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A QUANTITATIVE ISOTOPIC REPRESENTATION OF THE DEUTERON MAGNETIC MOMENT

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1. Statement of the Problem

A fundamental, still unresolved problem of nuclear physics is the old hypothesis that protons and neutrons experience an alteration of their intrinsic magnetic moments when they are members of a nuclear structure (see, e.g., [1], p.31). The hypothesis emerged rather naturally from experimental data on total nuclear magnetic moments, with particular reference to the few-body case which are known not to be interpretable via conventional quantum mechanical (QM) magnetic moments.

The physical basis of the hypothesis is so simple to appear trivial. Protons and neutrons are not point-like particles, but have an extended charge distribution with a radius of ≈ 1 fm. Since perfectly rigid bodies do not exist in the physical reality, we have to expect the possibility that these charge distributions can be deformed under sufficiently intense external forces (and/or collisions). In turn, since nucleons are spinning particles, ordinary Maxwell's electrodynamics implies that a possible deformation of their charge distribution necessarily implies an alteration of their intrinsic magnetic moments.

The hypothesis here considered is therefore reducible to the possible deformation of the charge distributions of nucleons due to the notoriously intense fields of the nuclear structure.

This effect is well established in classical, atomic and nuclear physics. However, at the deeper level of the individual nucleons, or hadrons at large, the effect has been only preliminarily confirmed by H.Rauch and his collaborators (see reviews [2] and quoted references) in a series of interferometric experiments on the 4π symmetry of neutrons. The best available measure for the angle of two spin-flips is given by

$$\theta = 715.87^{\circ} \pm 3.8^{\circ}, \quad \theta_{\text{Max}} = 719.67^{\circ}, \quad \theta_{\text{Min}} = 712.07^{\circ},$$
 (1.1)

which does not contain the value 720° needed for the perfect rigidity of the neutron. Nevertheless, the deviation, which is of the order of 1%, is smaller than the error and, therefore, measure must be re-run to acquire a final character (Sect.3).

Despite that, measures (1.1) are indeed plausible [3—5]. In fact, the experimenters filled up the electromagnetic gap with Mu-metal sheets to avoid stray fields. As a result, the neutron beam is under the action not only of the long range magnetic field causing the spin-flips, but also of short range, intense nuclear fields. Studies [3—5] therefore show that the intense electric and magnetic fields of the Mu-metal nuclei can cause an (average) 1% deformation of the charge distribution of the neutrons, with consequential small alteration of their intrinsic magnetic moment which, in turn, implies an angle of spin-flip different than the value 720° predicted by conventional theories.

Within the context of (relativistic) hadronic mechanics (IIM), originally submitted in [6] (see [7—9]), in this note we shall introduce the isotopies of Dirac's equation called isodirac equation, construct its isotopic $SL(2.C)\times T(3.1)$ -invariance, and prove its local isomorphism to the conventional symmetry.

We shall then show that the isodirac's equation permits the direct representation of: 1) nonspherical shapes of charge distributions: 2) their infinitely possible deformations; and 3) the consequential alteration of intrinsic magnetic moments.

The theory is applied, first, to a quantitative representation of Rauch's measures (1.1) and, second, to a resolution of the magnetic moments of the deuteron and of few-body nuclei.

The reader should be aware that the above isotopic methods provide an axiomatization of the so-called q-deformation for the more general case of Q operators [10].

It should be finally stressed that our study of the Maxwellian deformation of shape/alteration of the magnetic moment is of exterior-geometric character, i.e., independent of the constituents. It is evident that such a study must be complemented with corresponding studies of interior-structural character, which are already under way via the polarizability of quark orbits and other techniques.

2. Isotopies of Dirac's Equation

HM has been constructed as a step-by-step generalization of QM via new mathematical methods called *isotopies*. They essentially permit the achievement of nonlinear-nonlocal-noncanonical generalizations of any given linear-local-canonical structure, but in such a way to preserve the original axioms. In this sense, the isotopies are axiom-preserving maps. It then follows that HM and QM coincide, by construction, at the abstract, realization-free level.

