume, assumes the explicit values

$$\delta = \text{diag. } (1,1,1), \quad V = 0 \Rightarrow g = \text{diag. } (1.0028, 1.0028, 0.9944), \quad V \neq 0 \quad (18)$$

with consequential mutation of the magnetic moment of the order of  $6 \times 10^{-3}$ , i.e.,

$$\mu_n = -1.913 \text{ eh}/2\text{mpc} \Rightarrow \hat{\mu}_n = -1.902 \text{ eh}/2\text{mpc}, \quad (19)$$

which does indeed provide a first, approximate, but quantitative interpretation of Rauch's data (16).

Moreover, the conventional rotational symmetry is evidently broken for values (16). Nevertheless, our Galilei-isotopic relativities reconstruct the exact rotational symmetry for the

deformed neutrons. This is another aspect that we believe needs an identification, first, at the primitive Newtonian level, and then at the operator counterpart. We also believe that the issue deserves an analysis within the  $O(3)$  context and prior to  $SU(2)$  extensions, which will be considered elsewhere.

For this purpose, consider the subgroup of  $\hat{G}(3,1)$  given by our covering isorotational symmetries  $\hat{O}(3)$  [6]. As now familiar, the isotopes  $\hat{O}(3)$  provide the form-invariance of all possible ellipsoidal deformations of the sphere, Eqs. (11) or (14) while being locally isomorphic to the conventional rotational symmetry  $O(3)$ . This establishes the reconstruction of the exact rotational symmetry for deformed charge distributions (17) of course, at our isotopic level.

However, the mechanism of such reconstruction deserves a deeper inspection because important for experiments [11]. Consider our isorotation around the third axis, i.e.,

$$\hat{O}(3) : r' = \hat{R}(\theta_3) * r = (\exp \theta_3 \omega_{3\mu\nu} \sigma^{\mu\nu} / \sigma^3) r, \quad (20a)$$

$$\hat{1} = \text{diag.} (g^{-1}, g^{-1}), \quad (20b)$$

where  $g'$  is that of Eqs. (14), which is explicitly given by Eq. (39) of Ref. [6], i.e.,

$$r' = \hat{R}_i(\theta_3) * r = S_g(\theta_3) r = \begin{pmatrix} x \cos(\theta_3 b_1 b_2) - y \frac{b_2}{b_1} \sin(\theta_3 b_1 b_2) \\ x \frac{b_1}{b_2} \sin(\theta_3 b_1 b_2) + y \cos(\theta_3 b_1 b_2) \\ z \end{pmatrix}, \quad (21)$$

The reconstruction of the exact rotational symmetry is then based on the mechanism originating from the values  $b_1$  and  $b_2$  of Eq. (18) and Rauch's median angle (16)

$$\theta_3 = b_1 b_2 \theta_3 - 716^\circ \approx 720^\circ, \quad (22)$$

$$b_1 - b_2 = 1.0028$$

namely, our geometric isospace  $\hat{E}(r, g, \theta)$  reconstructs the angle  $\theta_3 = 720^\circ$  needed for the exact symmetry from an actual rotation of  $\theta_3 = 714^\circ$  in our physical space  $E(r, \delta, \theta)$ . For further comments, see Figure 1.

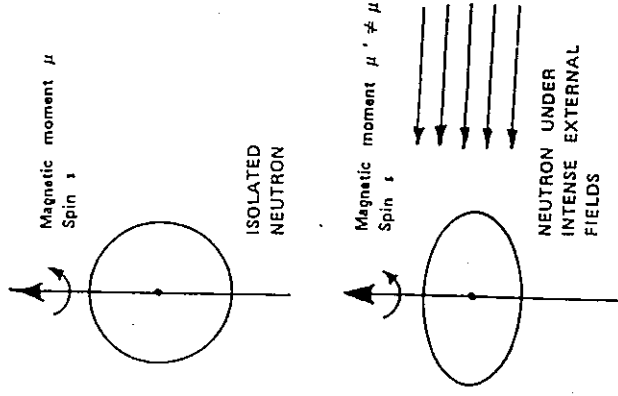


FIGURE 1: A schematic view of the ultimate physical origins of the notion of *isoparticle*, i.e., an extended particle characterized by our Galilei-isotopic relativities, which can experience *alterations of its intrinsic characteristics* called *mutations*, depending on the local physical conditions [2]. The physical origins of mutation are so simple, to appear trivial. In fact, the moment the *extended* character of a particle is admitted, the *deformation* of its shape becomes consequential under sufficiently intense external forces. But the deformation of shape implies a *necessary* alteration of the intrinsic magnetic moment, as well established in classical electrodynamics and atomic physics (it can be easily proven via the ordinary Maxwell's

