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TUTORIAL ON PARAMETERIZED RELATIVISTIC DYNAMICS

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Abstract

The Standard Model of electroweak and strong interactions relies on quantum field theory (QFT). QFT is based on two key assumptions: fields depend on space and time; and interactions between fields are local. The resulting view of nature has been very successful at explaining physical observations and is considered the conventional paradigm. Despite its record of success, some phenomena are anomalies that seem to require a modification of the Standard Model. The anomalies include neutrino oscillations, non-locality, and gravity. Most physicists are seeking to understand the anomalies within the context of QFT. Parameterized relativistic dynamics (PRD) is an alternative approach that extends two key assumptions of QFT. PRD is a non-local theory that allows fields to depend on space, time and an invariant evolution parameter. The study of PRD helps us understand how our view of nature can change when key assumptions of QFT are changed. This paper reviews parameterized relativistic dynamics and presents applications that can be directly compared with results from the conventional paradigm.

Contents

Section	Topic	Page
1	Introduction	353
2	Competing Paradigms and Scientific Revolutions	357
3	Classical Mass	363
4	Classical Precursor of a Parameterized Wave Equation	373
5	Postulates for Quantizing Classical Theory	377
6	Feynman's Path Integral Formulation	382
7	Symmetries in Relativistic Theories	385
8	Interpretations of Theoretical Formalisms	392
9	Probabilistic Formulation of Parameterized Relativistic Dynamics	394
10	Stueckelberg Equation for an N-Body System	402
11	Measuring the Evolution Parameter	405
12	Non-Locality	410
13	Experimental Test: Interference in Time	417
14	Experimental Test: K-meson Mass State Transition	420
15	Experimental Test: Neutrino oscillations	428
16	Experimental Test: Tachyon Physics	436
17	Conclusions	443
	Acknowledgements	444
Appendix	Notation and Metric	445
	References	447

1. Introduction

Isaac Newton developed a classical mechanics that treated time as a monotonically increasing evolution parameter. Newton's classical mechanics was invariant with respect to temporal translations in Newtonian time. The recognition that energy is quantized required the replacement of non-relativistic classical mechanics with non-relativistic quantum mechanics. The temporal symmetry of non-relativistic classical mechanics was preserved in the new non-relativistic quantum mechanics. Defining ρ as probability density and $\rho\vec{v}$ as probability flux, we can derive the non-relativistic probability conservation equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0 \tag{1.1}$$

from Schroedinger's equation. Albert Einstein rejected Newton's concept of time in the early 20th century in favor of an alternative view of time.

Physics was in a period of transition during the first 30 years of the 20th century. The theoretical frameworks for relativity and quantum theory were evolving. During this period, changes to fundamental concepts were being considered, including the relative merits of two temporal hypotheses. Most physicists are familiar with the hypothesis adopted by Einstein, namely that Newton's concept of absolute time must be replaced by the view that time was a component of a space-time four-vector. This view, which we call Hypothesis I, led to a probability conservation equation that could be expressed in covariant form

$$\partial_{\mu}(\rho V^{\mu}) = 0 \tag{1.2}$$

where $V^{\mu} = \{c, \vec{v}\}$. The notation and metric are defined in the Appendix. Equation (1.2) can be derived from both the Klein-Gordon and Dirac equations.

Acceptance of Hypothesis I led to the paradox that one time, Einstein's time, must satisfy two temporal concepts: time is a coordinate, and time is an arrow. The H-theorem in thermodynamics is often used to explain the apparent directionality of the time domain, but an alternative approach is possible.

An alternative temporal hypothesis was first considered by Fock [1937] and Stueckelberg [1941-42]. Fock and Stueckelberg introduced the idea that we should be working with two temporal variables. We formalize this idea as Hypothesis II, which says that relativistic quantum theory should include Einstein's coordinate time x^0, and an invariant evolution parameter s that is analogous to Newton's concept of absolute time. If we adopt Hypothesis II, we find a new term in the covariant probability conservation equation, thus

$$\frac{\partial \rho}{\partial s} + \partial_{\mu}(\rho V^{\mu}) = 0 \tag{1.3}$$

The new term is the derivative of probability density with respect to s. The implications of Hypothesis II are reviewed in this paper.

Hypotheses I and II are examples of competing hypotheses. We gain insight into the nuances of competing hypotheses by discussing them as key concepts of competing paradigms. A paradigm is the set of beliefs that underlie a particular world-view [Kuhn, 1962]. We call the paradigm that has been adopted by the current mainstream of practitioners the conventional paradigm, and we define normal science as the research program that is based on the conventional

paradigm. Normal science today is the conventional paradigm based on quantum field theory (QFT). QFT is the theoretical framework that underlies theories of the electroweak and strong interactions (the Standard Model). It also includes work by researchers who are attempting to develop a QFT theory of the gravitational interaction. Wilczek [1999] provides a succinct discussion of general principles, an outline of the intellectual development of QFT, and a few comments about the limitations of QFT. An authoritative presentation of QFT is the three-volume treatise written by Weinberg [1995, 1996, 2000]. We refer to the conventional paradigm of normal science as the conventional theory.

We focus in this paper on comparing the conventional paradigm with an alternative paradigm referred to here as Parameterized Relativistic Dynamics (PRD). PRD is a manifestly covariant quantum theory with invariant evolution parameter.

Proponents of the conventional paradigm, as well as all other readers, should approach the presentation of PRD with skepticism. After all, the conventional paradigm has been very successful. Section 2 places the conventional paradigm and PRD in the broad context of the advance of science. The reader will find that PRD provides insight into the meaning and value of assumptions that have been made in the development of the conventional paradigm. There are also some useful techniques and experimental predictions that can inspire future work.

This paper is organized as follows. We acknowledge the success of the conventional paradigm in Section 2, remind the reader of some of its limitations,

and discuss two key assumptions that distinguish the conventional paradigm and PRD. One common concept that changes as a result of a change in paradigm is the concept of mass, which is discussed from a classical perspective in Section 3. A classical precursor of a parameterized relativistic wave equation is developed in Section 4 using parameterized Hamilton-Jacobi theory. The postulates for quantizing PRD are presented in Section 5. Several wave equations are possible in PRD. The choice of wave equation is motivated by considering the parametric evolution of the wave function in a path integral formulation (Section 6) and a formulation in terms of symmetries (Section 7). Possible interpretations of the formalism are discussed in Section 8 and evaluated in the context of a probabilistic formulation in Section 9. In addition, the meaning of mass in the quantum theory is discussed in Section 9. The extension of the probabilistic formulation to describe an N-body system is presented in Section 10. This extension facilitates application of the formalism.

Several applications of PRD that replicate results of the conventional paradigm are documented in Fanchi [1993b] and Pavsić [2001]. We focus here on applications that lead to differences between PRD and the conventional paradigm because these differences serve to highlight the consequences of the assumptions that underlie each paradigm.

Section 11 presents the construction of a "clock" for measuring the invariant evolution parameter s. PRD is then shown to be a non-local theory in Section 12. Some applications that compare results of PRD with results from the conventional paradigm are discussed in Sections 13 through 16. The reader may

be surprised to learn that, despite the success of the conventional paradigm, it does have limitations and there are areas where experimental tests should be conducted. Concluding remarks are provided in Section 17.

2. Competing Paradigms and Scientific Revolutions

Parameterized Relativistic Dynamics may be viewed as a research program or paradigm that is competing with the conventional theory. The competition between paradigms is illustrated by the two hypotheses discussed in the previous section. In particular, Hypothesis I recognizes one time (Einstein's time) and is part of the conventional paradigm of normal science. By contrast, Hypothesis II recognizes two times (Einstein's time and an invariant evolution parameter) and is part of the PRD paradigm.

Practitioners of normal science test the predictions of the conventional paradigm, attempt to extend the applicability of the paradigm, and try to solve puzzles or anomalies that may arise. Periods of crisis occur when the existence of anomalies cannot be ignored. The existence of competing paradigm(s) can be useful because they may provide insight or an alternative perspective that will help resolve anomalies and advance science. Problems can occur between proponents of competing paradigms because there can be differences in concepts and terminology that can lead to a breakdown of communication within a discipline.

When anomalies cannot be resolved within normal science, it may become

necessary to replace the paradigm of normal science with a new paradigm. This paradigm shift is analogous to a gestalt shift. In psychology, gestalt refers to the way we perceive the environment. Figure 1 is an example of a picture that can look like two different images depending on the way you view the image. The picture may look like a young woman or an old lady. The change in perception is a change in world-view, or gestalt shift.

Figure 1. A change in perspective can show an old lady or a young woman [Hill, 1915]

Kuhn argued that scientific revolutions occur when the paradigm of normal science is replaced by a new paradigm. Acceptance of the paradigm shift may occur when resistance to the new paradigm is overcome. This may happen as

a result of convincing experiment(s) or convincing argument(s). In some cases, there will simply be attrition of adherents to the old paradigm. The resolution of anomalies and the closure of the period of crisis occurs when the new paradigm becomes the paradigm of normal science. Figure 2 summarizes Kuhn's view of scientific revolutions.

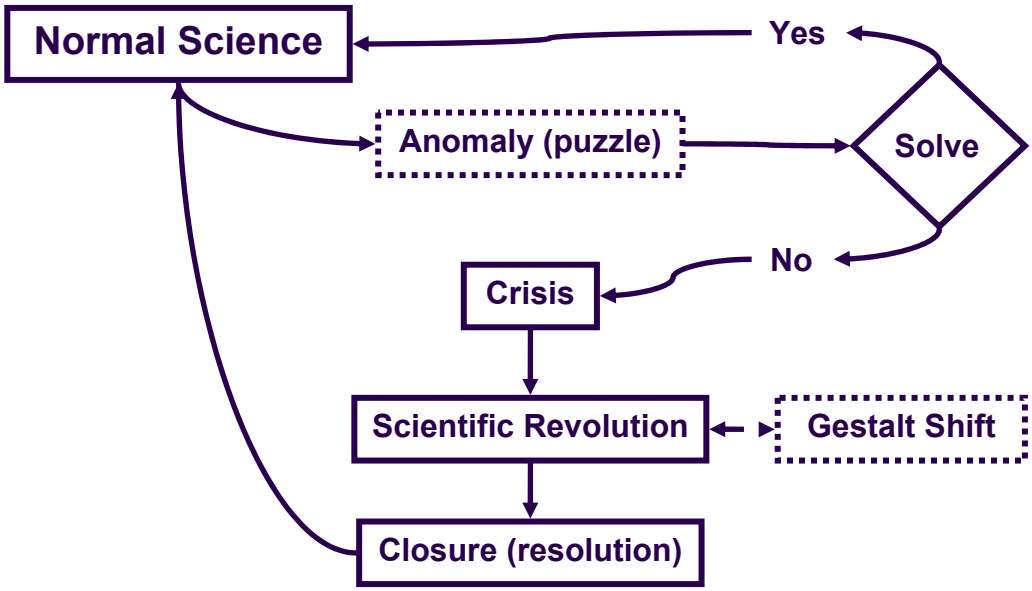

Figure 2. Schematic outline of Kuhn's view of scientific revolutions

The conventional paradigm has been quite successful at explaining a range of theoretical results. The Particle Data Group [Amsler, et al., 2008] periodically compiles experimental results and a selection of theoretical attempts to understand the results. Precision tests of quantum electrodynamics, such as the calculation of

the anomalous magnetic moment of the electron, agree with measured values to an accuracy of one part per billion or better.

Despite this record of success, there are anomalies. The Standard Model uses quarks and leptons as the building blocks of the particle zoo, yet no quark has ever been observed. The existence of quarks is inferred from a sizable body of experimental evidence. The masses of quarks and leptons are some of the parameters that must be entered into the Standard Model.

The Standard Model depends on the Higgs particle, which has yet to be discovered.

The Standard Model assumes that neutrinos are massless, yet neutrino oscillation experiments imply that neutrinos have mass. Neutrinos with mass can be included in an extension of the Standard Model, but this requires the use of more unexplained parameters.

The Standard Model does not account for gravity. A theory of gravity is needed that is consistent with QFT and is able to yield general relativity as the classical approximation. In a related context, proponents of the conventional paradigm are continuing to seek a better understanding of cosmological issues such as inflation, dark matter, and dark energy.

The Standard Model is local, yet experiments show that non-local effects occur in nature.

In addition to these anomalies, there are some practical issues associated with the formalism of the conventional paradigm. Historically, relativistic QFT has had conceptual and technical difficulties, such as divergences and infinities

that require renormalization, and the need to use perturbative techniques to solve problems. For example, it is difficult to solve relativistic bound state problems using the formalism of the conventional paradigm.

Proponents of the conventional paradigm view the anomalies as problems that will eventually be solved within the general framework of the conventional paradigm. Difficulties with the formalism are issues that can be overcome by technological advances in areas such as mathematics or computing.

It is possible that the perspective of proponents of the conventional paradigm is correct. If the practice of science was democratic, the conventional paradigm would win in a landslide. The history of science has shown, however, that progress sometimes depends on the development and eventual adoption of unconventional views. It is possible that the anomalies and formalistic difficulties are indicating that there is a problem with the conventional paradigm.

Two features of the conventional paradigm that are of interest here are the following:

C1. The conventional paradigm uses Einstein's concept of time. A consequence of this assumption is a dependency between energy and momentum. In the case of a free particle, the dependency is imposed in integrals by the delta function $\delta\left(m^2 - p^\mu p_\mu / c^2\right)$ where m is mass and p^μ is the energy-momentum four-vector.

C2. The conventional paradigm is a local theory. In the case of a free particle, only time-like solutions in which $p^\mu p_\mu > 0$ are allowed.

For comparison, the corresponding features in parameterized relativistic dynamics are extensions of assumptions C1 and C2, namely:

P1. PRD uses Einstein's concept of time and an invariant evolution parameter. A consequence of this assumption is that energy and momentum are independent.

P2. PRD is a non-local theory.

Figure 3 compares these features and shows that the PRD assumptions are an extension of the assumptions of the conventional paradigm.

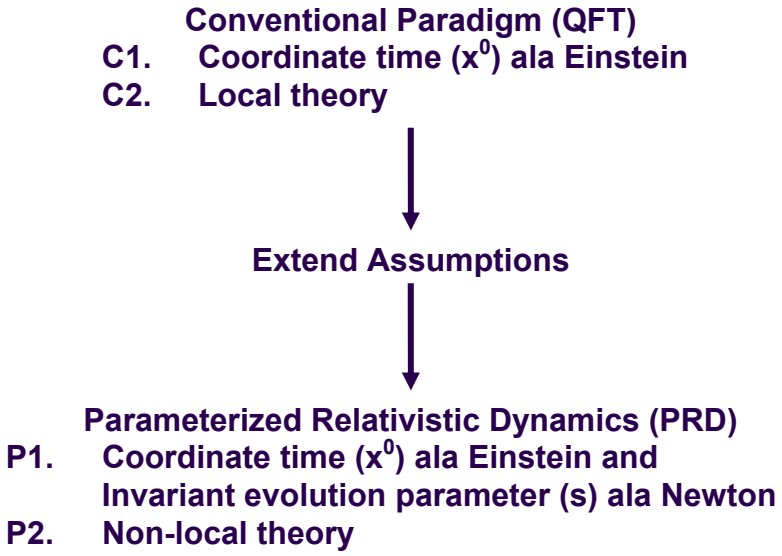

Figure 3. Comparison of Competing Assumptions

There are several ideas that are in competition between normal science and PRD. Differences between the assumptions in Figure 3 are the most fundamental. Other ideas are in competition, but depend on the details of the competing paradigm. To be specific, the competing paradigm considered here is the manifestly covariant quantum theory with invariant evolution parameter, or parameterized relativistic dynamics (PRD), that was first introduced by Fock and Stueckelberg, than rediscovered and extended by Horwitz and Piron [1973], and by Fanchi and Collins [1978].

3. Classical Mass

The concept of mass is fundamental to our understanding of nature, yet the meaning of the concept in PRD differs from the meaning in the conventional paradigm. We can begin to understand the difference by considering the concept of mass in the classical mechanics of PRD. This requires the development of relativistic classical mechanics with invariant evolution parameter.

Lagrangian Formulation of Parameterized Relativistic Classical Mechanics

We develop relativistic classical mechanics with invariant evolution parameter by invoking Hamilton's variational principle. First define a parameter-dependent action

$$A = \int_{s_1}^{s_2} L_s(q_1,...,q_n,\dot{q}_1,...,\dot{q}_n,s)ds \tag{3.1}$$

where the Lagrangian in the integrand is defined in terms of n generalized coordinates $(q_1,\ldots,q_n)$ and an invariant parameter s with the unit of time. A dot over a generalized coordinate denotes differentiation with respect to the parameter s. The subscript s in the symbol L_s has been written to remind us that L_s is the Lagrangian of special relativity. The instantaneous configuration of a system is described by the values of the n generalized coordinates $q_1,\ldots,q_n$ and corresponds to a point in an n-dimensional configuration space. The axes of the n-dimensional space are formed by the n generalized coordinates. The invariant parameter s traces the evolution of a system point in configuration space between fixed parameter end points s_1 and s_2. Notice that the evolution parameter s is not a fifth coordinate as in a Kaluza-Klein theory. It is a relativistic scalar that is mathematically independent of the generalized coordinates.