The fundamental isotopy is that of Planck's constant h = 1 which is mapped into the most general possible nonlinear integro-differential operator $I(t, x, \dot{x}, \ddot{x}, \psi, \partial \psi, \partial \partial \psi, ...)$ under the condition of preserving the original axioms of h, i.e., boundedness, smoothness, nondegeneracy, Hermiticity and positive definiteness.

The isotopy of the unit of QM, $1 \Rightarrow \hat{I}$, then imply that of the conventional associative product of operators $AB \Rightarrow A*B$, where the new product A*B must preserve the original axiom of associativity, i.e., $(AB)c = A(Bc) \Rightarrow (A*B)*C = A*(B*C)$. Also, recall that 1 is the unit of the enveloping operator algebra ξ of QM, i.e., $1A = A1 \equiv A \forall A \in \xi$. A second necessary condition for the isotopy $\xi : AB \Rightarrow \xi : A*B$ is that the new product A*B must admit \hat{I} as the left and right unit.

The latter condition is verified by the realization A*B = AQB, where Q is a fixed invertible operator such that $\hat{I} = Q^{-1}$, in which case $\hat{I}*A = Q^{-1}QA = A*\hat{I} = AQQ^{-1} = A \forall A \in \hat{\xi}$. Under these properties, the operator Q is called the *isotopic element* and \hat{I} the *isounit*.

The characterization of system in HM therefore requires two operators, the conventional Hamiltonian H = k + V (or Lagrangian) which represent all action-at-a-distance, potential interactions, and the isotopic operator Q which represents all contact nonlinear effects (e.g., nonlinear in the derivatives \dot{x} , $\partial \psi$, ...), nonlocal interactions (e.g., due to mutual wave penetration) and other events (e.g., the deformation of nonspherical charge distributions) which, as such, an beyond the representational capabilities of the Hamiltonian.

The isotopies $I \Rightarrow I$ and $AB \Rightarrow A*B$ then imply corresponding isotopies of all structures of QM into those of HM. For notational convenience, we here recall:

- 1) The isotopies of fields $F(n, +, \times)$ into the *isofields* $\hat{F}_Q(\hat{n}, +, *)$ (isomal \hat{R}_Q or isocomplex \hat{C}_Q for this note) with: isonumber $\hat{n} = n\hat{l}$; conventional sum +; isotopic product $\hat{n} * \hat{m} := \hat{n}Q\hat{m} = (nm)\hat{l}$; isonorm $\hat{l} : \hat{n} := n\hat{l}$; and a generalization of all operations of conventional numbers [7—9].
- 2) The isotopic lifting of the conventional Minkowski space $M(x, \eta, R)$ with familiar metric $\eta = \text{diag.}(1, 1, 1, -1)$ over the reals R, into the isominkowskian spaces $M(x, \hat{\eta}, R)$ originally submitted in [11].

$$\hat{M}(x, \hat{\eta}, \hat{R}): x = (r, x^4) = (r, c_0 t), \quad \hat{\eta} = Q \eta, \quad \hat{I} = Q^{-1},$$
 (2.1a)

$$Q = \operatorname{diag.}(b_1^2, b_2^2, b_3^2, b_4^2), \quad b_{\alpha} = b_{\alpha}(s, x, \dot{x}, \ddot{x}, \dot{\psi}, \dots) > 0, \tag{2.1b}$$

$$x^{2} = (x^{i} \hat{\eta} x)\hat{l} = (x^{\mu} \hat{\eta}_{uv}(x, \dot{x}, \ddot{x}, \hat{\psi}, \partial \hat{\psi}, \partial \partial \hat{\psi}, \dots)x^{\nu}]\hat{l} \in \hat{R}, \tau$$
(2.1c)

where c_0 is the speed of light in vacuum and the b's are called the *characteristic functions* of the medium considered.

The arbitrary functional dependence (2.1b) is useful for the *local-interior* description, e.g., the motion of an extended particle at a point within the *hadronic medium* [6] in the interior of hadrons. For the case under study in this paper, which is the *global-exterior* description of a hadron, the characteristic functions can be effectively averaged into constants b_{μ}^* [7—9].

In the latter case, the quantity b_4^* is a numerical characteristic of the hadron considered, which generally varies from hadron to hadron (because of the variation of their density with mass), and geometrizes the medium considered essentially along an isotopy of the conventional index of refraction (when the medium considered is water, $b_4^* = 1/n^*$, where n^* is the index of refraction and $x^2 = x^2/n^*$ [12]).