electrodynamics). In turn, this illustrates the concept of *mutation of the intrinsic magnetic moment*. The mutation of all remaining intrinsic characteristics can then follow from a number of direct and indirect arguments, e.g., of relativistic character (see the subsequent notes in this series). In conclusion, we can say that *the notion of extended-deformable isoparticle with its variable intrinsic characteristics is well established at the classical level*. Intriguingly, there are indications that the notion may well result to be founded also in particle physics. In fact, the notion is ultimately a representation of the old hypothesis that the intrinsic magnetic moments of protons and neutrons are altered (mutated) when the particles become members of a nuclear structure [1]. Operator studies conducted in ref. [1] via nonselfadjoint extensions of Dirac's equations (Sect. 4.19, ref. [2]) readily confirm such a plausible expectation (*nonselfadjoint coupling are velocity-dependent and, as such, they imply a necessary alteration of the gamma matrices of Dirac's equations, with consequential also necessary alteration of the magnetic moment and other characteristics of particles*). Additional operator studies were conducted in Refs. [8,9]. Particularly inspiring for physicists with young minds are the operator studies by Eder [10]. Intriguingly, a series of experiments on the  $4\pi$ -spinorial symmetry of thermal neutrons in interactions with external nuclear fields has been conducted by H.Rauch and his collaborators since the early 70's [11]. Even though still preliminary, the current best measures do indeed confirm a mutation of the magnetic moment because the angle  $710^\circ$  of two spin flips is not contained in the minimum and maximal values [16] [11]. In conclusion, there are sufficient conceptual, theoretical and experimental reasons to expect that *the concept of extended-deformable isoparticle is well founded also at the particle level*. In the final analysis it is appropriate here to stress that *the amount of deformation of the charge distribution of the neutrons under given external fields, with consequential mutation of the magnetic moment, is debatable at this writing. However, its existence could be simply out of any doubt because perfectly rigid bodies do not exist in the physical reality*. In the main text of this note we have provided a fully *Newtonian* but quantitative treatment of Rauch's experiment [11] which is capable of representing experimental angles (16) in first approximation. This should not be surprising. In fact, the ultimate physical structure of Rauch's experiment is purely classical: the deformation of the shape of the neutron depicted in this figure, with consequential mutation of the intrinsic magnetic moment. Also, Rauch's measures an angle, that of two spin flips, which occurs in our *classical*

environment. Operator extensions are only expected to provide refinements. As a final note, the reader should be aware that the conventional Galilei's (and Einstein's special) relativity is violated by the topic of this figure on numerous irreconcilable and independent counts. First, we have a violation caused by the representation of an actual nonspherical shape, e.g., Eq.s (11). Second, we have a violation caused by the deformation of the shape considered, Eq.s (14). Third, we have a violation because the theory is generally nonhamiltonian, e.g., Eq.s (24), (25), and (27) below, with the consequential loss of the algebraic and geometric foundations of Galilei's and Einstein's relativity. Fourth, we have a violation of their topological structure because the general origin of mutations is given by nonlocal-integral forces, and so on. Our covering Galilei-isotopic relativities (and, as we shall soon see in these notes, our Lorentz-isotopic relativities) provide a direct and quantitative representation of all the conditions of this note. The old relativities are violated because the underlying Galilei's (and Lorentz's) transformations are no longer symmetries of extended-deformable isoparticles. Nevertheless, the underlying fundamental space-time symmetries, the Galilei (and Lorentz) symmetry are preserved.

Remarkably, relativistic and operator treatments (to be presented elsewhere) appear to confirm in full the above rudimentary classical results. In particular, the mutation of magnetic moment (19) associates to shape mutations (18) appears to be able to represent Rauch's data [11]. As a matter of fact, mechanism (22) appears to be able to represent, not only the experimental data (16), but also to provide the physical origin of the "angle slow-down effect", i.e., the fact that all median angles measures by Rauch until now are lower than  $720^\circ$  [11] (see ref. [9]), as well as Eder's excellent accounts [10].