The parameterized Lagrangian has the functional form

$$L_s = L_s(q^\mu,\dot{q}^\mu,s) = L_s(q^0,q^1,q^2,q^3,\dot{q}^0,\dot{q}^1,\dot{q}^2,\dot{q}^3,s) \tag{3.2}$$

where

$$\dot{q} = \frac{dq}{ds} \tag{3.3}$$

As the invariant parameter s monotonically increases, the system point moves in configuration space. Hamilton's principle for conservative systems states that the motion of a system point in configuration space between fixed parameter end

points s_1 and s_2 must be such that the action is an extremum. A covariant formulation of Hamilton's principle requires that the variation of action A must vanish:

$$\delta A = 0 = \int_{s_1}^{s_2} \delta L_s \left(q^\mu, \dot{q}^\mu, s\right) ds \tag{3.4}$$

Variation of the Lagrangian gives

$$\delta L_s = \frac{\partial L_s}{\partial q^\mu} \delta q^\mu + \frac{\partial L_s}{\partial \dot{q}^\mu} \delta \dot{q}^\mu \tag{3.5}$$

where the Einstein summation convention applies. An integration by parts applied to the $\delta \dot{q}^\mu$ term on the right hand side of Eq. (3.5) yields

$$\int_{s_1}^{s_2} \frac{\partial L_s}{\partial \dot{q}^\mu} \delta \left[\frac{dq^\mu}{ds} \right] ds = \int_{s_1}^{s_2} \frac{\partial L_s}{\partial \dot{q}^\mu} \frac{d}{ds} \left(\delta q^\mu\right) ds = -\int_{s_1}^{s_2} \left[\frac{d}{ds} \frac{\partial L_s}{\partial \dot{q}^\mu} \right] \left(\delta q^\mu\right) ds \tag{3.6}$$

where the condition that the variation δq^μ vanish at the parameter end points has been invoked. Inserting Eq. (3.6) into Eq. (3.4) gives

$$\delta A = 0 = \int_{s_1}^{s_2} \left[\frac{\partial L_s}{\partial q^\mu} - \frac{d}{ds} \frac{\partial L_s}{\partial \dot{q}^\mu} \right] \left(\delta q^\mu\right) ds \tag{3.7}$$

Equation (3.7) is valid for arbitrary variations δq^μ only if the coefficients of the variations vanish. Consequently, the condition that Eq. (3.7) be valid requires that the coefficient of δq^μ vanish, thus

$$\frac{\partial L_s}{\partial q^\mu} - \frac{d}{ds} \frac{\partial L_s}{\partial \dot{q}^\mu} = 0; \mu = 0,1,2,3 \tag{3.8}$$

Equation (3.8) is the parameterized Euler-Lagrange equation.

Hamiltonian Formulation of Parameterized Relativistic Classical Mechanics

A procedure for transforming from the classical Lagrangian formulation to the Hamiltonian formulation is demonstrated here for a single particle Lagrangian with the functional form in Eq. (3.2). The generalized coordinates are elements of the space-time four-vector. The Euler-Lagrange equations for Eq. (3.2) are given in Eq. (3.8).

Canonical equations can be derived using the Legendre transformation

$$\begin{aligned} K\left(q^{\mu}, p_{\mu}, s\right) &= \dot{q}^{\mu} p_{\mu} - L_s\left(q^{\mu}, \dot{q}^{\mu}, s\right) \\ &= K\left(q^0, q^1, q^2, q^3, p_0, p_1, p_2, p_3, .s\right) \end{aligned} \tag{3.9}$$

where K is the Hamiltonian function and the Einstein summation convention applies. The differential of the Hamiltonian K is calculated directly from the left hand side of Eq. (3.9) as

$$dK = \frac{\partial K}{\partial q^{\mu}} dq^{\mu} + \frac{\partial K}{\partial p_{\mu}} dp_{\mu} + \frac{\partial K}{\partial s} ds \tag{3.10}$$

The differential of the right hand side of Eq. (3.9) is

$$dK = \dot{q}^{\mu} dp_{\mu} + p_{\mu} d\dot{q}^{\mu} - \frac{\partial L_s}{\partial q^{\mu}} dq^{\mu} - \frac{\partial L_s}{\partial \dot{q}^{\mu}} d\dot{q}^{\mu} - \frac{\partial L_s}{\partial s} ds \tag{3.11}$$

Equating Eqs. (3.10) and (3.11) leads to the relationships

$$\frac{\partial K}{\partial s} = -\frac{\partial L_s}{\partial s} \tag{3.12}$$

$$p_\mu = \frac{\partial L_s}{\partial \dot{q}^\mu} \tag{3.13}$$

$$\dot{q}^\mu = \frac{\partial K}{\partial p_\mu} \tag{3.14}$$

$$\frac{\partial K}{\partial q^\mu} = -\frac{\partial L_s}{\partial q^\mu} \tag{3.15}$$

Differentiating Eq. (3.13) with respect to s gives

$$\dot{p}_\mu = \frac{d}{ds}\frac{\partial L_s}{\partial \dot{q}_\mu} = \frac{\partial L_s}{\partial q^\mu} = -\frac{\partial K}{\partial q^\mu} \tag{3.16}$$

The resulting canonical equations are Hamilton's equations:

$$\dot{p}_\mu = -\frac{\partial K}{\partial q^\mu} \tag{3.17}$$

and

$$\dot{q}^\mu = \frac{\partial K}{\partial p_\mu} \tag{3.18}$$

Poisson Bracket

The rate of change of an invariant function $F(q^\lambda, p^\lambda, s)$ with respect to the invariant evolution parameter s may be evaluated using Hamilton's equations.

We begin with the expansion

$$\frac{d}{ds}F\left(q^{\lambda},p^{\lambda},s\right)=\frac{\partial F}{\partial s}+\frac{\partial F}{\partial q^{\lambda}}\frac{dq^{\lambda}}{ds}+\frac{\partial F}{\partial p^{\lambda}}\frac{dp^{\lambda}}{ds} \tag{3.19}$$

Substituting Eqs. (3.17) and (3.18) into (3.19) gives

$$\frac{dF}{ds}=\frac{\partial F}{\partial s}+\{F,K\} \tag{3.20}$$

where $\{F,K\}$ is the Poisson bracket

$$\{F,K\}\equiv\frac{\partial F}{\partial q^{\lambda}}\frac{\partial K}{\partial p_{\lambda}}-\frac{\partial K}{\partial q_{\lambda}}\frac{\partial F}{\partial p^{\lambda}} \tag{3.21}$$

The Poisson Bracket is useful for studying the conservation properties of a physical variable. The invariant function F is constant, or conserved, with respect to s if $dF/ds=0$. Thus, if F represents a physical variable that does not depend explicitly on s, that variable is conserved with respect to s if the Poisson bracket equals zero, that is, $\{F,K\}=0$.

Mass of a Classical Free Particle

We can use the Lagrangian and Hamiltonian formulations to discuss the mass of a classical free particle. The Hamiltonian for a free particle with constant rest mass M can be written as

$$K_f=\frac{g^{\mu\nu}}{2M}p_{\mu}p_{\nu} \tag{3.22}$$

The state of the particle is characterized by eight independent numbers

$$q^{\mu} = \left(q^{0}, q^{1}, q^{2}, q^{3}\right) = \left(ct, \vec{q}\right)$$
$$p_{\mu} = \left(p_{0}, p_{1}, p_{2}, p_{3}\right) = \left(E/c, -\vec{p}\right) \tag{3.23}$$

where c is the speed of light in vacuum. The q's are elements of the space-time four-vector and the p's are elements of the momentum-energy four-vector. Substituting Eq. (3.22) into the canonical equations (3.17) and (3.18) gives

$$\dot{q}^{\mu} = p^{\mu}/M \tag{3.24}$$

and

$$\dot{p}_{\mu} = 0 \tag{3.25}$$

The Lagrangian corresponding to Eq. (3.22) follows immediately, thus

$$L_{f} = \frac{g^{\mu\nu}}{2M}\left(M\dot{q}_{\mu}\right)\left(M\dot{q}_{\nu}\right) = \frac{M}{2}\dot{q}^{\nu}\dot{q}_{\nu} \tag{3.26}$$

Some familiar results of special relativity follow from Eqs. (3.24) and (3.25). For example, from Eq. (3.25) we find that four-momentum is a constant:

$$p_{\mu} = p_{0\mu} \tag{3.27}$$

Substituting Eq. (3.27) into (3.24) and integrating with respect to s gives

$$q^{\mu} = q_{0}^{\mu} + \frac{p_{0}^{\mu} s}{M} \tag{3.28}$$

where q_{0}^{μ} is an integration constant. Equation (3.28) shows that the trajectory of a free relativistic particle depends linearly on the evolution parameter s. Varying the space-time four-vector and forming its inner product gives

$$\delta q^{\mu}\delta q_{\mu} = \frac{p_{0}^{\mu} p_{0\mu}}{M^{2}}\left(\delta s\right)^{2} \tag{3.29}$$

Taking the initial condition as

$$M^2 = \frac{p_0^{\mu} p_{0\mu}}{c^2} \tag{3.30}$$

yields the usual relation between classical invariant "proper time" and the space-time four-vector:

$$\delta q^{\mu} \delta q_{\mu} = c^2 (\delta s)^2 \tag{3.31}$$

Remember that this "proper time" is the independent evolution parameter s associated with the action (3.1). Furthermore, we see that the mass of the classical free particle is a constant of the motion. Pavsic showed that mass appears in PRD as a constant of motion in classical mechanics [Pavsic, 1991a, 2001], and in both first and second quantized formulations [Pavsic, 1991b, 2001].

Mass of a Classical Charged Particle in an Electromagnetic Field

The Hamiltonian for a particle with electric charge e in an external electromagnetic field described by the four-vector potential A^{μ} is obtained from Eq. (3.31) by introducing the concept of "minimal coupling", i.e.

$$K = \frac{g^{\mu\nu}}{2M} \left[p_{\mu} - \frac{e}{c} A_{\mu} \right] \left[p_{\nu} - \frac{e}{c} A_{\nu} \right] \tag{3.32}$$

where the four-vector potential $A^{\mu}(q)$ may depend on q^{μ} but not on s. Substituting Eq. (3.32) into the canonical equations (3.17) and (3.18) gives

$$\dot{q}_\mu = \frac{\left[p_\mu - \frac{e}{c} A_\mu \right]}{M} = \frac{\pi_\mu}{M} \tag{3.33}$$

and

$$\dot{p}_\mu = \frac{e}{Mc}\left[p^\nu - \frac{e}{c} A^\nu \right] \partial_\mu A_\nu = \frac{e}{Mc} \pi^\nu \partial_\mu A_\nu \tag{3.34}$$

The kinetic momentum π^μ is

$$\pi^\mu = p^\mu - \frac{e}{c} A^\mu \tag{3.35}$$

The Lorentz force is derived from the canonical equations by substituting Eq. (3.33) into (3.34) to get

$$\dot{p}_\mu = \frac{e}{c} \dot{q}^\nu \partial_\mu A_\nu \tag{3.36}$$

Differentiating Eq. (3.35) with respect to s gives

$$\dot{\pi}_\mu = \dot{p}_\mu - \frac{e}{c}\frac{dA_\mu}{ds} = \dot{p}_\mu - \frac{e}{c}\frac{\partial A_\mu}{\partial q^\nu}\frac{dq^\nu}{ds} = \dot{p}_\mu - \frac{e}{c}\partial_\nu A_\mu \dot{q}^\nu \tag{3.37}$$

Rearranging Eq. (3.37) and combining with Eq. (3.36) lets us write

$$\dot{p}_\mu = \dot{\pi}_\mu + \frac{e}{c}\partial_\nu A_\mu \dot{q}^\nu = \frac{e}{c}\dot{q}^\nu \partial_\mu A_\nu \tag{3.38}$$

Solving for the rate of change of kinetic momentum with respect to s yields the Lorentz force equation

$$\dot{\pi}_\mu = \frac{e}{c} F_{\mu\nu} \dot{q}^\nu \tag{3.39}$$

where $F_{\mu\nu}$ is the anti-symmetric field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{3.40}$$

The mass of the charged particle in an electromagnetic field is found by forming the inner product of the Lorentz force with π^μ:

$$\dot{\pi}_\mu \pi^\mu = \frac{1}{2}\frac{d}{ds}\left(\pi_\mu \pi^\mu\right) = \frac{e}{c} F_{\mu\nu} \dot{q}^\nu \pi^\mu \tag{3.41}$$

Using Eq. (3.33) in (3.41) lets us write

$$\frac{1}{2}\frac{d}{ds}\left(\pi_\mu \pi^\mu\right) = \frac{e}{Mc} F_{\mu\nu} \dot{q}^\nu \dot{q}^\mu \tag{3.42}$$

Summation over μ, ν in the latter term leads to

$$F_{\mu\nu} \dot{q}^\nu \dot{q}^\mu = 0 \tag{3.43}$$

since $F_{\mu\nu}$ is an anti-symmetric tensor. Equation (3.42) becomes

$$\frac{d}{ds}\left(\pi_\mu \pi^\mu\right) = 0 \tag{3.44}$$

which implies

$$\pi_\mu \pi^\mu = \pi_{0\mu} \pi_0^\mu \equiv \kappa^2 \tag{3.45}$$

where κ is a constant of integration with respect to s. To be consistent with the initial condition specified in Eq. (3.30) for arbitrary A^μ, including $A^\mu = 0$ for all μ, we must have

$$\kappa^2 = M^2 c^2 \tag{3.46}$$

The constant of the motion corresponds to classical mass.

The important point to emphasize here is that energy and momentum – not mass – are the primary variables. In accelerator experiments, energy can be measured by a calorimeter and momentum can be measured using the trajectory of

a charged particle in a magnetic field. Mass is not measured; mass is inferred from energy and momentum.

It is well known that taking energy and momentum as the observables in the Lagrangian lets us include light-like particles with $M = 0$ as a viable solution. Physical examples of light-like particles are photons. In the $M = 0$ case, the Hamiltonian K in Eq. (3.32) would have to be changed. A parameterized extension of electrodynamics is discussed by Saad, et al. [1989] and Land [2005a, b and references therein].

4. Classical Precursor of a Parameterized Wave Equation

Historically Hamilton-Jacobi theory has served as a classical stepping stone to wave mechanics. By surmising that the Hamilton-Jacobi equation was the short wavelength limit of a wave equation for matter, Schrödinger was led to his formulation of wave mechanics. An analogous approach can be used to develop a parameterized relativistic Hamilton-Jacobi formulation.

Before we can proceed to the Hamilton-Jacobi formulation, we must first introduce the concept of a canonical transformation. For ease of reference, let us recall Hamilton's equations of motion:

$$\dot{q} = \frac{\partial K}{\partial p}, \dot{p} = -\frac{\partial K}{\partial q} \tag{4.1}$$

We are interested in transformations of coordinates and momenta from the set $\{q, p\}$ to a new set $\{Q, P\}$ which preserves the form of Hamilton's equations; thus

$$\dot{Q} = \frac{\partial K}{\partial P}, \dot{P} = -\frac{\partial K}{\partial Q} \tag{4.2}$$

Transformations which preserve the form of Hamilton's equations are canonical transformations.

Another way to think about canonical transformations is to insist that the form of Hamilton's principle be preserved. Thus, if Hamilton's principle has the form

$$\delta \int_{s_1}^{s_2} \left[\sum_i p_i \dot{q}_i - H(q, p, s) \right] ds = 0 \tag{4.3}$$

for the set $\{q, p\}$ of generalized coordinates, then $\{Q, P\}$ are related to $\{q, p\}$ by a canonical transformation if Hamilton's principle has the form

$$\delta \int_{s_1}^{s_2} \left[\sum_i P_i \dot{Q}_i - K(Q, P, s) \right] ds = 0 \tag{4.4}$$

The integrands must be equal to within the total derivative of a generating function F such that

$$\delta \int_{s_1}^{s_2} \frac{dF}{ds} ds = 0 \tag{4.5}$$

Possible forms of the generating function include $F_1(q, Q, s)$, $F_2(q, P, s)$, $F_3(p, Q, s)$ and $F_4(p, P, s)$.