The space components b_k^* permit the desired direct representation of the generally nonspherical shapes of hadrons, as well as of their deformations. For applications of the isominkowskian geometry (2.1) and its verification with available experimental data, one may consult: refs. [7] on the nonlocal-isotopic treatment of the Bose-Einstein correlation and its verification [13] via the UA1 experiment; refs. [14] on the isominkowskian representation of available (anomalous and nonanomalous) experimental data on the behavior of the meanlife of unstable hadrons with speed; and others [9].

The isotopic character (as well as novelty) of the generalization is established by the fact that, under the *joint* lifting of the metric $\eta \Rightarrow \hat{\eta} + Q\eta$ and of the field $R \Rightarrow \hat{R}, \hat{I} = Q^{-1}$, all infinitely possible isospaces $\hat{M}(x, \hat{\eta}, \hat{R})$ are locally isomorphic to the original space $M(x, \eta, R)$ under the sole condition of positive-definiteness of the isounit \hat{I} [11]. Such a local isomorphism then sets the foundation for the expected isomorphism of the corresponding symmetries. Note that separation (2.1c) is the most general possible nonlinear, nonlocal and noncanonical generalization of the original separation $x'\eta x$ under the condition of preserving the original topology, i.e., sig. $\eta = \text{sig. } \hat{\eta} = (+, +, +, -)$.

3) The preceding isotopies then imply those of Hilbert spaces $\mathcal{H} \langle \psi | \phi \rangle \in C$ into the isohilbert spaces \mathcal{H}_O with isoinner product and isonormalization

$$\hat{\mathcal{H}}_{O}: \langle \hat{\psi} \, \hat{l} \, \hat{\phi} \rangle := \langle \hat{\psi} \, | \, \mathcal{Q} \, | \, \hat{\phi} \rangle \, \hat{I} \in \, \hat{C}_{O}; \quad \langle \hat{\psi} \, \hat{l} \, \hat{\psi} \rangle = \hat{I}, \tag{2.2}$$

under which operators that are originally Hermitean (observable) for QM remain Hermitean (observable) for HM [7—9].

- 4) The liftings of the Hilbert space then require corresponding isotopies of all conventional operations [7—9]. We here mention isounitarity $\hat{U}*\hat{U}^{\hat{T}}=\hat{U}^{\hat{T}}*\hat{U}=\hat{I}$; the isoeigenvalue equations $H*|\hat{\Psi}\rangle = HQ|\hat{\Psi}\rangle = \hat{E}*|\hat{\Psi}\rangle \equiv E|\hat{\Psi}\rangle, E \neq E^*$; etc.
- 5) The lifting of the unit, base field and carrier space then require, for mathematical consistency, the lifting of the entire structure of Lie's theory, that is, the isotopies of enveloping associative algebras ξ , Lie algebras L, Lie groups G_{λ} representation theory, etc. [7—9]. Here we mention the isoassociative enveloping operator algebras ξ_Q : $A*B \equiv AQB$, Q = fixed; the Lie-isotopic algebras L_Q with basic product

$$\hat{L}_{Q}: [A, B]_{\xi_{Q}} = [A, B] = A*B - B*A = AQB - BQA;$$

$$(2.3)$$

the (connected) Lie-isotopic groups \hat{G}_Q of isolinear isolantary transforms on $\hat{M}_Q(x,\hat{\eta},\hat{R})$

$$x' = \hat{U}(w) * x = \hat{U}Qx = \hat{U}Q(x, \dot{x}, \ddot{x}, \psi, ...)x,$$
 (2.4a)

$$\hat{U}(w) = e^{iX * \hat{w}}_{l\xi_{\Omega}} = \left\{ e^{iXQw}_{l\xi} \right\} \hat{I}. \tag{2.4b}$$

We are now equipped to study the desired isotopies of the Dirac equation. They were first submitted in [6], Sect. 4.20, via the addition of variationally nonselfadjoint (nonlagrangian) interactions to represent deep mutual overlapping of wavepackets/charge distributions of hadrons which, as such, are expected to be dependent on the velocities. Nonlagrangian couplings then imply an alteration of the gamma-matrices which, in turn, provides the desired alteration of the magnetic moment called mutation. Subsequent isotopies of Dirac equation have been studied in [15], although with conventional gamma matrices.