In conclusion, the rotational and Galilei symmetries characterize a *theory of rigid bodies*, as well known. Our rotation-isotopic and Galilei-isotopic symmetries characterize instead a *theory of deformable bodies* without violating the abstract  $O(3)$  and  $G(3,1)$  symmetries, but by realizing them instead in their most general possible form. This completes our consideration for an isoparticle under a conventional, external, arbitrary *potential* field.

ISOPARTICLE UNDER EXTERNAL NONSELFADJOINT INTERACTIONS. The next example is that of an extended-deformable isoparticle under, this time, *nonpotential* external fields caused by motion within a physical medium. Note that this



class of interactions is strictly excluded by the conventional Galilei relativity, but it is rather natural for our covering Galilei-isotropic relativities.

In this case,  $N = 1$ , the selfadjoint interactions can be assumed to be null ( $V = 0$ ) for simplicity, but we have nontrivial nonselfadjoint interactions which, are represented in Birkhoffian mechanics by a generalized functional dependence of the Lie-isotopic (symplectic-isotopic) tensor of the theory.

A first simple case in one space-dimension is given by a particle moving within a resistive medium under a quadratic damping force

$$m\ddot{r} + \gamma \dot{r}^2 = 0, \quad (23)$$

with the Birkhoffian representation [3]

$$\hat{R}^\circ = (pe^{\gamma r}, 0), \quad H = pe^{\gamma r} p / 2m, \quad (24)$$

which provides a first approximation of systems such as a satellite penetrating Jupiter's atmosphere or, along similar conceptual grounds, a proton moving within the core of a star.

The interested reader can readily enlarge the above example to three dimensions, e.g., for motion along the third axis

$$\hat{R}^\circ = (p_1 g_{ij}, 0), \quad g = \text{diag.} (b_1^2, b_2^2, b_3^2 e^{\gamma r}), \quad H = p_j g_{ij} p_j / 2m, \quad (25)$$

with a deformation of shape, this time, due to contact interactions.

Along similar lines, one can have an isoparticle subject to selfadjoint ( $V \neq 0$ ) and nonselfadjoint ( $g = g(r)$ ) interactions. In this case, a simple example is given by the quadratically damped oscillator

$$\ddot{r} + r + \gamma \dot{r}^2 + \lambda \gamma r^2 = 0, \quad m = k = 1, \quad (26)$$

with the Birkhoffian representation [3]

$$\hat{R}^\circ = (pe^{\gamma r}, 0), \quad H = \lambda pe^{\gamma r} p + \lambda re^{\gamma r}, \quad (27)$$

Numerous additional examples can be worked by the interested reader in any desired combination of selfadjoint and nonselfadjoint forces, the latter being local-differential or nonlocal-integral, as desired.

In all the above cases, the Galilei-isotropic relativities permit the explicit construction of the generalized invariance  $\hat{G}(3,1)$  via the computation of the Lie-isotopic tensor (7), and its use in exponentiations (1), all in a way which reconstructs the exact Galilei symmetry in isospaces  $\mathfrak{R}_T \times T^*E_2(r, G, \mathfrak{R})$ , while the conventional symmetry is manifestly broken in  $R_T \times T^*E(r, \delta, \mathfrak{R})$ .

Under the assumed topological conditions, the existence theory for expansions (1) assures their convergence into a finite form (although, one should not expect easy series of simple computation).

A first understanding is that, to have a full  $\hat{G}(3,1)$ -invariant model, the coordinates  $r$  of the preceding examples have to be interpreted as relative coordinates, otherwise one has the full  $\hat{O}(3)$ -invariance.

Another understanding, stressed earlier during the course of our analysis, is that the total energy of the isoparticle here considered is *nonconserved* by assumption. In this case, the conserved Birkhoffian merely represents a first integral of the equations of motion, and not a physical quantity (see the examples of ref. [3]).

This completes our examples of Galilei-isotropic symmetries for *one* isoparticle under the most general, known, nonlinear, nonlocal and nonhamiltonian external forces. Examples of closed-isolated bound states of isoparticles were previously given in note [12].

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