An example that helps guide a search for a parameterized wave equation begins with the generating function F_1. The integrands are related by the expression

$$\sum_i p_i \dot{q}_i - H = \sum_i P_i \dot{Q}_i - K + \frac{d}{ds} F_1(q, Q, s) \tag{4.6}$$

Expanding dF_1/ds gives

$$\frac{dF_1}{ds} = \sum_i \frac{\partial F_1}{\partial q_i} \dot{q}_i + \sum_i \frac{\partial F_1}{\partial Q_1} \dot{Q}_i + \frac{\partial F_1}{\partial s} \tag{4.7}$$

Recognizing that the coordinates q_i, Q_i are independent, the coefficients of dq_i/ds and dQ_i/ds must vanish. Combining terms yields

$$p_i = \frac{\partial F_1}{\partial q_i}, P_i = -\frac{\partial F_1}{\partial Q_i} \tag{4.8}$$

so that

$$K = H + \frac{\partial F_1}{\partial s} \tag{4.9}$$

Equation (4.9) applies to all generating functions, i.e.

$$K = H + \frac{\partial F_j}{\partial s}, j = \{1,2,3,4\} \tag{4.10}$$

If we write

$$F_2(q, P, s) = F_1(q, Q, s) + \sum_i P_i Q_i \tag{4.11}$$

than we can use the above procedure to derive the transformation equations

$$p_i = \frac{\partial F_2}{\partial q_i}, Q_i = \frac{\partial F_2}{\partial P_i} \tag{4.12}$$

and Eq. (4.10) with $j = 2$. We are especially interested in F_2 because of its relation to Hamilton's principal function.

HAMILTON'S PRINCIPAL FUNCTION

Given the above mathematical tools, we want to find a canonical transformation to a coordinate system that has variables Q, P that are constant in s, i.e.

$$\begin{aligned} \frac{\partial K}{\partial P} &= \dot{Q} = 0, \\ -\frac{\partial K}{\partial Q} &= \dot{P} = 0 \end{aligned} \tag{4.13}$$

Such a transformation can be found if the generating function F_2 is chosen to be the principal function S_{HJ} satisfying

$$K = H + \frac{\partial S_{HJ}}{\partial s} = 0 \tag{4.14}$$

or

$$H(q, p, s) + \frac{\partial S_{HJ}}{\partial s} = 0 \tag{4.15}$$

The principal function S_{HJ} is the solution of Eq. (4.10) with the additional constraint that the transformed Hamiltonian $K = 0$. If the transformation exists, then K is the constant 0 and Eqs. (4.13) are satisfied.

We recast Eq. (4.15) by recalling the relation

$$p = \frac{\partial F_2}{\partial q} \tag{4.16}$$

from Eq. (4.12). The result is the Hamilton-Jacobi equation expressed in terms of the principal function:

$$H\left(q, \frac{\partial S_{HJ}}{\partial q}, s\right) + \frac{\partial S_{HJ}}{\partial s} = 0 \tag{4.17}$$

As an example, the Hamilton-Jacobi equation for a charged particle in an electromagnetic field becomes

$$\frac{g^{\mu\nu}}{2M}\left[\partial_\mu S_{HJ} - eA_\mu\right]\left[\partial_\nu S_{HJ} - eA_\nu\right] + \frac{\partial S_{HJ}}{\partial s} = 0 \tag{4.18}$$

where the Hamiltonian is given by Eq. (3.32). Recalling that the non-relativistic Hamilton-Jacobi description was a precursor to a wave mechanical description of matter in the short wavelength limit, Eq. (4.18) gives us an idea of what to expect from a wave equation with an invariant evolution parameter for the given Hamiltonian.

As a historical note, Fock [1937] used a Hamilton-Jacobi equation of the form of Eq. (4.18) as the basis for a quantum theory. He added the constraint that all of his final solutions must be stationary (independent of s). Fock's work was the first attempt to incorporate an evolution parameter into a relativistic formulation of quantum theory.

5. Postulates for Quantizing Classical Theory

The modern formulation of parameterized relativistic dynamics can be expressed in several different ways. Each formulation provides a different

perspective and gives us insight into the meaning of parameterized relativistic dynamics and its relationship to the conventional theory. The formulation considered here is a formulation in terms of postulates that were introduced by Horwitz and Piron [1973].

Conventional Postulates

Horwitz and Piron [1973] presented three conventional postulates for quantizing classical theory.

P1. The states of a physical system are characterized by unit rays in a normed, linear vector space (Hilbert space)

P2. Superposition applies: if two states of a system are represented by ψ_1 and ψ_2, then $\psi = a_1\psi_1 + a_2\psi_2$ is also a state of the system.

P3. Associate a Lie algebra with a set of transformations.

- Physical measurements are associated with generators of transformations
- Each physical observable is represented by a linear Hermitian operator

They then introduced a fourth postulate that applied to the evolution of a state. Is there a need for a fourth postulate? We demonstrate the need for a fourth postulate that alters our understanding of time by conducting a thought experiment.

Suppose we want to measure the particle world-line of a non-interacting particle in terms of space-time coordinates. We design an experiment with clocks and detectors at points A and B as shown in Figure 4. A particle source at the left of the figure sends a particle between points A and B. The clocks and detectors

measure the space-time coordinates for each event. We repeat the measurements a statistically significant number of times to obtain an ensemble of measurements.

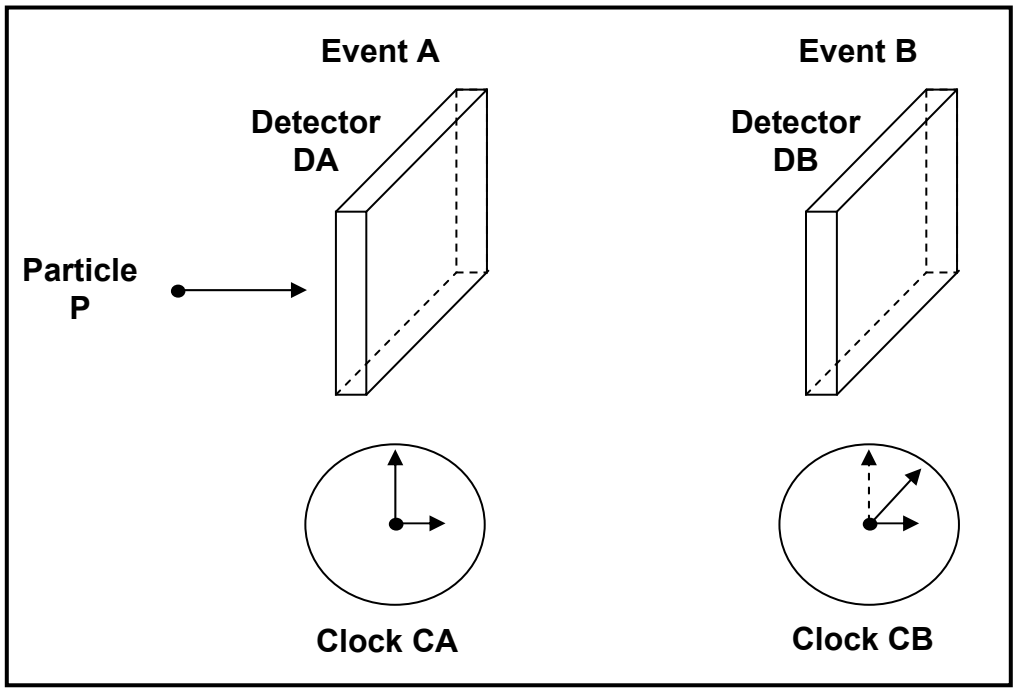

Figure 4. Schematic of an apparatus for measuring a particle world-line

The results of the space-time measurements are shown in Figure 5. The figures present the space-time locations in terms of the means of the space and time measurements, and the error bars display the uncertainty in the measurements. The primed frame F' is moving with respect to the unprimed frame F. Each space-time location is an event.

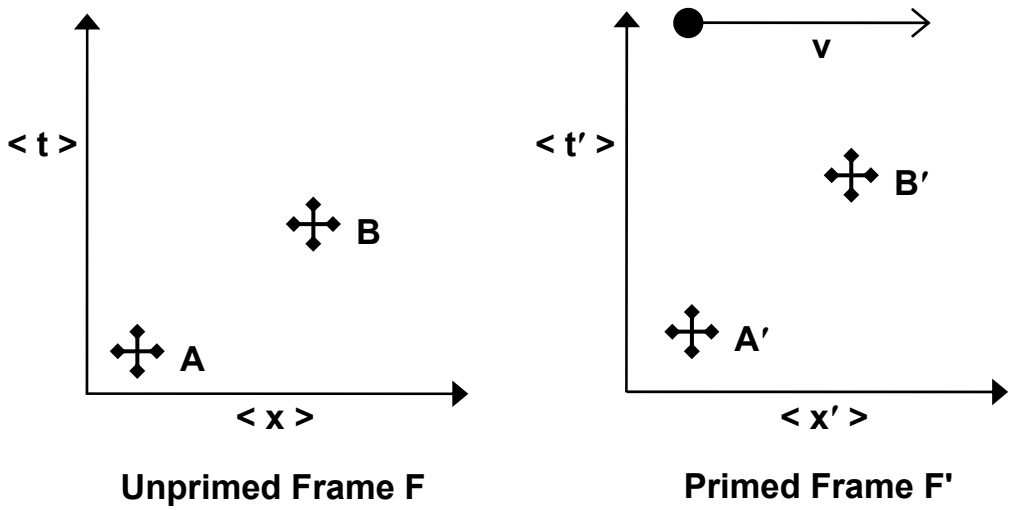

Figure 5. Plot space-time observations

We can interpolate between the events by introducing a parameter s that lets us parameterize the space-time observations. Figure 6 displays a parameterized particle world-line. The system is deterministic if we only allow a single world-line, and is probabilistic if we allow many possible world-lines. The quantities x, t, and s are independent variables, while the means <x> and <t> may be correlated using the parameter s. To retain the manifestly covariant character of the theory, the parameter s needs to be an invariant parameter.

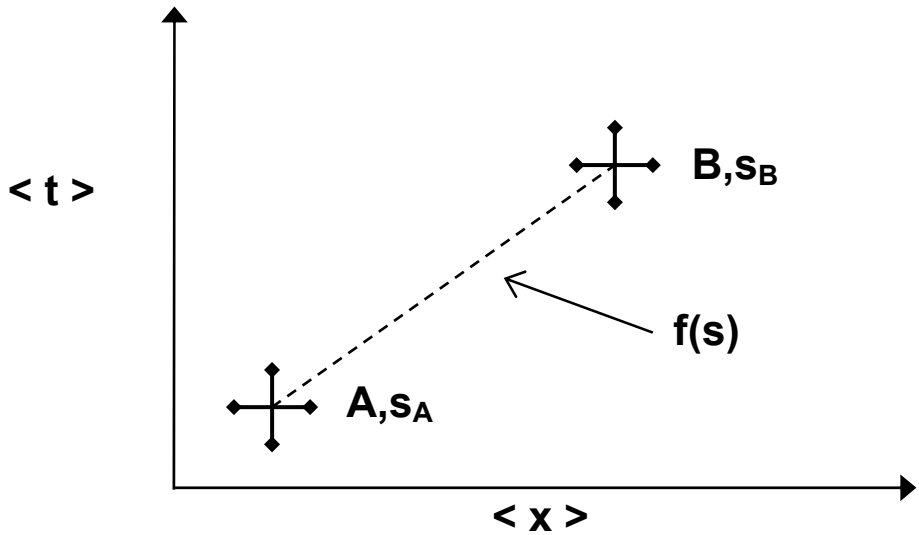

Figure 6. Parameterize space-time observations

The thought experiment demonstrates that it may be empirically useful to have a scalar evolution parameter and motivates the introduction of a fourth postulate for quantizing classical theory:

P4. The dynamical evolution of the system is given by a unitary operator that is a function of a scalar evolution parameter.

Figure 7 illustrates the evolution of two space-time (x_0, x_1) diagrams A and B with respect to a scalar evolution parameter s. A single space-time diagram is a static system since the world-line is fixed. Multiple space-time diagrams allow

world-lines to vary with respect to the scalar evolution parameter. This can be viewed as a dynamic system since the world-line can vary, and motivates the term "parameterized relativistic dynamics" as a descriptor of manifestly covariant quantum theory with invariant evolution parameter.

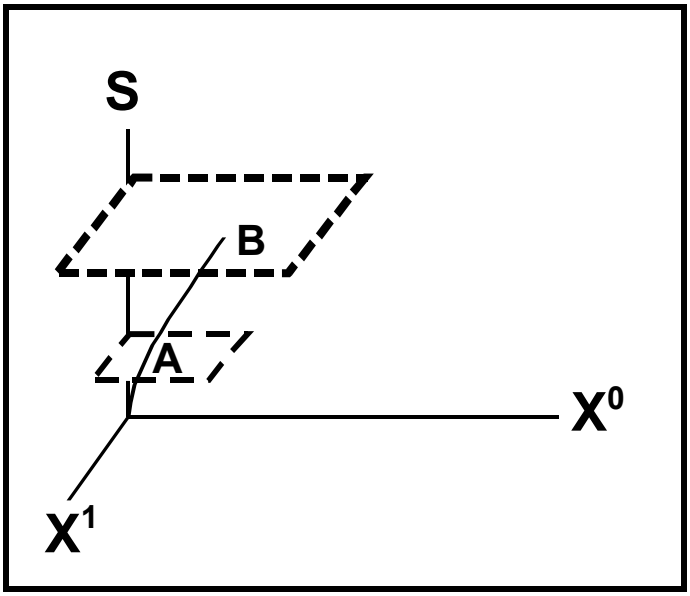

Figure 7. Parameterized evolution of two space-time diagrams

6. Feynman's Path Integral Formulation

We can see what a relativistic dynamical quantum theory could look like by applying path integral concepts. Feynman presented a path integral formulation of quantum mechanics in his 1942 Princeton University Ph.D. Dissertation

[Feynman, 1942]. Feynman postulated that the probability of a particle following a path between two points in a region of space is the absolute square of the sum of probability amplitudes representing all possible paths in the region. The paths contribute equally in magnitude, but the phase of their contribution is the classical action. The Feynman path integral approach provides another perspective on the relationship between the competing paradigms. We consider a relativistic example from Feynman [1948].

We calculate the probability amplitude at space-time x_{i+1}, t_{i+1} and evolution parameter $s+\varepsilon$ from the probability amplitude at x,t,s. The probability amplitude at $x_{i+1}, t_{i+1}, s+\varepsilon$ is the sum of the probability amplitudes for all possible paths weighted by a factor that depends on the action:

$$\Phi(x_{i+1}, t_{i+1}, s+\varepsilon) = \frac{1}{A}\int e^{iS(x_i,t_i)/\hbar}\Phi(x_i, t_i, s)dx_i dt_i \tag{6.1}$$

Action is defined in terms of the Lagrangian

$$S(x_i, t_i) = \int L(\dot{x}_i, \dot{t}_i)ds \tag{6.2}$$

The Lagrangian for a relativistic free particle is

$$L(\dot{x}, \dot{t}) = \frac{m}{2}\left[\left(\frac{dt}{ds}\right)^2 - \left(\frac{dx}{ds}\right)^2\right] \tag{6.3}$$

Solving the integral for action using the trapezoidal rule gives the action for the free particle as

$$S(x_i, t_i) = \varepsilon\frac{m}{2}\left[\left(\frac{t_{i+1}-t_i}{\varepsilon}\right)^2 - \left(\frac{x_{i+1}-x_i}{\varepsilon}\right)^2\right] \tag{6.4}$$

Substituting the action into the integral for probability amplitudes lets us construct a relativistic wave equation that depends on the scalar evolution parameter s, thus

$$\frac{\partial \Phi}{\partial s} = \frac{i\hbar}{2m}\left[\frac{\partial^2 \Phi}{\partial t^2} - \frac{\partial^2 \Phi}{\partial x^2}\right] \tag{6.5}$$

Stueckelberg was the first to use parameterized wave equations of this type in a context that foreshadowed the postulates presented in Section 5, so parameterized wave equations with first order derivatives in s, and second order derivatives in space-time are called the Stueckelberg equations. Equation (6.5) is the Stueckelberg equation for a non-interacting particle.

We relate the Stueckelberg equation for a free particle to the Klein-Gordon equation by writing the probability amplitude in terms of a stationary state solution with respect to the invariant evolution parameter s. The probability amplitude has the form

$$\Phi(x,t,s) = \Phi_{KG}(x,t) exp\left[-i\frac{M^2 s}{2m\hbar}\right] \tag{6.6}$$

where we have introduced a separation of variables. Substituting Eq. (6.6) into (6.7) and simplifying gives the Klein-Gordon equation

$$M^2 \Phi_{KG} = -\hbar^2 \partial^{\mu} \partial_{\mu} \Phi_{KG} \tag{6.7}$$

where the eigenvalue M^2 is interpreted as the square of a mass. According to this interpretation, the Klein-Gordon equation is an equation for the stationary state solution of the Stueckelberg equation. Stationary state in this context refers to stationary with respect to the invariant evolution parameter.