Via the isolinearization of the second-order isoinvariant [9], in this note we submit the following isodirac equation for the case $\partial b/\partial x = 0$ (or for $b_{\mu}(x, \dot{x}, \ddot{x}, \dot{\psi}, \partial \dot{\psi}, \partial \partial \dot{\psi}, ...)$ averaged to constants b_{μ} ")

$$(\hat{\gamma}_{\mu} * p^{\mu} + i\hat{m}) * \hat{\psi}(x) = (\hat{\eta}^{\mu\nu}\hat{\gamma}_{\mu}QP_{\nu} + i\hat{m}) Q \hat{\psi} = 0, \quad \hat{m} = m\hat{l} \in \hat{R},$$
(2.5)

where the isogamma matrices $\hat{\gamma}_{\mu}$ are characterized by

$$\{\hat{\gamma}_{\mu}, \hat{\gamma}_{\nu}\} = \hat{\gamma}_{\mu} Q \hat{\gamma}_{\nu} + \hat{\gamma}_{\nu} Q \hat{\gamma}_{\mu} = 2\hat{\eta}_{\mu\nu} \hat{I}, \quad \hat{I} = T^{-1}, \tag{2.6a}$$

$$\hat{\gamma}_{k} = b_{3} \begin{pmatrix} 0 & \sigma_{k} \\ -\sigma_{k} & 0 \end{pmatrix} \hat{I}, \quad \hat{\gamma}_{4} = ib_{4} \begin{pmatrix} I_{s} & 0 \\ 0 & -I_{s} \end{pmatrix} \hat{I}, \tag{2.6b}$$

 I_s = diag. (1,1) and γ_u , σ_k are the conventional forms.

Eq. (2.5) is based on the covariance under the isopoincare symmetry (see below) plus the momentum isoquantization rule $p_{\mu} * \hat{\psi} = -i \hat{l}_{\mu}^{\nu} (\partial/\partial x_{\nu}) \hat{\psi} = -i b_{\mu}^{-2} (\partial/\partial x_{\mu}) \hat{\psi} = -i (\partial/\partial x^{\mu}) \hat{\psi}$, as established by

the fundamental isotopy $h \Rightarrow \hat{l}$, the isotopies of the classical Hamilton — Jacobi equations [12] and of conventional quantization methods [7].

It should be stressed that Eq. (2.5) is one of the simplest possible realizations of the isodirac equation and that more complex realizations exist, including an intriguing cases with nondiagonal isotopic elements [9].

The reader not familiar with isotopic techniques should be aware that electromagnetic interactions can be represented via their embedding in the generalized Lie-tensor (the b-functions) and the use of the kinetic energy only for Lagrangian or Hamiltonian [12]. Thus, rather than a free particle, Eq. (2.5) represents a spinor under the most general known combination of linear and nonlinear, local and nonlocal, as well as Lagrangian and nonlagrangian interactions.

Note also that Eq. (2.5) is not unitarily equivalent to the conventional equation. In fact, there exist no unitary transform $UU^{\uparrow} = U^{\uparrow}U = I$ such that $U^{\uparrow} \gamma_{\mu}U^{\uparrow} = Ub_{\mu}\gamma_{\mu}\hat{I}U^{\uparrow} = \gamma_{\mu}$, $\mu = 1, 2, 3, 4$.

From the viewpoint of symmetries, the lifting of the Dirac into the isodirac equation implies the isotopies of the spinorial covering of the Poincare group $\mathcal{P}(3.1) = SL_Q(2.\hat{C}) \times T(3.1)$ into the isopoincare group $\hat{\mathcal{P}}_Q(3.1) = SL_Q(2.\hat{C}) \times \hat{T}_Q(3.1)$ constructed with respect to the isounit $\hat{I} = Q^{-1}$. Despite the loss of unitary equivalence of the equations, we shall now show that $\mathcal{P}(3.1)$ and $\hat{\mathcal{P}}_Q(3.1)$ are locally isomorphic. In fact, the generators of the isotopic $\hat{SU}(2)$ symmetry