7. Symmetries in Relativistic Theories

A specific example of a parameterized wave equation was constructed in Section 6. A more general approach is to consider the role of the invariant evolution parameter in the context of a symmetry transformation.

Scientific theories consist of a mathematical formalism with a set of rules for relating mathematical quantities and operations to physical quantities and measurements. Relationships between data measurements are described by equations that are expected to remain unchanged under some types of transformations. The transformations may be represented by symmetry groups, such as the Poincaré group for space-time transformations, and the SU(2)⊗U(1) group used in electroweak theory. The Poincaré group is called an external symmetry because space-time coordinates change when the symmetry transformation is applied, while the SU(2)⊗U(1) group is called an internal symmetry because space-time coordinates do not change when the symmetry transformation is applied. Most research in the conventional theory has focused on understanding internal symmetry groups. By contrast, scalar evolution parameter theories represent a change in the external symmetry that is motivated in part by the thought experiment discussed in Section 5.

Symmetry Transformation

The symmetry transformation associated with manifestly covariant

quantum theory is put into context by first discussing the Lorentz transformation associated with special relativity. The principle of relativity says that relationships between physically significant quantities should remain invariant with respect to transformations between different reference frames. Special relativity focuses on reference frames that do not accelerate with respect to one another. Invariance requirements may be represented mathematically as transformations between coordinate systems. The homogeneous Lorentz transformation of special relativity may be written as

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} \tag{7.1}$$

where the 4x4 matrix Λ contains real elements. If we include a spatial translation we obtain the inhomogeneous Lorentz transformation

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} + \alpha^{\mu} \tag{7.2}$$

The simplest way to include an invariant evolution parameter is to add a scalar evolution parameter translation to the inhomogeneous Lorentz transformation, thus [Fanchi, 1979]

$$\begin{aligned} x'^{\mu} &= \Lambda^{\mu}_{\nu} x^{\mu} + \alpha^{\mu} \\ s' &= s + \Delta s \end{aligned} \tag{7.3}$$

An extension of Eq. (7.3) that included a boost b was first presented by Johnson [1969], and Aghassi, Roman and Santilli [1970]:

$$\begin{aligned} x'^{\mu} &= \Lambda^{\mu}_{\nu} x^{\nu} + \alpha^{\mu} + b^{\mu} s \\ s' &= s + \Delta s \end{aligned} \tag{7.4}$$

The transformation in Eq. (7.4) is called the JARS transformation. Santilli extended the mathematics further when he developed hadronic mathematics,

which is beyond the scope of this paper [for a recent overview, see Santilli, 2006, Chapter 1 and associated references].

It is instructive to compare the JARS transformation to the Galilei transformation [Aghassi, Roman and Santilli, 1970], which may be written as

$$\begin{aligned} x'^{\,j}_G &= R^j_i (x_G)^i + (a_G)^j + (\nu_G)^j t_G \\ t'_G &= t_G + \Delta t_G \end{aligned} \tag{7.5}$$

where R is the rotation matrix, a_G is a spatial translation, and v_G is a non-relativistic "boost". In terms of transformations, the invariant evolution parameter s of manifestly covariant quantum theory is the relativistic analog of Newtonian time in the Galilei transformation.

Recognizing that Newtonian time is an evolution parameter in a Euclidean three-dimensional space and not a temporal coordinate in a Lorentzian four-dimensional space gives us insight into how to interpret the invariant evolution parameter s, and can guide a search for experimental tests of the competing paradigms. For example, we know that a potential that depends on Newtonian time in the non-relativistic quantum theory can represent systems in which there are transitions between energy states, e.g. the laser. Is there an analogue in parameterized relativistic dynamics?

If we allow the potential to depend on the invariant evolution parameter s in manifestly covariant quantum theory, we find that the theory can represent systems in which there are transitions between mass states [Fanchi, 2003a]. This interpretation depends on our interpretation of the factors that appear in the stationary state solution of the parameterized wave equation, such as the

Stueckelberg equation discussed above. Given this interpretation, several experimental tests of the competing paradigms are possible. Before discussing the implications of the interpretation, we need to delve more deeply into the symmetry of the system to further develop the formalism.

The JARS Group

We have introduced the JARS group as a symmetry group for transformations in four-dimensional space-time with an invariant evolution parameter. We can develop an understanding of how a state changes with respect to transformations by using the properties of the JARS group to find generators of infinitesimal transformations. First we specify the properties of the group. We then study how the state changes with respect to a canonical transformation. We are interested in canonical transformations because they leave Hamilton's equations of motion invariant. Our study of canonical transformations leads to a set of generators of infinitesimal transformations and commutation relations between the state and the generators. The commutation relation between the state and the generator of translations with respect to the invariant parameter s gives a parameterized field equation. We find that the Stueckelberg equation is one form of a parameterized field equation. Other possible parameterized field equations are discussed.

The JARS group is a 15-parameter continuous group: 6 parameters represent rotations Λ in four-dimensional space-time; 4 parameters a^{μ} represent

translations in four-dimensional space-time; 4 parameters b^{μ} represent a boost, and 1 parameter represents a translation of the evolution parameter. We can represent the group as {a, b, Λ, Δs}. The group has the identity element {0, 0, 1, 0} and the inverse element $\{-\Lambda^{-1}(a-b\Delta s), -\Lambda^{-1}b, \Lambda^{-1}, -\Delta s\}$. The composition law of two group elements is

$$\{a_2, b_2, \Lambda_2, \Delta s_2\}\{a_1, b_1, \Lambda_1, \Delta s_1\} = \{a_2 + \Lambda_2 a_1 + (\Delta s_1) b_2, b_2 + \Lambda_2 b_1, \Lambda_2 \Lambda_1, \Delta s_1 + \Delta s_2\} \tag{7.6}$$

Coordinate transformations are represented by continuous symmetry groups, notably Lie groups. An r-parameter Lie group is a set of transformations with the properties of a group and labeled by continuously varying parameters. The law of group multiplication can be linearized if we consider infinitesimal transformations.

Infinitesimal JARS Transformations

Infinitesimal transformations must exist arbitrarily close to the identity element of the group. The parameters for an infinitesimal JARS transformation with first order infinitesimals $\varepsilon_x^{\mu}, \varepsilon_p^{\mu}, \varepsilon_{\mu}^{\nu}, \varepsilon$ are given by

$$\begin{aligned} \Lambda_{\mu}^{\nu} &= g_{\mu}^{\nu} + \varepsilon_{\mu}^{\nu} \\ a_{\mu} &= \varepsilon_{x\mu} \\ b_{\mu} &= \varepsilon_{p\mu} \\ \Delta s &= \varepsilon \end{aligned} \tag{7.7}$$

Infinitesimal generators may be constructed from the infinitesimal transformations

to linearize the law of group multiplication. The generators form a vector space called a Lie algebra.

Canonical Transformation and Unitary Operator Representation

A canonical transformation of the state ψ in manifestly covariant quantum theory is

$$\psi'(x,s) = U\psi(x,s)U^{-1} \tag{7.8}$$

with a unitary operator

$$U = 1 + iF_J \tag{7.9}$$

The 15 infinitesimal generators associated with the 15-parameter continuous group are

$$F_J = \frac{1}{2}J^{\mu\nu}\varepsilon_{\mu\nu} + P^{\mu}(\varepsilon_p)_{\mu} + X^{\mu}(\varepsilon_x)_{\mu} + K\varepsilon \tag{7.10}$$

where

J = antisymmetric angular momentum tensor (6 infinitesimal generators)

P = energy-momentum four-vector (4 infinitesimal generators)

X = space-time four-vector (4 infinitesimal generators)

K = generator of s translations (1 infinitesimal generators)

The generators of infinitesimal transformations are operators that satisfy the following commutation relations with the field ψ:

$$
\begin{aligned}
i[J^{\nu\mu},\psi] &= \Sigma^{\nu\mu}\psi + (x^{\nu}\partial^{\mu} - x^{\mu}\partial^{\nu})\psi \\
i[P^{\nu},\psi] &= -\partial^{\nu}\psi \\
i[X^{\nu},\psi] &= -s(\partial^{\nu}\psi) \\
i[K,\psi] &= -\partial_{s}\psi
\end{aligned}
\tag{7.11}
$$

The last commutation relation is the equation that describes how the field evolves with respect to evolution parameter translations. The operator K is the generator of translations in s.

The commutation relation $i[K,\psi] = -\partial_s\psi$ can be used to derive the following parameterized wave equation:

$$i\frac{\partial\psi}{\partial s} = K_j\psi \tag{7.12}$$

Several forms for K have been proposed. Nambu [1950] first proposed the Dirac-like form

$$K_1 = \alpha_1\gamma_\mu P^\mu, P^\mu \equiv -i\partial^\mu = -i\frac{\partial}{\partial x_\mu} \tag{7.13}$$

Fock [1937] and Stueckelberg [1941-42] introduced the Schroedinger-like form

$$K_2 = \alpha_2 P_\mu P^\mu \tag{7.14}$$

Johnson [1969] suggested the form

$$K_3 = \alpha_3\sqrt{P_\mu P^\mu} \tag{7.15}$$

Choice of K generators

The Dirac-like generator and the Schroedinger-like generator have

received more attention than the square root form in Eq. (7.15). Theories with Dirac-like generators are more analogous to the Standard Model and have been used for many modern applications [Fanchi, 1993a], but are ambiguous in their definition of the normalization condition and other probability concepts. Theories with Schroedinger-like generators can be constructed from conventional probability concepts, and there is a straightforward correspondence to classical theory as illustrated by the path integral formulation and development of a relativistic quantum potential by analogy to Bohm's non-relativistic formalism. How do we decide which generator of parameter translations is correct? One way is to consider different interpretations of the formalism.

8. Interpretations of Theoretical Formalisms

The validity of the paradigm of parameterized relativistic dynamics depends on how well the formalism and associated interpretation can represent observations. Many interpretations are possible, such as the Copenhagen interpretation, the statistical interpretation, the many-worlds interpretation, an interpretation in terms of Bohm's quantum potential, a consistent histories interpretation, and so on. There is overlap between some of the ideas represented by these interpretations. We are interested in areas of disagreement that make these interpretations examples of competing paradigms and that can lead to experimental tests of the paradigms they represent. We consider two of these interpretations here to see how an interpretation can help us find the correct

generator of parameter translations.

The Copenhagen interpretation has been associated with the conventional theory for decades. A key component of the Copenhagen interpretation is the concept of the collapse of the wave function when an observation is made. Fröhner [1998] remarked that the wave function collapses instantaneously throughout physical space if new information is taken into account, and called the collapse a logical change rather than a physical change. The collapse of the wave function implies non-locality, which is discussed in more detail later.

Another point of view arises when we apply Born's interpretation of the wave function as a probability amplitude to the wave function of parameterized relativistic dynamics. In this case, the formalism applies to an ensemble of observations, as we illustrated in Section 5 with a thought experiment. The statistical interpretation is consistent with experimental observations, as vividly demonstrated by the set of images of electron interference in a fine slit experiment presented in Electron Holography [Tonomura, 1993].

Tonomura was looking for an interference pattern associated with the wave-like character of the electron. The interference pattern was not apparent until a statistically significant number of events was observed. Electron interference in a fine slit experiment is a statistical effect. The interference pattern does not appear when a single observation is made, but it does appear when a statistically significant number of observations are made.

These ideas have been used to formulate parameterized relativistic dynamics from fundamental probability concepts. The formulation has been

extended to particles with internal variables such as spin or isospin [Fanchi, 1993b, and references therein; Fanchi, 2003a]. The formalism in Section 9 is limited to scalar particles to illustrate the approach.

9. Probabilistic Formulation of Parameterized Relativistic Dynamics

We begin by assuming that a conditional probability density $\rho(x|s)$ can be found that represents the physical system of interest. The symbol x denotes a set of space-time coordinates. The μth component of the position four-vector is written as x_a^μ where $\mu = 0,1,2,3$. Indices 1, 2, 3 signify space components and index 0 signifies the geometric time component. The symbol s is an invariant evolution parameter that conditions the probability density $\rho(x|s)$.

The conditional probability distribution $\rho(x|s)$ is capable of representing the disappearance or reappearance of particles in space-time. In the case of a single particle, the conditional probability density can be expressed as the product $\rho(x^1, x^2, x^3|x^0, s)\rho(x^0|s)$. The distribution $\rho(x^0|s)$ is the marginal probability density in time and is conditioned by the evolution parameter s. When $\rho(x^0|s)$ is zero, the probability of observing a particle at time x^0 given parameter s is zero. In other words, the particle cannot be detected anywhere in space when $\rho(x^0|s)$ is zero. By contrast, when $\rho(x^0|s)$ is nonzero, there is a nonzero probability of observing a particle at time x^0 given parameter s.

According to probability theory, the conditional probability density must be positive definite and normalizable. The Born representation of the positive definite requirement is expressed in terms of the probability amplitude Ψ as

$$\rho(x|s) = \Psi^*(x,s)\Psi(x,s) \geq 0 \tag{9.1}$$

with the normalization condition

$$\int_D \rho(x|s)d^4x = 1 \tag{9.2}$$

where D denotes the space-time volume. The probability amplitude Ψ is also referred to as the field or wave function of the system. The normalization integral in Eq. (9.2) implies that the particle could be observed somewhere in space at some point in time; it does not require the particle to exist for all time. This is a significant difference relative to 3-space normalization because it means PRD can be used to describe particles with finite lifetimes.

The probability amplitude is specified only to within a gauge transformation. Consequently, we have the decomposition

$$\Psi(x,s) = \sqrt{\rho(x|s)}\, e^{i\xi(x,s)} \tag{9.3}$$

where the scalar function ξ represents the gauge transformation.

Field equations that exhibit a "minimal coupling" electromagnetic interaction are derived from the above assumptions and probability conservation. Conservation of probability implies the continuity equation

$$\frac{\partial\rho}{\partial s} + \frac{\partial}{\partial x_\mu}(\rho V^\mu) = 0 \tag{9.4}$$

Using the Born representation of probability density and the continuity equation

gives the probability flux

$$\rho V^{\mu} = -\frac{i\hbar}{2m}\left[\Psi^* \frac{\partial \Psi}{\partial x_{\mu}} - \Psi \frac{\partial \Psi^*}{\partial x_{\mu}}\right] - \frac{eA^{\mu}}{mc}\Psi^* \Psi \tag{9.5}$$

with four-velocity

$$V^{\mu}(x,s) = \frac{\hbar}{m}\frac{\partial \xi(x,s)}{\partial x_{\mu}} - \frac{e}{mc}A^{\mu}(x,s) \tag{9.6}$$

and electromagnetic four-vector potential A^{μ}.

The above equations are used to derive the parameterized field equation

$$i\hbar \frac{\partial \Psi}{\partial s} = K\Psi \tag{9.7}$$

with mass operator

$$K = \frac{\pi^{\mu}\pi_{\mu}}{2m} + V \tag{9.8}$$

where V is potential energy. The four-vector potential is contained in the four-momentum operator with minimal coupling

$$\pi^{\mu} = \frac{\hbar}{i}\frac{\partial}{\partial x_{\mu}} - \frac{e}{c}A^{\mu} \tag{9.9}$$

The parameterized field equation in Eq. (9.7) is called the Stueckelberg equation for a single particle. Application of the four-vector potential to the covariant Stark effect was discussed by Land and Horwitz [2001].

In this formulation, the definition of the expectation value of an observable Ω is

$$\langle \Omega \rangle = \int \Psi^* \Omega \Psi dx \tag{9.10}$$

The space-time uncertainty principle for this formulation is

$$\left|\Delta x_{\mu}\right|\left|\Delta p_{\mu}\right| \geq \frac{\hbar}{2} \tag{9.11}$$

where summation over repeated indices is not implied in this equation. Unlike the conventional theory, the uncertainty principle for both energy and three-momentum is a consequence of the manifestly covariant probabilistic formulation.

The Meaning of Mass

We illustrate the formalism and develop more insight into the meaning of m by examining the free particle. The Stueckelberg equation for the free particle is

$$i\hbar \frac{\partial \psi_f}{\partial s} = -\frac{\hbar^2}{2m} \partial_{\mu} \partial^{\mu} \psi_f \tag{9.12}$$

Equation (9.12) has the general solution

$$\begin{aligned} \Psi_f(x,s) &= \int \psi_{f\kappa}(x,s) dk_f \\ &= \int \eta_{f\kappa}\, exp\left[i\kappa_f(k_f)s + ik_{f\mu}x^{\mu}\right] dk_f \end{aligned} \tag{9.13}$$

where the integral is over energy-momentum, $\eta_{f\kappa}$ denotes normalization coefficients, and

$$\kappa_f(k_f) = -\frac{\hbar}{2m} k_{f\mu} k_f^{\mu} \tag{9.14}$$

It is possible to construct a wave packet using Eq. (9.13) [e.g. Fanchi and Collins, 1978; Fanchi, 1993b]. Figure 8a illustrates a wave packet localized in space-time,

and Figure 8b shows that a world line is the result of the wave packet evolving with respect to invariant evolution parameter s.

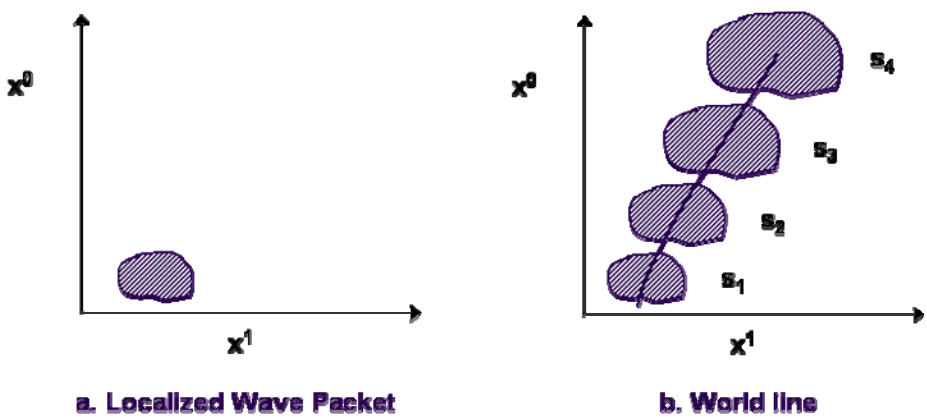

Figure 8. Wave Packet in Space-Time [after Pavsic, 2001, Figure 1.2]

The expectation value of the four-velocity of the free particle is

$$\left\langle V_f^{\mu} \right\rangle = \frac{d\left\langle x_f^{\mu} \right\rangle}{d\,s} = \frac{\left\langle p_f^{\mu} \right\rangle}{m} \tag{9.15}$$

Integrating Eq. (9.15) from s to $s + \delta s$ gives

$$\delta\left\langle x_f^{\mu} \right\rangle = \frac{\left\langle p_f^{\mu} \right\rangle}{m}\delta\,s \tag{9.16}$$

Equation (9.16) expresses the most probable trajectory of the free particle.

The observable world-line of the free particle is given by the inner product

$$\delta\left\langle x_f^{\mu} \right\rangle \delta\left\langle x_{f\mu} \right\rangle = \frac{\left\langle p_f^{\mu} \right\rangle \left\langle p_{f\mu} \right\rangle}{m^2}(\delta\,s)^2 \tag{9.17}$$

Solving for m^2 gives

$$m^2 = \frac{\langle p_f^\mu \rangle \langle p_{f\mu} \rangle}{\delta \langle x_f^\mu \rangle \delta \langle x_{f\mu} \rangle} (\delta s)^2 \tag{9.18}$$

Equation (9.18) applies to both time-like (bradyonic) motion and space-like (tachyonic) motion. We can see this by observing that in parameterized relativistic dynamics, the invariant evolution parameter increases monotonically so that $\delta s > 0$. On the other hand, the terms $\langle p_f^\mu \rangle \langle p_{f\mu} \rangle$, $\delta \langle x_f^\mu \rangle \delta \langle x_{f\mu} \rangle$ can be either time-like $\langle p_f^\mu \rangle \langle p_{f\mu} \rangle > 0$, $\delta \langle x_f^\mu \rangle \delta \langle x_{f\mu} \rangle > 0$ or space-like $\langle p_f^\mu \rangle \langle p_{f\mu} \rangle < 0$, $\delta \langle x_f^\mu \rangle \delta \langle x_{f\mu} \rangle < 0$. Consequently, m^2 is positive for both time-like and space-like motion because negative signs associated with space-like motion cancel. Free tachyons in parameterized relativistic dynamics have real mass.

The conventional theory imposes the constraint

$$m_s^2 c^2 = p_f^\mu p_{f\mu} \tag{9.19}$$

on allowed values of energy-momentum, where we have added subscript s to distinguish the mass of the conventional theory from the mass of parameterized relativistic dynamics. A relationship between mass and energy-momentum exists in parameterized relativistic dynamics, but we must consider the classical limit of negligible dispersion as follows.

Energy-momentum dispersion is

$$\Delta^2 p_f \equiv \langle p_f^\mu p_{f\mu} \rangle - \langle p_f^\mu \rangle \langle p_{f\mu} \rangle \tag{9.20}$$

Substituting Eq. (9.20) into (9.17) gives

$$\delta\left\langle x_f^{\mu}\right\rangle\delta\left\langle x_{f\mu}\right\rangle = \frac{\left\langle p_f^{\mu} p_{f\mu}\right\rangle - \Delta^2 p_f}{m^2}\left(\delta s\right)^2 \tag{9.21}$$

In the classical limit, dispersion is negligible and we find that m^2 is statistically related to the independent variables energy and momentum by

$$m^2c^2 = \left\{\frac{c^2\left(\delta s\right)^2}{\delta\left\langle x_f^{\mu}\right\rangle\delta\left\langle x_{f\mu}\right\rangle}\right\}\left\langle p_f^{\mu} p_{f\mu}\right\rangle \tag{9.22}$$

We have inserted the term c^2 so that the bracketed term is dimensionless. In the classical limit, the magnitude of the bracketed term is one, but we need to account for the sign of $\delta\left\langle x_f^{\mu}\right\rangle\delta\left\langle x_{f\mu}\right\rangle$, so Eq. (9.22) becomes

$$m^2c^2 = \frac{\left\langle p_f^{\mu} p_{f\mu}\right\rangle}{sgn\left\{\delta\left\langle x_f^{\mu}\right\rangle\delta\left\langle x_{f\mu}\right\rangle\right\}} \geq 0 \tag{9.23}$$

where "sgn" says that we are just using the sign of the bracketed term. Equation (9.23) applies to both time-like and space-like free particles. It says that m may be interpreted as on-shell mass for a free particle moving at any speed.

The role of mass is a significant difference between parameterized relativistic dynamics and the conventional theory. This difference is the basis for some of the experimental tests discussed in later sections.

Stueckelberg-Feynman Interpretation

Equation (9.15) is the parameterized basis of the Stueckelberg-Feynman interpretation. The space components of $\left\langle V_f^{\mu}\right\rangle$ are the space components of the

phase velocity, and $\langle V_f^0 \rangle$ is the temporal component. Recalling that s is monotonically increasing, Eq. (9.15) shows that the particle propagates into the temporal coordinate "future" when $d\langle x_f^0 \rangle$ is positive, and into the temporal coordinate "past" when $d\langle x_f^0 \rangle$ is negative. If we substitute Eq. (9.13) into Eq. (9.15) we find

$$\langle V_f^\mu \rangle = \frac{\hbar}{m} \langle k_f^\mu \rangle \tag{9.24}$$

where we now express $\langle V_f^\mu \rangle$ in terms of wavenumbers $\langle k_f^\mu \rangle$. Recognizing that k_f^0 is proportional to the frequency of the wave $(k_f^\mu = \omega/c)$, Eq. (9.24) says that a positive frequency particle propagates forward in time and a negative frequency particle propagates backward in time. This result is the Stueckelberg-Feynman interpretation. Furthermore, in the non-relativistic limit when

$$m^2 c^2 >> \langle \vec{p} \cdot \vec{p} \rangle \tag{9.25}$$

we have the expected result

$$\left| \langle V_f^0 \rangle \right| \to c \tag{9.26}$$

The Stueckelberg-Feynman interpretation appears naturally in the PRD formulation and provides an acceptable physical interpretation of positive and negative energy solutions within the framework of a single particle theory. It eliminates the need for Dirac's sea of electrons, which is now considered unnecessary in conventional quantum field theory [Weinberg, 1995, pg. 14]. Furthermore, the Stueckelberg-Feynman interpretation has been used to resolve

Klein's paradox [Fanchi, 1993, Chapters 9 and 10 and references therein].

10. Stueckelberg Equation for an N-Body System

The probabilistic formalism in Section 9 has been used to derive field equations that exhibit the "minimal coupling" electromagnetic interaction can be derived for an N-body system [Fanchi, 1993 and references therein]. Let $\underline{\Psi}$ be a column vector with elements $\underline{\Psi}^T = [\psi_1, \psi_2, \ldots, \psi_\Lambda]$ where Λ represents the N particles and their associated internal variables, such as spin and isospin. The field equations have the form

$$i\hbar \frac{\partial \underline{\Psi}}{\partial s} = \underline{\underline{K_N}}\, \underline{\Psi} = \left\{ \sum_{a=1}^{N} \frac{\pi_a^\mu \pi_{a\mu}}{2m_a} \underline{\underline{I}} + \underline{\underline{V}} \right\} \underline{\Psi} \tag{10.1}$$

where the Hermitian operator $\underline{\underline{K_N}}$ is interpreted as an N-body mass operator, $\underline{\underline{I}}$ is the identity matrix, and $\underline{\underline{V}}$ represents interactions. The four-momentum operator in the bracketed term is

$$\pi_a^\mu = p_a^\mu - \frac{e}{c} A_a^\mu \tag{10.2}$$

with

$$p_a^\mu = \frac{\hbar}{i} \frac{\partial}{\partial x_{a\mu}} \tag{10.3}$$

The expectation value of an observable in the N-body formalism is

$$\langle \Omega \rangle = \int \underline{\Psi}^+ \Omega \underline{\Psi} dx \tag{10.4}$$

where the integral is over the hypervolume D^N and where $\underline{\Psi}^+$ is the conjugate transpose of $\underline{\Psi}$.

Non-interacting N-Body System

The N-body formalism can be used to describe the thought experiment in Section 5. A non-interacting system corresponds to the conditions $\left\{A_a^\mu = 0, \underline{\underline{V}} = 0\right\}$ so that Eq. (10.1) becomes

$$i\hbar \frac{\partial \psi_f}{\partial s} = \sum_{a=1}^{N} \left(-\frac{\hbar^2}{2m_a} \frac{\partial^2 \psi_f}{\partial y_a^\mu \partial y_{a\mu}} \right) \tag{10.5}$$

The column vector eigenfunction $\underline{\Psi}$ is replaced by a scalar ψ_f since we are neglecting any interactions associated with internal variables like spin in this case. Equation (10.5) has the solution

$$\psi_f(y,s) = \prod_{a=1}^{N} \eta_a^{1/2} \exp\left[-\frac{i\hbar}{2m_a}\left(k_a^\mu k_{a\mu}\right)s + ik_{a\mu} y_a^\mu \right] \tag{10.6}$$

where η_a is the normalization constant for particle a.

The expectation value of the four-velocity for particle a is given by

$$\left\langle V_a^\mu \right\rangle = \frac{1}{m_a} \int \psi_f^* \left[\frac{\hbar}{i} \frac{\partial}{\partial y_{a\mu}} \right] \psi_f \, dy \tag{10.7}$$

Substituting Eq. (10.6) into (10.7) gives

$$\frac{d\langle y_a^\mu \rangle}{ds} = \langle V_a^\mu \rangle = \frac{\hbar \langle k_a^\mu \rangle}{m_a} \tag{10.8}$$

Integrating Eq. (10.8) from s to $s + \delta s$ gives

$$\delta \langle y_a^\mu \rangle = \frac{\hbar \langle k_a^\mu \rangle}{m_a} \delta s \tag{10.9}$$

Equation (10.9) expresses the expected value of the relativistic free particle a.

The observable world-line of particle a is

$$\delta \langle y_a^\mu \rangle \delta \langle y_{a\mu} \rangle = \frac{\hbar^2 \langle k_a^\mu \rangle \langle k_{a\mu} \rangle}{m_a^2} \delta s^2 \tag{10.10}$$

In the classical limit of negligible dispersion, we have

$$\delta \langle y_a^\mu \rangle \delta \langle y_{a\mu} \rangle = \frac{\hbar^2 \langle k_a^\mu k_{a\mu} \rangle}{m_a^2} \delta s^2 \tag{10.11}$$

The observable on-shell mass of particle a is

$$\langle p_a^\mu p_{a\mu} \rangle = \hbar^2 \langle k_a^\mu k_{a\mu} \rangle \equiv m_a^2 c^2 \tag{10.12}$$

Substituting Eq. (10.12) into (10.11) gives the light cone constraint of classical special relativity:

$$\delta \langle y_a^\mu \rangle \delta \langle y_{a\mu} \rangle = c^2 \delta s^2 \tag{10.13}$$

Equation (10.13) assures macroscopic causality for each particle of the N-body system in the sense that the classical world-line of the ath particle is its expected trajectory. Furthermore, Eq. (10.13) implies that the rest frame clocks of every particle run at the same rate, i.e. they have the same classical proper time intervals. This calibration of rest frame clocks is a result of assuming a probability

distribution with a single evolution parameter.

11. Measuring the Evolution Parameter

The single evolution parameter formalism for N-body systems presented in Section 10 can be combined with the observation in Section 7 that the invariant evolution parameter is the relativistic analogue of Newtonian time to develop a physically meaningful evolution parameter clock [Fanchi, 1986]. Let N_c denote the number of objects needed to represent the evolution parameter clock, and N_e denote the number of objects in the experimental system of interest. The N-body system contains $N_c + N_e$ objects. There should be negligible interaction between the evolution parameter clock and the experimental system of interest. As an illustration of the procedure for designing a system that includes an evolution parameter clock and an experimental system, let us consider the behavior of two scalar particles: the interacting particle in the experimental system, and the "s-clock" particle. The system is illustrated in Figure 9.

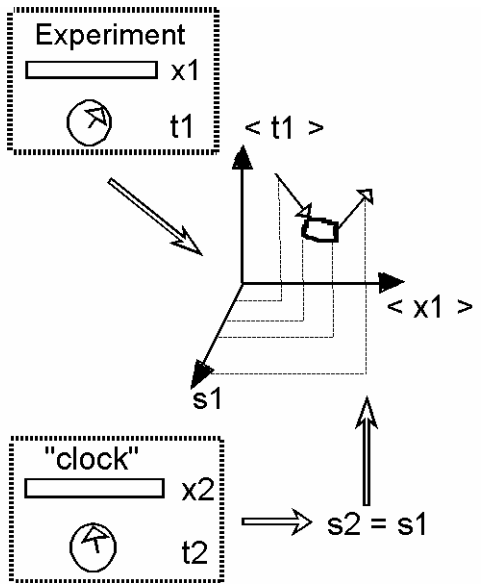

Figure 9. Evolution Parameter Clock

The “s-clock” particle trajectory may be used to replace the evolution parameter s with familiar observables. The non-interacting “s-clock” particle is introduced because it serves to independently define the evolution parameter s. The “s-clock” particle in this design propagates without interaction from a source to a detector. Its trajectory is used to quantify evolution parameter s and parameterize the trajectory of the interacting particle in the experimental system. We measure the “s-clock” particle’s world-line using a rod with spatial coordinate x2 and clock with temporal coordinate t2. The world-line obeys an equation like Eq. (10.13). The value of the evolution parameter is then used to parameterize the experimental system. The assumption here is that the parameter of the evolution

parameter clock can be equated to the parameter characterizing the experimental system. The same assumption is made in the non-relativistic Schroedinger theory using Newtonian time [Fanchi, 1986] and is typical of the operational procedure we use to make "time" measurements in the non-relativistic domain. Parameterized relativistic dynamics allows us to extend the operational procedure to the relativistic domain.

The “s-clock” analysis described above depends on the following assumptions:

a. The rest frame clocks of the two particles are calibrated.

b. Particle 1 does not interact with particle 2, but may interact with other particles through a potential V_I.

c. Particle 2 is a free, scalar particle.

Assumption (a) is a fundamental assumption of the single evolution parameter formalism. The remaining two assumptions serve to define the system of interest. Notice that particle 1 and its interaction comprise the experimental system of interest. Particle 2 is an additional particle that serves as the evolution parameter clock.