$$\hat{S}_k = \frac{1}{2} \, \varepsilon_{kij} \, \hat{\gamma}_i * \hat{\gamma}_j = \frac{1}{2} \, \{ \varepsilon_{kij} \, b_i \, b_j \, \gamma_i \, \gamma_j \} \, \hat{I}, \tag{2.7a}$$

$$\hat{S}_{1} = b_{2}^{-1} b_{3}^{-1} \hat{S}_{1}, \quad \hat{S}_{2} = b_{1}^{-1} b_{3}^{-1} \hat{S}_{2}, \quad \hat{S}_{3} = b_{1}^{-1} b_{2}^{-1} \hat{S}_{3}, \tag{2.7b}$$

verify isocommutation rules with the conventional structure constants of SU(2),

$$[\widetilde{S}_{i}^{\hat{}}\widehat{S}_{i}^{\hat{}}] = \widehat{S}_{i}^{\hat{}}Q\widehat{S}_{i}^{\hat{}} - \widehat{S}_{i}^{\hat{}}Q\widehat{S}_{i}^{\hat{}} = i\epsilon_{ijk}\widehat{S}_{k}. \tag{2.8}$$

The generators for the isotopic $\hat{SL}_O(2.\hat{C})$ symmetry

$$J_{\mu\nu} = \{\hat{J}_{ij}, \hat{L}_{k4}\}, \hat{L}_{k4} = \frac{1}{2}\hat{\gamma}_k * \hat{\gamma}_4 = \frac{1}{2} \{b_k b_4 \gamma_k \gamma_4\} \hat{I}, \hat{L} \equiv L$$
 (2.9)

plus the linear momentum vefiry the isocommutators

$$\begin{split} [J_{\mu\nu}, J_{\alpha\beta}] * |\psi\rangle &= i(\eta_{\nu\alpha} J_{\beta\mu} - \eta_{\mu\alpha} J_{\beta\nu} - \eta_{\nu\beta} J_{\alpha\mu} + \eta_{\mu\beta} J_{\alpha\nu}) * |\psi\rangle, \\ [J_{\mu\nu}, P_{\alpha}] * |\psi\rangle &= i(\eta_{\mu\alpha} P_{\nu} - \eta_{\nu\alpha} P_{\mu}) * |\psi\rangle, \\ [J_{\mu\nu}, P_{\nu}] * |\psi\rangle &= 0, \quad \mu, \nu = 1, 2, 3, 4, \end{split}$$
 (2.10)

which coincide with the isocommutation rules of the isopoicaré symmetry $\hat{P}(3.1) = \hat{O}(3.1) \times \hat{T}(3.1)$, Eqs. (7.42) of [7], and prove the isomorphism at the spinorial level, $\mathcal{P}(3.1) = \mathcal{P}_{O}(3.1)$.

The basic isocasimirs are given by [7]

$$C^{(0)} = \stackrel{\wedge}{l}, \quad C^{(1)} = \stackrel{\wedge}{P^2} = P * P = P_{ij} \stackrel{\wedge}{g}^{\mu\nu} P_{\nu},$$
 (2.11a)

$$C^{(2)} = \hat{W}^{\frac{1}{2}} = \hat{W}_{\mu} \hat{g}^{\mu\nu} \hat{W}_{\nu}, \quad \hat{W}_{\mu} = \epsilon_{\mu\alpha\beta\rho} J^{\alpha\beta} * P^{\rho}.$$
 (2.11b)

The isopoincaré group can then be written

$$\hat{S}: \hat{\psi}' = \left\{ \Pi_k e^{i\hat{S}_k \theta_k} \right\} * \psi = \left\{ \Pi_k e^{i\frac{1}{2}\theta_k \xi_{kij} b_i b_j \gamma_i \gamma_j} \right\} \hat{\psi}, \tag{2.12a}$$

$$\hat{L}: \hat{\psi}' = \left\{ \Pi_k e^{i\hat{L}_k w_k} \right\} * \hat{\psi} = \left\{ \Pi_k e^{i\frac{1}{2}w_k b_k b_4 \gamma_k \gamma_4} \right\} \hat{\psi}, \tag{2.12b}$$

$$\hat{T}(3.1): e_{i\xi}^{iP\eta x^*} = e^{iPT\eta x^*} = e^{iP\mu \eta^* x^*} v.$$
(2.12c)