Assumptions (b) and (c) simplify the problem. The appropriate Stueckelberg equation for the two-particle system is

$$\left\{ i\hbar \frac{\partial}{\partial s} - \left[\frac{p_1^{\mu} p_{1\mu}}{2m_1} + \frac{p_2^{\mu} p_{2\mu}}{2m_2} + V_I \right] \right\} \psi(1,2,s) = 0 \qquad (11.1)$$

where $\psi(1,2,s)$ denotes the s-dependent eigenfunction of the two-particle system with 4-space components $y_{1\mu}, y_{2\mu}$ for particles 1 and 2 respectively. Equation

(11.1) may be separated by writing

$$\psi(1,2,s) = \psi(1,s)\psi(2,s) \tag{11.2}$$

Dividing Eq. (11.1) by (11.2), simplifying and using the separation constant α_R yields the two equations

$$\left\{ i\hbar \frac{\partial}{\partial s} - \left[\frac{p_1^{\mu} p_{1\mu}}{2m_1} + V_1 + \alpha_R \right] \right\} \psi(1,s) = 0 \tag{11.3}$$

and

$$\left\{ i\hbar \frac{\partial}{\partial s} - \left[\frac{p_2^{\mu} p_{2\mu}}{2m_2} - \alpha_R \right] \right\} \psi(2,s) = 0 \tag{11.4}$$

The value of the separation constant α_R is determined in principle once the interactions and the boundary conditions of the physical system are specified. In our case, particles 1 and 2 are physically independent so that the separation constant α_R can be set to zero, i.e. $\alpha_R = 0$.

Particle 2 was introduced as a means of removing the indefiniteness of the evolution parameter s. Equation (11.4) with $\alpha_R = 0$ is the field equation for a free particle:

$$i\hbar \frac{\partial \psi_f(2,s)}{\partial s} = \left[-\frac{\hbar^2}{2m_2} \frac{\partial^2}{\partial y_2^{\mu} \partial y_{2\mu}} \right] \psi_f(2,s) \tag{11.5}$$

Equation (11.5) has a particular solution

$$\psi_f(2,s) = \eta_f^{1/2}\, exp\left[-\frac{i\hbar}{2m_2} \left(k_2^{\mu} k_{2\mu} \right) s + i k_{2\mu} y_2^{\mu} \right] \tag{11.6}$$

where η_f is a normalization constant.

In the limit of negligible dispersion (the classical limit), the most probable trajectory of particle 2 is given by

$$(\delta s)^2 = (s - s_0)^2 = \frac{1}{c^2}\delta\langle y_2^\mu\rangle\delta\langle y_{2\mu}\rangle \tag{11.7}$$

where s_0 is the evolution parameter at which the rest frame clocks of both particles were calibrated. We can arbitrarily define the s axis so that $s_0 = 0$ and subsequently find from Eq. (6.7) an expression for evolution parameter in terms of the measurable space-time trajectory of particle 2:

$$s^2 = \frac{1}{c^2}\delta\langle y_2^\mu\rangle\delta\langle y_{2\mu}\rangle \tag{11.8}$$

Neglecting statistical variations and writing $\{y_2^\mu\} = \{t, x\}$ for linear motion, we obtain

$$s^2 = \delta t^2 - \frac{\delta x^2}{c^2} = \delta t^2\left[1 - \beta^2\right] \tag{11.9}$$

where

$$\beta = \frac{v}{c}, v \equiv \frac{\delta x}{\delta t} \tag{11.10}$$

If we write L as the distance δx traveled by particle 2 in the interval δt, then Eq. (11.9) can be written as

$$s = \frac{L}{v}\left[1 - \beta^2\right]^{1/2} = \frac{L}{c}\frac{\left[1 - \beta^2\right]^{1/2}}{\beta} \tag{11.11}$$

If we measure the space-time trajectory of particle 2, we obtain an operational determination of the invariant evolution parameter s. This parameterization can be used in the Stueckelberg equation describing the physically interesting system

associated with particle 1. Some applications are presented in later sections.

12. Non-Locality

The formalism described in previous sections is useful for addressing another difference between the conventional theory and its parameterized competitor: non-locality. Definitions from the Stanford Encyclopedia of Philosophy help clarify what we mean when we discuss non-locality and the related concept of action-at-a-distance. Non-locality refers to the correlation of particles with space-like separation. The following hypotheses are possible explanations for how non-local systems influence each other.

1. Influence propagates continuously in space-time (superluminal or faster-than-light propagation).
2. Influence due to action-at-a-distance.
3. Influence due to a quantum potential.
4. Influence due to non-separability of states of composite system, e.g. entangled states.

Action-at-a-distance in the second hypothesis occurs if a change of the intrinsic properties in one system instantaneously changes the intrinsic properties in a distant system without the existence of a process that propagates this influence contiguously in space and time. The third hypothesis relies on a quantum potential. The non-relativistic analogue for representing non-classical action-at-a-distance was Bohm's non-relativistic quantum potential [see Bohm

and Hiley, 1993]. An approach based on entangled states is introduced in the fourth hypothesis. The concept of entangled states can be thought of from an operational perspective. Suppose we have two systems that are represented by separate and distinct quantum states. Let us allow the 2 systems to interact and then separate the systems. The states of the two separated systems are entangled if canonically conjugate variables (e.g. position and momentum) are correlated. For example: the composite wave function of a multi-particle system is entangled if it is not a product of single-particle wave functions. Quantum entanglement is present if the linkage between quantum states of two systems results in correlations between observable properties of the systems, even if they have space-like separation.

Given the idea of entangled states, we can differentiate between a local and a non-local quantum theory. A local quantum theory is a theory in which a composite wave function of two quantum systems with space-like separation can be factored into a product of wave functions for each system. The composite wave function of local quantum theory is said to be factorizable. A non-local quantum theory is a theory in which a composite wave function cannot be factored. The composite wave-function of non-local quantum theory is said to be non-factorizable.

These concepts are important in differentiating the conventional theory from parameterized relativistic dynamics. The parameterized theory uses an expectation value over all space-time, and allows off-mass shell wavefunctions to contribute to the probability density. This means that the parameterized theory is

an action-at-a-distance theory that allows the probability density to be affected by non-local interactions. Consequently, we need to consider the possibility of non-locality, quantum entanglement, and faster-than-light motion. First, however, we must answer the question: is non-locality real?

The EPR Experiment

Einstein, Podolsky and Rosen [1935] first recognized that quantum theory could be non-local and designed a thought experiment that was intended to test the theory. The EPR (Einstein-Podolsky-Rosen) experiment begins with the preparation of two systems (e.g. particles) in a certain quantum state, and then lets them separate. EPR required the following two conditions.

1. Separability: each system should be characterized by its own properties (e.g. position, momentum).
2. Locality: it should not be possible to alter properties of a distant system instantaneously by acting on a local system.

Bell [1994] extended the EPR experiment to correlations between 2-valued dynamical variables (e.g. spin).

The EPR/B (Einstein-Podolsky-Rosen/Bell) apparatus shown in Figure 10 illustrates non-locality for a system of two particles with spin [Greenstein and Zajonc, 1997]. Alice can deduce the spin of Bob's particle by observing the spin of her particle, even if Alice and Bob are separated by spacelike distances. This appears to give observers Bob and Alice a means of exchanging information

across spacelike distances at superluminal velocities and raises an issue of fundamental importance to the interpretation of quantum theory.

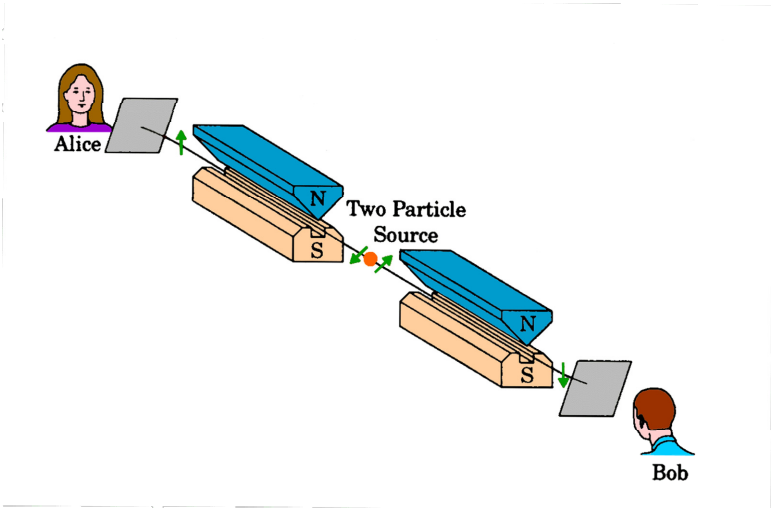

Figure 10. EPR Apparatus

Quantum mechanics, through Bell's inequalities [Bell, 1964 and 1987] prohibits spatially separated quantum particles from physically influencing each other at seemingly instantaneous velocities. However, Bell's inequalities do not prohibit non-locality in which the correlation between quantum states influences the outcome of measurements. Aspect, et al. [1982] were the first to show experimentally that entanglement and non-locality are real. They performed the EPR/B experiment and showed that entanglement is non-local.

Bohm expressed the non-local connection between quantum particles through the non-relativistic wave function and the non-relativistic quantum potential [Bohm, 1952, 1953]. The extension of Bohm's ideas in the conventional

theory of relativistic quantum mechanics is problematic, however [Bohm and Hiley, 1993]. The difficulties associated with a relativistic quantum potential can be overcome by using parameterized relativistic dynamics. The calculation of relativistic quantum potentials is relatively straightforward, as we illustrate with two examples [Fanchi, 2000]: the relativistic scalar particle and the relativistic N-particle system.

The Relativistic Scalar Particle

The Stueckelberg equation for a relativistic particle interacting with a scalar potential V is

$$i\hbar \frac{\partial \psi}{\partial s} = \frac{-\hbar^2}{2m} \partial_\mu \partial^\mu \psi + V\psi \tag{12.1}$$

where m is an observable mass defined in terms of the expectation values of energy-momentum four-vectors. Following Bohm, we let $\psi(x^\mu, s) = \rho(x^\mu, s)^{1/2} \, exp[i\xi(x^\mu, s)]$ where ρ and ξ are invariant scalar functions that depend on space-time $\{x^\mu\}$, and the evolution parameter s. The probability density ρ is normalized over space-time.

A relativistic continuity equation is obtained by defining the relativistic action $S = \hbar\xi$ to find

$$\frac{\partial \rho}{\partial s} + \frac{1}{m} \partial_\mu \left(\rho \partial^\mu S \right) = 0 \tag{12.2}$$

Equation (12.2) has the form of a relativistic continuity equation in parameterized

relativistic dynamics when the 4-velocity is

$$v^{\mu} = \frac{1}{m}\partial^{\mu} S \tag{12.3}$$

If the observable mass m does not depend on space-time coordinates, Eq. (12.2) becomes

$$\frac{\partial \rho}{\partial s} + \partial_{\mu}\left(\rho v^{\mu}\right) = 0 \tag{12.4}$$

Introducing the relativistic action $S = \hbar\xi$ gives the relativistic Hamilton-Jacobi equation

$$\frac{\partial S}{\partial s} + \frac{1}{2m}\left(\partial_{\mu} S\right)\left(\partial^{\mu} S\right) + Q + V = 0 \tag{12.5}$$

with a relativistic quantum potential

$$Q = \frac{\hbar^2}{2m}\left[\frac{1}{4\rho^2}\partial^{\mu}\rho\partial_{\mu}\rho - \frac{1}{2\rho}\partial_{\mu}\partial^{\mu}\rho\right] \tag{12.6}$$

The term Q is a relativistic scalar because the probability density ρ, the observable mass m, and the derivative operators are relativistic scalars. The relativistic probability density is normalized over space-time and depends on space-time coordinates and a temporal parameter s. Both t and s serve as evolution parameters in their respective domains. Notice that this analysis does not rely on nor even allude to hidden variables. From a purely formal analysis, the relativistic quantum potential does not depend on hidden variables, nor does it imply that hidden variables must exist.

The Relativistic N-Particle System

The Stueckelberg equation for N spin-0 particles is

$$\left\{ i\hbar \frac{\partial}{\partial s} - \left[\sum_{n=1}^{N} \frac{\pi_n^{\mu} \pi_{n\mu}}{2m_n} + V_N \right] \right\} \psi(1,2,\ldots,N,s) = 0 \tag{12.7}$$

where V_N is a scalar interaction potential representing the interactions of all N particles, and the canonical four-momentum with minimal electromagnetic coupling is

$$\pi_n^{\mu} = \frac{\hbar}{i} \frac{\partial}{\partial x_n^{\mu}} - \frac{e}{c} A_n^{\mu} \tag{12.8}$$

We make the assumption that there is no electromagnetic interaction to simplify the following analysis. In this case the 4-vector potentials are zero and Eq. (12.7) becomes

$$\left\{ i\hbar \frac{\partial}{\partial s} - \left[\sum_{n=1}^{N} \left(-\frac{\hbar^2}{2m_n} \partial_n^{\mu} \partial_{n\mu} \right) + V_N \right] \right\} \psi(1,2,\ldots,N,s) = 0 \tag{12.9}$$

Following the usual procedure, we write the wave function as

$$\psi(1,2,\ldots,N,s) = \rho^{1/2}(1,2,\ldots,N,s) exp[i\zeta(1,2,\ldots,N,s)] \tag{12.10}$$

where ρ and ζ are real, scalar functions. Within the context of parameterized relativistic dynamics, ρ and ζ are relativistic scalars. Given Eqs. (12.9) and (12.10), we can derive the relativistic continuity equation

$$\frac{\partial \rho}{\partial s} + \sum_{n=1}^{N} \partial_{n\mu} \left(\rho v_n^{\mu} \right) = 0 \tag{12.11}$$

for the N-particle system with 4-velocity

$$v_n^{\mu} = \frac{1}{m_n} \partial_n^{\mu} S \tag{12.12}$$

and $S = \hbar\zeta$ is the relativistic action for the N-particle system. The relativistic Hamilton-Jacobi equation is

$$\frac{\partial S}{\partial s} + \sum_{n=1}^{N} \frac{1}{2m_n} \left(\partial_{n\mu} S\right)\left(\partial_n^{\mu} S\right) + Q_N + V_N = 0 \tag{12.13}$$

with relativistically scalar quantum potential

$$Q_N = \sum_{i=1}^{N} \frac{\hbar^2}{2m_n} \left[\frac{1}{4\rho^2} \left(\partial_{n\mu}\rho\right)\left(\partial_n^{\mu}\rho\right) - \frac{1}{2\rho} \partial_{n\mu} \partial_n^{\mu} \rho \right] \tag{12.14}$$

The quantum potential Q_N depends on the joint probability distribution $\rho(1,2,\ldots,N,s)$ of all N particles in space, time and the evolution parameter s. The change in the probability distribution associated with one particle, say particle 1, can change the quantum potential Q_N. We illustrate this concept by noting that the marginal probability distribution for particle 1 can be written as $\rho(1,s) = \rho(1,2,\ldots,N,s)/\rho(2,\ldots,N,s)$ where $\rho(2,\ldots,N,s)$ is a conditional probability distribution. The change in Q_N is non-local if particles with space-like separation relative to particle 1 are affected by the change in $\rho(1,s)$.

13. Experimental Test: Interference in Time

In principle, the scientific method requires that we conduct experiments to determine the validity of competing paradigms. In practice, it is difficult to secure resources to support the experiments, or technology may not be sufficiently

advanced for conducting suitable experiments to the level of accuracy that is needed to test predictions. Nevertheless, it is incumbent upon researchers studying competing paradigms to seek experimental tests that could test the validity of the paradigms. Several experimental tests are now possible. We consider two classes of tests here: interference in time, and mass state transitions.

Interference in time

Wave-particle duality is a key concept in quantum theory. We observed in Section 8 that a fine slit experiment could be used to demonstrate the interference of electron matter waves. This interference is a result of superpositioning matter waves in the space-momentum domain. The dependence of wave functions on space and time coordinates implies that interference should also be observable in the time-energy domain. Results from pulsed laser experiments have been reported by authors such as Linder, et al. [2005] and Wollenhaupt, et al. [2002]. The results demonstrate interference of electron matter waves in the time-energy domain. Lindner, et al. [2005] explained the interference results of their attosecond laser double-slit experiment by numerically solving the time-dependent, non-relativistic Schroedinger equation.

Horwitz [2006, 2007] presented several arguments that show that non-relativistic quantum mechanics in the conventional theory, which includes the Schroedinger equation, is not a proper theory for explaining the results of experiments that measure the interference of matter waves in the time-energy

domain. For example, Dirac [1930, pp 34, 36] argued that if time t in the non-relativistic quantum theory was an operator, then the commutation relation (t, E) between t and energy of the system E would imply that E is not bounded below. This implies that negative energy states should be possible in the conventional theory of non-relativistic quantum mechanics, which is not consistent with the conventional theory. Horwitz pointed out that negative energy states are allowed in relativistic quantum theories. Experiments that show the temporal interference of matter waves at low energies cannot be adequately described by the conventional theory of non-relativistic quantum mechanics. Horwitz argued that the experiments are significant because they show the need for new theoretical tools.