We now restrict our attention to the global-exterior treatment with constant $b_{\mu}^* = \text{Aver. } \{b_{\mu}(x, \dot{x}, \ddot{x}, \dot{\psi}, \dot{\psi}^{\uparrow}, \partial \dot{\psi}, \partial \dot{\psi}^{\uparrow}, \ldots)\}$. It is easy to see that realization (2.7b) implies the preservation of the conventional spin $\frac{1}{2}$ eigenvalues,

$$\hat{S}^{2} * \hat{\psi} = \{ -\Sigma_{k} \hat{S}_{k} \hat{Q} \hat{S}_{k} \} * \hat{\psi} = \{ (3/4)\hat{I} \} * \hat{\psi} = (3/4) \hat{\psi},$$

$$\hat{S}_{3} * \hat{\psi} = \pm \frac{1}{2} \hat{\psi}. \tag{2.13}$$

Despite that, the shape of the particle is not the perfect sphere with semiaxes (1, 1, 1), but the ellipsoids with semiaxes $(b_1^{2}, b_2^{2}, b_3^{2})$. Via a simple isotopy of conventional derivation (here omitted for brevity), the isodirac equation then implies the desired mutation of the intrinsic magnetic moment which, when printed along the third axis, is given by

$$\hat{\mu} = \frac{b_3^*}{b_4^*} \mu, \quad \hat{m} = \frac{b_3^*}{b_4^*} m, \tag{2.14}$$

as first empirically introduced in [6], Eqs. (4.20, 16), p.803. This concludes our study of the isodirac equation (see [9] for details).

The advances presented in this section are the following. First, we have constructed the most general known, nonlinear, nonlocal and noncanonical realization of the spinorial covering of the Poincare symmetry, and shown the local isomorphism $\mathcal{P}(3.1) = \mathcal{P}_Q(3.1)$ (only realizations of P(3.1) were previously known [7]).

Second, we have generalized the conventional notion of Dirac spinor under local-differential-Lagrangian interactions into a particular class of isospinors with nonlocal-integral-nonlagrangian interactions isoinvariant under $\mathcal{P}_O(3.1)$.

Third, among the infinite possible isospinors, we have selected in this note those permitting a direct representation of nonspherical shapes, their possible deformations, and the consequential mutation of the intrinsic magnetic moment, under the condition of preserving conventional value of spin $\frac{1}{2}$.

Moreover, all these results are derived in a form directly applicable to q-deformations in Q-operator realization [10].

These results are important for applications because they confirm that the mutation of the intrinsic magnetic moment can be represented as a purely geometric-exterior event which, in its simplest possible form, does indeed preserve Pauli's exclusion principle [16] and other nuclear laws [9].

3. Applications to Nuclear Physics

In this section we shall show that isodirac equation (2.5) permits novel applications in nuclear physics, such as a numerical representation of Rauch's interferometric measures (1.1) and a quantitative resolution of the total magnetic moment of few-body nuclear structures.

3.A. Application to Rauch's Measures. Let us recall that experiments [2] essentially test the familiar transformations

$$\psi' = R(\theta_{1})\psi = e^{iS_{1}\theta_{3}/2}\psi = e^{i\gamma_{1}\gamma_{2}\theta_{3}/2}\psi.$$
(3.1)

However, measures (1.1) indicate an apparent 1% deviation from law (3.1). In [12], Vol. II, Sect.VII.2, we provided a nonrelativistic treatment of the problem. In this section we present, apparently for the first time, a relativistic operator treatment of measures (1.1) via Eq. (2.5).

The basic physical event studied in this note is the deformation of shape of the charge distribution of nucleons under sufficiently intense external fields, and the consequential alteration (called mutation) of their intrinsic magnetic moments.

We are here evidently referring to an average deformation for all members of the neutron beam while passing through the electromagnet gap, and definitely not to a constant deformation. It is also evident that, as it occurs in similar events at the classical, atomic and nuclear levels, the original shape is regained after removal of the intense fields. Thus, the neutrons reacquire their original shape and conventional intrinsic magnetic moment, soon after passing through the electromagnet.

The above deformation of shapemutation of magnetic moments is represented in Eq.(2.5) via the transition from the perfect sphere I = diag.(1, 1, 1) to the infinitely-possible ellipsoids $\hat{I} = Q^{-1} = \text{diag.}(b_1^{*-2}, b_2^{*-2}, b_3^{*-2})$ with semiaxes b_k^{*-2} . This lifting essentially expresses the transition from the trivial unit I of O(3) to the isounit \hat{I} of its isotopic covering $\hat{O}_O(3) = O(3)[12]$.

Since the deviation of the mean angle 716° from 720° is small, it is reasonable to assume that it is proportional to the mutation $\hat{\mu}_n$ of the magnetic moment of the neutron μ_n in the intense fields of the Mu-metal nuclei

$$716^{\circ}/720^{\circ} \cong \hat{\mu}_{n}/\mu_{n}.$$
 (3.2)

Eq.(2.5) characterized the isotopies $SU(2) \Rightarrow \hat{SU}_Q(2)$, Eqs.(2.7), with covering isorotations from Eqs.(2.12a)

$$\hat{\Psi}' = \hat{R}(\theta) * \hat{\Psi} = e^{ib_1 b_2 \hat{\gamma}_1 \hat{\gamma}_2 \theta_3/2} \hat{\Psi}. \tag{3.3}$$

Moreover, the isorotational symmetry predicts the following connection between the measured angle and the angle of the exact symmetry

$$\hat{\theta}_3 = b_1^* b_2^* \theta_{3_{|\theta_3| = 715}} = 720^* \tag{3.4}$$

namely, the isotopic methods reconstruct the exact rotational symmetry via a mechanism based in the deformation of the carrier space in such a way to reproduce the angle 720° of the exact symmetry. This is a rather general occurrence for the isorotational symmetry and it is not reviewed here for brevity (see, e.g., [12] or [9], Vol.II, Chapter III).

Since the neutron is a spinning particle, it is rather natural to assume that it possesses a cylindrical symmetry, i.e., $b_1^* = b_2^* \neq b_3^*$. The shape will then be a prolate (oblate) spheroidal ellipsoid if $b_3^{*-2} > b_1^{*-2} = b_2^{*-2}(b_3^{*-2} < b_1^{*-2} = b_2^{*-2})$.

By using as median value $\theta_3 = 716^\circ$, Eq.(3.4) yields $b_1^* = b_2^* = 1.0028$. To compute the mutated magnetic moment from (2.14), we need an independent value of b_4^* . Its best available value is given by $b_4^* = 1.653$ numerically predicted in ref. [7] for the Bose-Einstein correlation and confirmed in ref. [13] via the UA1 data from CERN. This value can effectively be assumed for the neutron because the term b_4^* is a geometrization of the density of the particle considered, while the density of the neutron is of the same order of magnitude as that of the fireball of the $p-\bar{p}$ annihilation.

Thus, the isodirac equation (2.5) provides a direct interpretation of measures (1.1) via the following numerical values of the characteristic b *-quantities

$$b_1^* = b_2^* \approx 1.0028, \quad b_3^* \approx 1.662, \quad b_4^* = 1.653.$$
 (3.5)

with mutated magnitic moments (along the third axis)

$$\mu_n = -1.913 \Rightarrow \hat{\mu}_n = \mu_n b_3^* / b_4^* = -1.902.$$
 (3.6)

Note the oblate character of the deformed neutrons with semiaxes (0.9944, 0.9944, 0.362) which represents a decrease of its magnetic moment. In turn, such a decrease is necessary to represent the angle slow-down effect [5], that is, the fact that all average angles have been systematically lower than 720° for all measures conducted in experiments [2].

Values (3.5) can also be understood from the fact that all unperturbed, spinning charge distributions, thus including the neutron, *are not* expected to be perfectly spherical, but be precisely of oblate spheroidal type, as confirmed by preliminary studies [17] via HM.