Thirty years earlier, Horwitz and Rabin [1976] used Stueckelberg's relativistic theory, the forerunner of parameterized relativistic dynamics (PRD), to predict interference in time. PRD treats time as an observable, has a commutation relation for time and energy, and allows both positive and negative energy states. PRD can be used to compute interference phenomena in the space-momentum and time-energy domains in a manner that is analogous to calculations of matter wave interference in the space-momentum domain in the conventional theory of non-relativistic quantum mechanics. Horwitz [2006, 2007] used PRD to calculate a diffraction formula to explain the results of experiments that measure interference of matter waves in the time-energy domain.

14. Experimental Test: K-meson Mass State Transition

The conditional probability distribution ρ defined in parameterized relativistic dynamics is mathematically capable of representing the disappearance or reappearance of a particle in space-time. Our purpose here is to present the formalism for describing a two-state particle, and then use the formalism to analyze the stability of the neutral K-meson (K^0). The results are then compared with results of the conventional theory.

Theory of Two-State Particle Decay

Consider a system with 2 particles: an s-clock particle and the 2-state particle. The 2-state particle is the primary system of interest in this treatment of particle stability. It is worthwhile to include the s-clock particle to illustrate how to use parameterized relativistic dynamics to relate the evolution parameter s to a physical system. We begin with the N-particle Stueckelberg equation

$$i\hbar \frac{\partial \underline{\Psi}}{\partial s} = \left[\sum_{a=1}^{N} \frac{\pi_a^{\mu} \pi_{a\mu}}{2m_a} \underline{\underline{I}} + \underline{\underline{V}} \right] \underline{\Psi} \tag{14.1}$$

and rewrite it for a 2-state particle and a scalar particle, thus:

$$i\hbar \frac{\partial \underline{\Psi}}{\partial s} = \left[\frac{\pi_1^{\mu} \pi_{1\mu}}{2m_1} \underline{\underline{I}} + \underline{\underline{V}}_1 \right] \underline{\Psi} + \left[\frac{p_2^{\mu} p_{2\mu}}{2m_2} \underline{\underline{I}} + V_2 \underline{\underline{I}} \right] \underline{\Psi} \tag{14.2}$$

We have assumed for simplicity that the s-clock particle, particle 2, is not

experiencing an electromagnetic interaction $\left\{A_2^{\mu}=0\right\}$ but may be subjected to a scalar potential V_2. The matrix potential experienced by particle 1 is assumed to depend on the coordinates of particle 1 such that $\underline{\underline{V_1}}=V_1\underline{\underline{I}}$ where $V_1\left(x_1^{\mu}\right)$ is a scalar potential. Particle 1 does not interact with the s-clock particle, particle 2.

The trial solution for this system is the column vector

$$\underline{\Psi}=\begin{bmatrix}\Psi_1(1,2,s)\\ \Psi_2(1,2,s)\end{bmatrix}=\phi(2,s)\begin{bmatrix}\psi_1(1,s)\\ \psi_2(1,s)\end{bmatrix} \tag{14.3}$$

where we have simplified the notation by denoting the coordinates of particle 1 by index 1 and the coordinates of particle 2 by index 2. The inner product of the trial solution is

$$\begin{aligned}\underline{\Psi}^{+}\underline{\Psi}&=\Psi_1^{*}\Psi_1+\Psi_2^{*}\Psi_2\\ &=\phi^{*}(2,s)\phi(2,s)\left\{\psi_1^{*}(1,s)\psi_1(1,s)+\psi_2^{*}(1,s)\psi_2(1,s)\right\}\end{aligned} \tag{14.4}$$

The normalization condition is

$$\iint\underline{\Psi}^{+}\underline{\Psi}dx_1dx_2=\int\phi^{*}\phi dx_2\int\left[\psi_1^{*}\psi_1+\psi_2^{*}\psi_2\right]dx_1=1 \tag{14.5}$$

which implies

$$\int\phi^{*}\phi dx_2=1 \tag{14.6}$$

and

$$\int\left[\psi_1^{*}\psi_1+\psi_2^{*}\psi_2\right]dx_1=1 \tag{14.7}$$

Substituting the trial solution into the Stueckelberg equation for this system gives the following equations:

$$i\hbar\frac{\partial\psi_j}{\partial s}=\left[\frac{\pi_1^{\mu}\pi_{1\mu}}{2m_1}+V_1\right]\psi_j\,;\,j=1,2 \tag{14.8}$$

for the 2-state particle, and

$$i\hbar\frac{\partial\phi}{\partial s}=\left[\frac{p_2^{\mu}p_{2\mu}}{2m_2}+V_2\right]\phi \tag{14.9}$$

for the s-clock particle.

Non-interacting 2-State Particle

It is easier to understand the analysis of particle decay by first analyzing a non-interacting particle with 2 states. Equation (14.8) for the free 2-state particle is

$$i\hbar\frac{\partial\underline{\Phi}}{\partial s}=-\frac{\hbar^2}{2m}\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right]\underline{\Phi} \tag{14.10}$$

where $\{x_1^{\mu}\}=\{\vec{x},ct\}$. It has the free particle solution

$$\underline{\Phi}=\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix}\sim\begin{bmatrix}exp\left(-i\omega_1 t+i\vec{k}_1\cdot\vec{x}-i\Omega_1 s\right)\\ exp\left(-i\omega_2 t+i\vec{k}_2\cdot\vec{x}-i\Omega_2 s\right)\end{bmatrix} \tag{14.11}$$

The constants $\{\Omega_j\}$ multiplying the evolution parameters are

$$\Omega_j=\frac{m_j^2c^2}{2m\hbar} \tag{14.12}$$

where m_j is the mass eigenvalue of state j and m is an effective mass that is

determined below. The value of m_j is found by substituting $\{\Psi_j\}$ into the field equation to find

$$\hbar\Omega_j = \frac{\hbar^2}{2m}\left[\frac{\omega_j^2}{c^2} - \vec{k}_j \cdot \vec{k}_j\right] = \frac{m_j^2 c^2}{2m} \tag{14.13}$$

and

$$m_j^2 = \frac{\hbar^2}{c^2}\left[\frac{\omega_j^2}{c^2} - \vec{k}_j \cdot \vec{k}_j\right] \tag{14.14}$$

The normalization condition is

$$\int \underline{\Phi}^+ \underline{\Phi} d^4x = 1 = \int \left(\Phi_1^* \Phi_1 + \Phi_2^* \Phi_2\right) d^4x \tag{14.15}$$

The effective mass m is given by

$$m^2 c^2 = \left\langle p^\mu p_\mu \right\rangle = \int \underline{\Phi}^+ p^\mu p_\mu \underline{\Phi} d^4x \tag{14.16}$$

The effective mass depends on the contribution of each state $\{\Phi_j\}$ to the total wave function $\underline{\Phi}$. For example, if the particle can be found in either state Φ_1 or Φ_2 with equal probability, then

$$\int \Phi_1^* \Phi_1 d^4x = \int \Phi_2^* \Phi_2 d^4x = \frac{1}{2} \tag{14.17}$$

and Eq. (14.16) becomes

$$m^2 c^2 = \frac{1}{2}\hbar^2\left(\frac{\omega_1^2}{c^2} - \vec{k}_1 \cdot \vec{k}_1\right) + \frac{1}{2}\hbar^2\left(\frac{\omega_2^2}{c^2} - \vec{k}_2 \cdot \vec{k}_2\right) \tag{14.18}$$

Recalling that

$$m_j^2 = \frac{\hbar^2}{c^2}\left[\frac{\omega_j^2}{c^2} - \vec{k}_j \cdot \vec{k}_j\right] \tag{14.19}$$

is interpreted as the mass of state j, then the effective mass is

$$m^2 = \frac{1}{2}\left(m_1^2 + m_2^2\right) \tag{14.20}$$

If $m_1 = m_2$, then the effective mass m of the 2-state particle is equal to the mass of the particle in either state Φ_1 or state Φ_2. If $m_1 \neq m_2$, then m is the average given above.

Expectation values can be used to relate the 4-position of the 2-state particle with s. We begin by evaluating

$$\delta\langle x^\mu \rangle = \frac{1}{m}\langle p^\mu \rangle \delta s \tag{14.21}$$

Performing the operator calculation and again assuming states Φ_1, Φ_2 are equally probable as in the previous example, we find

$$\delta\langle x^\mu \rangle = \frac{\hbar}{m}\left(\frac{k_1^\mu + k_2^\mu}{2}\right)\delta s \tag{14.22}$$

and

$$\delta\langle x^\mu \rangle \cdot \delta\langle x_\mu \rangle = \frac{\hbar^2}{m^2}\left(\frac{k_1^\mu + k_2^\mu}{2}\right)\cdot\left(\frac{k_{1\mu} + k_{2\mu}}{2}\right)\delta s^2 \tag{14.23}$$

If $k_1^\mu = k_2^\mu = k^\mu$, then we obtain the familiar relationship

$$\delta\langle x^\mu \rangle \cdot \delta\langle x_\mu \rangle = \frac{\hbar^2}{m^2} k^\mu \cdot k_\mu \delta s^2 \tag{14.24}$$

that describes the special relativistic trajectory of a free particle. If we measure

time x^0 in the rest frame, Eq. (14.24) becomes

$$c^2\left(\delta\langle t\rangle_{rest}\right)^2 = \frac{\hbar^2 k^\mu k_\mu}{m^2}\delta s^2$$

or $\delta\langle t\rangle_{rest} = \delta s$ for a free particle with $m^2c^2 = \hbar^2 k^\mu k_\mu$.

Interacting 2-State Particle

We are now ready to consider the stability of a particle with two mass states. The field equations for the two states Ψ_1, Ψ_2 of interest here are represented by Eq. (14.8). We highlight the essential aspects of this problem by assuming $\{A_1^\mu\} = 0$ so that Eq. (14.8) becomes

$$i\hbar\frac{\partial\Psi_j}{\partial s} = \left\{-\frac{\hbar^2}{2m_1}\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right] + V_1\right\}\Psi_j;\ j = 1,2 \tag{14.25}$$

where $\{x_1^\mu\} = \{ct, \vec{x}\}$. Equation (14.25) has the solution

$$\Psi_j \sim exp\left[-\Omega_j s - \frac{\lambda_j t}{2} + i\vec{k}_j \cdot \vec{x}\right];\ j = 1,2 \tag{14.26}$$

The term Ω_j is given by

$$\Omega_j = \frac{M_j^2 c^2}{2m_1\hbar} \tag{14.27}$$

where M_j is the mass of state j. The 3-momentum of state j is $\hbar\vec{k}_j$, and

$$\lambda_j^2 = \frac{8m_1c^2}{\hbar^2}V_1 - \frac{4c^2}{\hbar^2}\left[M_j^2c^2 + \hbar^2\vec{k}_j \cdot \vec{k}_j\right] \tag{14.28}$$

The normalization condition for the state $\underline{\Psi}^T = [\Psi_1, \Psi_2]$ is

$$\int \underline{\Psi}^+ \underline{\Psi} d^4x = 1 = \int \left(\Psi_1^* \Psi_1 + \Psi_2^* \Psi_2\right) d^4x \tag{14.29}$$

The formalism presented above is readily applicable to the neutral K-meson. In the conventional theory the neutral kaon state K^0 and its antiparticle state $\overline{K}^0$ are considered superpositions of states. We adopt this interpretation and write the symmetric state K^0 and anti-symmetric state $\overline{K}^0$ in terms of the states Ψ_1 and Ψ_2, thus

$$K^0 \sim \Psi_1 + \Psi_2 \tag{14.30}$$

and

$$\overline{K}^0 \sim \Psi_1 - \Psi_2 \tag{14.31}$$

The intensity of a K^0 beam is

$$\begin{aligned} K^{0*}K^0 &\sim \left(\Psi_1 + \Psi_2\right)^* \left(\Psi_1 + \Psi_2\right) \\ &= \Psi_1^*\Psi_1 + \Psi_2^*\Psi_2 + \Psi_1^*\Psi_2 + \Psi_2^*\Psi_1 \end{aligned} \tag{14.32}$$

Substituting Eq. (14.26) into (14.32) gives

$$\begin{aligned} K^{0*}K^0 \sim & e^{-\lambda_1 t} + e^{-\lambda_2 t} + e^{-\frac{\lambda_1+\lambda_2}{2}t} e^{i(\Omega_1-\Omega_2)s} e^{-i\left(\vec{k}_1-\vec{k}_2\right)\vec{x}} \\ & + e^{-\frac{\lambda_1+\lambda_2}{2}t} e^{-i(\Omega_1-\Omega_2)s} e^{i\left(\vec{k}_1-\vec{k}_2\right)\vec{x}} \end{aligned} \tag{14.33}$$

A similar result is obtained for the intensity of a $\overline{K}^0$ beam.

The expression for $K^{0*}K^0$ in Eq. (14.33) can be compared to the result of the conventional theory by writing Eq. (14.33) in the rest frame where three-

momenta vanish and $t \to s$. The result is

$$K^{0*}K^{0} \sim \left[e^{-\lambda_1 s} + e^{-\lambda_2 s} + e^{-\frac{\lambda_1+\lambda_2}{2}s} 2\,cos(\Omega_1 - \Omega_2)s \right] \tag{14.34}$$

The difference in mass between states Ψ_1, Ψ_2 is contained in the argument

$$(\Omega_1 - \Omega_2)s = \left(\frac{M_1^2 c^2}{2m_1\hbar} - \frac{M_2^2 c^2}{2m_1\hbar} \right) s = \frac{1}{2m_1\hbar}\left(M_1^2 c^2 - M_2^2 c^2\right)s \tag{14.35}$$

Comparison of Theoretical Predictions

The change in mass between state K_1 and state K_2 in the conventional theory is

$$\Delta m_{12} = \frac{\hbar}{c^2}\left|\omega_1 - \omega_2\right| \tag{14.36}$$

where $\hbar\omega_1$ is the energy of the long lived state K_L^0 and $\hbar\omega_2$ is the energy of the short lived state K_S^0.

The change in mass between state K_1 and state K_2 in parameterized relativistic dynamics is given by Eq. (14.35):

$$\Delta M_{12} = \frac{\hbar}{c^2}\frac{1}{2m_1\hbar}\left[M_1^2 c^2 - M_2^2 c^2\right] = \frac{1}{2m_1}\left[M_1^2 - M_2^2\right] \tag{14.37}$$

where M_1 is the mass of the long lived state K_L^0 and M_2 is the mass of the short lived state K_S^0. The mass m_1 for the 2-state system is the average given in Eq. (14.20), or

$$m_1^2 = \frac{1}{2}\left[M_1^2 + M_2^2\right] \tag{14.38}$$

Substituting Eq. (14.38) into Eq. (14.37), and defining the mass difference $\delta_{12} = M_1 - M_2$, we have to first order

$$\Delta M_{12} \approx \frac{2\delta_{12} M_1}{2\left[M_1^2 - \delta_{12} M_1\right]^{1/2}} = \frac{\delta_{12}}{\left[1 - \frac{\delta_{12}}{M_1}\right]^{1/2}} \tag{14.39}$$

Recognizing that $\delta_{12} << M_1$, we have

$$\Delta M_{12} \approx \delta_{12}\left[1 + \frac{1}{2}\frac{\delta_{12}}{M_1}\right] \tag{14.40}$$

The mass differences predicted by the conventional theory in Eq. (14.36) and parameterized relativistic dynamics in Eq. (14.40) are very similar. Differences between the two theories arise because the theories treat time differently. The difference is highlighted in the analysis of neutrino oscillations presented next. Independent, accurate, and direct measurements of neutral K-meson masses are needed to test the theories.

15. Experimental Test: Neutrino oscillations

Neutrino oscillation by flavor mixing has been described by the mixing of either two or three neutrino flavors. We outline here the two-state flavor mixing calculation within the context of parameterized relativistic dynamics [Fanchi, 1998b], and show that the result is an expression for the probability of transition

from one neutrino flavor to another that has similarities with the result of the conventional theory, but differs in details that are experimentally testable.

The evolution equation for a state may be written in terms of the evolution operator as

$$i\hbar \frac{\partial}{\partial s}\left|\nu_j\right\rangle = K_j\left|\nu_j\right\rangle \tag{15.1}$$

where K_j is the eigenvalue of the mass operator for mass state j. We restrict our discussion to two mass states $\left\{\left|\nu_j\right\rangle\right\}$ and two neutrino flavor states $\left\{\left|\nu_\varepsilon\right\rangle\right\}$, both of which may be written as 2-component column vectors:

$$\left\{\left|\nu_j\right\rangle\right\} = \begin{bmatrix}\left|\nu_1\right\rangle \\ \left|\nu_2\right\rangle\end{bmatrix}, \left\{\left|\nu_\alpha\right\rangle\right\} = \begin{bmatrix}\left|\nu_e\right\rangle \\ \left|\nu_\mu\right\rangle\end{bmatrix} \tag{15.2}$$

We relate the mass basis $\left|\nu_j\right\rangle$ to the flavor basis $\left|\nu_\alpha\right\rangle$ with a unitary transformation U such that

$$\left|\nu_\alpha\right\rangle = U\left|\nu_j\right\rangle \tag{15.3}$$

where

$$\left|\nu_\alpha\right\rangle = U\left|\nu_j\right\rangle \tag{15.4}$$

and θ is the mixing angle of mass states in vacuum. We hypothesized as in the conventional theory that the mixing angle is not zero so that it is meaningful to speak of flavor state mixing.