In symmary, the isodirac equation permits a direct representation of: 1) nonspherical charge distributions of hadrons via the basic isounit $\hat{l} = \text{diag.}(b_1^{*-2}, b_2^{*-2}, b_3^{*-2}, b_4^{*-2})$, where b_k^{*-2} represents the semiaxes and b_4^{*-2} geometrizes the density of the particle; 2) all possible deformations of these shapes via a dependence of the isounit, e.g., on the intensity E of the external fields $b^* \Rightarrow b(E)$; and 3) the «angle slow-down effect» because predicting a decrease of the intrinsic magnetic moment for the physical conditions considered.

From the viewpoint of the isominkowskian geometry (2.1) a most fundamental aspect is the confirmation of the property that data (3.7) characterize a physical medium in the interior of the neutron of the highest possible Type 9 [7], p.104, which has been consistently obtained for all hadrons with a density equal or bigger than that of the kaons [5,7,9].

Needless to say, there is a possibility that the value $b_4^* = 1.653$ needs adjustments for the neutron, e.g., because of the possible difference in densities with the $p-\bar{p}$ fireball. These, and other improvements, must be deferred to some future time.

3B. Applications to Total Nuclear Magnetic Moments. As well known (see, e.g., [1]), total nuclear magnetic moments are computed via the familiar expressions

$$\mu^{(S)} = g^{(s)}(eh/2m_p c_0) S, \quad g_p^{(s)} = 5.585, \quad g_n^{(s)} = -3.816, \quad e\hbar/2m_p c_0 = 1,$$

$$\mu^{(L)} = g^{(L)}L, \quad g_n^{(L)} = 1, \quad g_n^{(L)} = 0. \tag{3.7}$$

In this note we have provided a quantitative treatment of the old hypothesis (Sect.1) that the intrinsic magnetic moments of nucleons are altered (mutated) when these particles are members of a nuclear structure.

In principle, we have to expect different deformations of the charge distributions of nucleons for different nuclei, different positions in the same nucleus, etc. Our model therefore implies that the total isominkowskian space is the tensorial product $\hat{M}_{\text{tot}} = 11_k^* \hat{M}_k$, k = 1, 2, ..., A, with individual isounits $\hat{I}_k = Q_k^{-1} = \text{diag.}(b_{k1}^{*-2}, b_{k2}^{*-2}, b_{k3}^{*-2}, b_{k4}^{*-2})$, where the densities of p and n, and b_4^* , can be approximately the same for all nucleons.

An isotopy of the conventional QM treatment (here omitted for brevity, see [9] for details), then leads to the following HM model of total nuclear magnetic moments here submitted apparently for the first time

$$\mu_{\text{tot}}^{\text{HM}} = \Sigma_k \left(\hat{g}_k^{(L)} L_{k3} + \hat{g}_k^{(S)} S_{k3} \right),$$

$$\hat{g}_k^{(L)} = 0.605 \ b^*_{k3} g_k^{(L)}, \quad \hat{g}_k^{(S)} = 0.605 \ b^*_{k3} g_k^{(S)}, \quad b_4^* = 1.653.$$
(3.8)

It is easy to see that the above model provides a quantitative resolution of the old problem of total magnetic moments, particularly for few-body nuclear structures. As an illustration, consider the case of the deuteron, which is a p-n bound state in triplet S-state (L=0), with a very small mixture from D-states (L=2), the states with L=1 being unallowed by parity [1]. By ignoring very small corrections, we have the theoretical and experimental values

$$\mu_D^{\text{QM}} = g_p + g_n = 0.879, \quad \mu_D^{\text{Exp}} = 0.857 \ (\mu_p = 1).$$
 (3.9)

Numerous studies of relativistic, orbital and other types have been conducted, but none of them has achieved a numerical representation of the deviation $\Delta \mu_D = \mu_D^{QM} - \mu_D^{Exp} = 0.022$ in a final form, to our best knowledge.

The resolution of the above problem is an ideal application of HM, in general, and of the isodirac equation, in particular. Assume in first the approximation that the deformations of shape of the proton and neutron are the same. Application of (3.8) then yields the representation

$$\mu_D^{\text{HM}} = 0.605 \ b_3^* (g_p + g_n) \equiv \mu_D^{\text{Exp}} = 0.857, \ b_3^* = 1.611,$$
 (3.10)

where the value b_3^* is considerably in line with that of data (3.5).

Needless to say, calculations (3.10) have been presented to the capability of the isotopic methods. More accurate and computerized calculations are forthcoming.

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