The evolution parameter dependent solution of Eq. (15.1) in the mass basis is

$$\begin{bmatrix} |v_1(s)\rangle \\ |v_2(s)\rangle \end{bmatrix} = \begin{bmatrix} e^{-iK_1 s/\hbar} & 0 \\ 0 & e^{-iK_2 s/\hbar} \end{bmatrix} \begin{bmatrix} |v_1(0)\rangle \\ |v_2(0)\rangle \end{bmatrix} \tag{15.5}$$

where

$$K_j = \hbar^2 k_j^{\mu} k_{j\mu} / 2m_j = \hbar^2 \left[(\omega_j / c)^2 - k_j \cdot k_j \right] / 2m_j \tag{15.6}$$

The energy-momentum four-vector is k_j^{μ} and m_j is the mass of state j. In parameterized relativistic dynamics, the components of the energy-momentum four-vector are observables. The treatment of neutrino oscillations as flavor mixing within the conventional theory assumes the 3-momentum equality $\mathbf{k}_j = \mathbf{k}$ for both mass states.

In an oscillation process, we begin with a pure beam of electron neutrino ν_e particles and calculate the probability for formation of muon neutrino ν_μ particles. For two flavors of neutrinos and the unitary transformation U we have

$$\begin{aligned} |v_e\rangle &= cos\,\theta\, |v_1\rangle + sin\,\theta\, |v_2\rangle \\ |v_\mu\rangle &= -sin\,\theta\, |v_1\rangle + cos\,\theta\, |v_2\rangle \end{aligned} \tag{15.7}$$

The evolution parameter dependent state for the electron neutrino is

$$|v_e(s)\rangle = e^{-iK_1 s/\hbar}\, cos\,\theta\, |v_1\rangle + e^{-iK_2 s/\hbar}\, sin\,\theta\, |v_2\rangle \tag{15.8}$$

The above expressions are used to form the matrix element for the transition from state ν_e to ν_μ:

$$\begin{aligned} \langle v_\mu | v_e(s)\rangle = &\left[-\langle v_1 | \sin\theta + \langle v_2 | \cos\theta \right] \times \\ &\left[e^{-i\kappa_1 s} \cos\theta\, |v_1\rangle + e^{-i\kappa_2 s} \sin\theta\, |v_2\rangle \right] \end{aligned} \tag{15.9}$$

where $\kappa_j = K_j / \hbar$. Expanding and simplifying the transition matrix element gives

$$\langle \nu_\mu | \nu_e(s) \rangle = sin\,\theta\, cos\,\theta \left(e^{-i\kappa_2 s} - e^{-i\kappa_1 s} \right) \tag{15.10}$$

where we have used the orthonormality condition

$$\langle \nu_i | \nu_j \rangle = \delta_{ij} \tag{15.11}$$

for {i,j}={1,2}. Using a trigonometric identity lets us write

$$\langle \nu_\mu | \nu_e(s) \rangle = \frac{1}{2} sin\,2\theta \left(e^{-i\kappa_2 s} - e^{-i\kappa_1 s} \right) \tag{15.12}$$

We calculate the square of the magnitude of the exponential term for later use in calculating the probability. From Euler's formula we find

$$e^{-\kappa_2 s} - e^{-\kappa_1 s} = cos\,\kappa_2 s - cos\,\kappa_1 s - i\left(sin\,\kappa_2 s - sin\,\kappa_1 s \right) \tag{15.13}$$

The square of the magnitude of this term is

$$\left| e^{-i\kappa_2 s} - e^{-i\kappa_1 s} \right|^2 = 2\left[1 - cos(\kappa_2 - \kappa_1)s \right] \tag{15.14}$$

The probability of forming state ν_μ is

$$P(\nu_e \to \nu_\mu) = \tfrac{1}{2} sin^2\, 2\theta \cdot \left[1 - cos(\kappa_2 - \kappa_1)s \right] \tag{15.15}$$

The corresponding probability of remaining in state ν_e for the 2-state system is

$$P(\nu_e \to \nu_e) = 1 - P(\nu_e \to \nu_\mu) = 1 - \tfrac{1}{2} sin^2\, 2\theta \cdot \left[1 - cos(\kappa_2 - \kappa_1)s \right] \tag{15.16}$$

The probability $P(\nu_e \to \nu_e)$ may be written in the form

$$P(\nu_e \to \nu_e) = 1 - \tfrac{1}{2} sin^2\, 2\theta \left[2\, sin^2 \frac{(K_2 - K_1)s}{2\hbar} \right] \tag{15.17}$$

Simplifying $P(\nu_e \to \nu_e)$ gives

$$P(v_e \to v_e) = 1 - sin^2\, 2\theta\, sin^2 \frac{(K_2 - K_1)s}{2\hbar}$$
$$P(v_e \to v_\mu) = 1 - P(v_e \to v_e) = sin^2\, 2\theta\, sin^2 \frac{(K_2 - K_1)s}{2\hbar} \tag{15.18}$$

Disappearance Experiment

The model developed above is explicitly related to experiment by applying it to the disappearance of an electron antineutrino produced by a source, such as a nuclear reactor or supernova. Evidence of electron neutrino disappearance is obtained by counting coincidence events in which positrons and neutrons are simultaneously created by inverse β decay of the neutron, i.e., the interaction of an electron antineutrino and a proton: $\bar{v}_e + p \to e^+ + n$. According to the flavor-mixing hypothesis, the electron antineutrino state may transform into an alternative neutrino species as it propagates. In principle, detectors can be placed along the electron antineutrino flight path to determine the probability of disappearance of electron antineutrinos as a function of distance L from their source.

The relativistic energy difference between the initial and final flavor states is

$$K_2 - K_1 = \frac{\hbar^2 k_2^\mu k_{2\mu}}{2m_2} - \frac{\hbar^2 k_1^\mu k_{1\mu}}{2m_1} \tag{15.19}$$

In the case of negligible dispersion, we have

$$K_2 - K_1 \approx \frac{m_2^2 c^2}{2m_2} - \frac{m_1^2 c^2}{2m_1} = \frac{(m_2 - m_1)c^2}{2} \tag{15.20}$$

For comparison with the conventional theory, we express K_2 - K_1 in terms of the rest mass difference such that

$$K_2 - K_1 \approx \frac{\Delta m}{2} c^2, \Delta m \equiv m_2 - m_1 \tag{15.21}$$

Substituting the above expression into the probability of forming the final state ν_μ from initial state ν_e gives

$$P(\nu_e \to \nu_\mu) = sin^2 2\theta \, sin^2 \frac{(K_2 - K_1)s}{2\hbar} \tag{15.22}$$

To complete the derivation, we substitute Eq. (11.11) into Eq. (15.22) to find

$$P(\nu_e \to \nu_\mu) = sin^2 2\theta \, sin^2 \frac{\Delta m c^2}{4\hbar} \frac{L}{c} \frac{\left[1 - \beta^2\right]^{1/2}}{\beta} \tag{15.23}$$

Model Comparison

The parameterized relativistic dynamics result for the probability of forming the final state ν_μ from initial state ν_e is

$$\begin{aligned} P_{PRD}(\nu_e \to \nu_\mu) &= \sin^2 2\theta \sin^2 \left\{ \frac{(m_2 - m_1)c^2}{4\hbar} \frac{L}{c} \frac{\left[1 - \beta^2\right]^{1/2}}{\beta} \right\} \\ &\equiv \sin^2 2\theta \sin^2 \alpha_{PRD} \end{aligned} \tag{15.24}$$

Dynamical factors are collected in the term α_{PRD}. We make a direct comparison

of Eq. (15.24) with the conventional theory by substituting $E_\nu = m_1 c^2 \Big/ \sqrt{1-\beta^2}$ into the conventional theory result to find

$$P_{Std}\left(\nu_e \to \nu_\mu\right) = \sin^2 2\theta \sin^2 \left\{ \frac{(m_2 - m_1)c^2}{4\hbar} \frac{L}{c} \left[1-\beta^2\right]^{1/2} \frac{(m_2+m_1)}{m_1} \right\} \qquad (15.25)$$
$$\equiv \sin^2 2\theta \sin^2 \alpha_{Std}$$

Comparing P_{PRD}, P_{Std} and the dynamical factors $\alpha_{PRD}, \alpha_{Std}$ shows that the parameterized relativistic dynamics model and the conventional theory have the same dependence on the flavor mixing angle θ, but their dependence on dynamical factors differs significantly.

Discussion of Systematic Bias

Ideally, neutrino oscillation experiments should provide direct measurements of the energy and momentum of the incident electron neutrino and the product muon neutrino. In addition, the experiments should provide the probability of disappearance $P\left(\nu_e \to \nu_\mu\right)$ of electron neutrinos and the distance of separation L between the electron neutrino source and detector. In practice, experimentalists have been unable to provide all of this information.

The ratio of probabilities in Eqs. (15.24) and (15.25) is

$$\frac{P_{Std}}{P_{PRD}} = \frac{\sin^2 \alpha_{Std}}{\sin^2 \alpha_{PRD}} \qquad (15.26)$$

This ratio exhibits a complicated relationship that depends on poorly known

physical parameters, namely (m_1, m_2, $\beta = v/c$). Fanchi [1998] presents examples for the ratios of the two probabilities and figures illustrating the distinctively different oscillatory behavior of the probabilities for some arbitrarily assumed parameters.

In principle, it should be possible to test the validity of the conventional theory and parameterized relativistic dynamics as more experimental data becomes available. The possibility of experimental testing has been corrupted by a systematic bias in which results of neutrino oscillation experiments are presented as plots of $\Delta m^2 \equiv m_2^2 - m_1^2$ versus $\tan\theta$. The factor Δm^2 allows a simplified presentation of the conventional theory result given by Eq. (15.25), but Δm^2 does not simplify the presentation of data for the parameterized relativistic dynamics result shown in Eq. (15.24). A typical representation of experimental data in the literature is a theory-dependent plot of $\Delta m^2 \equiv m_2^2 - m_1^2$ versus $\tan\theta$.

Another view of neutrino oscillation experimental results is obtained by rearranging Eq. (15.25) to find

$$\left(m_2^2 - m_1^2\right) = \left[sin^{-1} \sqrt{\frac{P_{Std}\left(v_e \to v_\mu\right)}{sin^2 2\theta}} \right] \Bigg/ \left[\frac{c^4}{4\hbar} \frac{L}{cE_\nu} \right] \tag{15.27}$$

The terms on the right hand side of Eq. (15.27) are all observable given a mixing angle. The usual plots of experimental data in the literature may be viewed as theory-independent plots of $\left[\sin^{-1} \sqrt{\frac{P_{Std}\left(v_e \to v_\mu\right)}{\sin^2 2\theta}} \right] \Bigg/ \left[\frac{c^4}{4\hbar} \frac{L}{cE_\nu} \right]$ versus $\tan\theta$.

This view removes the systematic bias associated with plots of $\Delta m^2 \equiv m_2^2 - m_1^2$

versus $\tan\theta$, but it does not explicitly present the crucial data $P(v_e \to v_\mu)$, and it continues to present data as a function of the unknown mixing angle.

An alternative plot that facilitates the comparison of competing theories without introducing a systematic bias is to plot $P(v_e \to v_\mu)$ versus L because both $P(v_e \to v_\mu)$ and L are directly measurable. We can have more confidence in the objective validity of masses and mixing angles obtained from theory-independent techniques if the masses and mixing angles are calculated from a presentation of data that does not depend on a particular theory.

16. Experimental Test: Tachyon Physics

In our discussion of non-locality in Section 12, we observed that faster-than-light (FTL) or superluminal travel was a possible mechanism for explaining non-locality. Many physicists have given serious consideration to FTL travel ever since Bilaniuk et al. [1962] showed that the concept was not necessarily precluded by special relativity. Bilaniuk et al. introduced the term "luxon" to denote a particle traveling at the speed of light. To distinguish between FTL particles and everything else, Feinberg [1967] called FTL particles "tachyons," which he coined from the Greek word for swift. Bilaniuk and Sudarshan [1969] used the term "tardyons" to denote slower-than-light particles. "Tardyon" has since been replaced by "bradyon," which was coined from the Greek word for slow [Recami, 1987]. We use the terms bradyon, luxon, and tachyon to denote subluminal, luminal, and superluminal particles, respectively.

An important feature of parameterized relativistic dynamics is that all values of energy and momentum must be considered, and not just those values that yield on-shell masses. One consequence of this feature is that we can describe free particle motion for both bradyons and tachyons (see Section 9). This has significant physical implications.

Classical special relativity does not allow subluminal particles to become superluminal particles. The mass m of a classical particle moving at speed v is related to the rest mass m_0 by the transformation

$$m = m_0 \Big/ \sqrt{1 - \left(\frac{v}{c}\right)^2} \tag{16.1}$$

The mass m approaches infinity as $v \to c$ from below or from above. Is there a way to get around this restriction?

In non-relativistic quantum theory, time-dependent potentials allow for transitions between energy states, which are observed in a variety of physical systems, such as spectra and lasers. Some classically forbidden transitions can occur in non-relativistic quantum theory because of the quantum tunneling effect. Is there an analogue in parameterized relativistic dynamics?

In parameterized relativistic dynamics, we have seen that s-dependent potentials allow for transitions between mass states. We now describe a system that allows for quantum transitions across the light cone.

Consider mass state transitions at a scattering vertex [Fanchi, 1990, 1993b]. We postulate the following hypothetical physical picture:

$$\text{Projectile } (\Psi) + \text{Target } (\Phi_T) \to \text{Product} \tag{16.2}$$

A field equation for this system is

$$i\hbar \frac{\partial \Psi}{\partial s} = -\frac{\hbar^2}{2m_\Psi} \frac{\partial^2 \Psi}{\partial x^\mu \partial x_\mu} + g(\Phi_T + \Phi_T^*)\Psi \tag{16.3}$$

where g is a postulated coupling constant. Note that the s-dependence occurs in the wave function interaction term. This problem can be solved using iterative techniques. An iterative solution to the s-dependent perturbation problem is obtained by analogy to the non-relativistic t-dependent perturbation calculation. In the non-relativistic theory, the transition amplitudes refer to transitions between energy states, as in the laser. In PRD, the transition amplitudes refer to transitions between mass states.

s-Dependent Perturbation Theory

The field equation for an s-dependent perturbation is

$$i\hbar \partial_s \Psi = K_0 \Psi + K_1 \tag{16.4}$$

where K_0 refers to the unperturbed mass generator and K_1 refers to the s-dependent interaction term. The perturbation calculation is subject to the constraint that assures the perturbation is Hermitian:

$$\int [K_1^* \Psi - \Psi * K_1] d^4 x = 0 \tag{16.5}$$

An approximate solution to the perturbation problem is obtained by writing the eigenfunction expansion

$$\Psi(x,s) = \int a_\alpha(s) \psi_\alpha(x,s) d\alpha \tag{16.6}$$

where ψ_α are solutions for the unperturbed system and $a_\alpha(s)$ are expansion coefficients. This expansion may be viewed as a superposition of unperturbed mass states.

Mass State Transitions

The transition probability amplitude in PRD is

$$a_\alpha = a_\alpha^0 - \frac{i}{\hbar}\int_0^s [\int \psi_\alpha^* K_I d^4x] ds' \tag{16.7}$$

The transition probability density to state α is

$$P_\alpha = a_\alpha^* a_\alpha \tag{16.8}$$

and the transition rate density to state α is

$$R_\alpha = \frac{\partial P_\alpha}{\partial s} \tag{16.9}$$

If we apply the s-dependent perturbation theory formalism to the interaction in Eq. (16.2), we find the usual four-momentum constraints

$$\begin{aligned} &Four - Momentum \\ &k_\alpha = k_a + K_b \\ &k_\alpha = k_a - K_b \end{aligned} \tag{16.10}$$

where subscript a denotes the projectile, subscript b denotes the target, and subscript α denotes the product particle. In addition, we find a set of equations that constrain the allowed masses. The mass constraint equations are

Mass

$$q_\alpha = q_a + Q_b \qquad (16.11)$$
$$q_\alpha = q_a - Q_b$$

where the free particle masses are

$$q_n = \frac{\hbar^2 k_n \cdot k_n}{2m_n \hbar} \qquad (16.12)$$

Possible mass state transitions are given in Table 1. Letter B denotes Bradyon and letter T denotes Tachyon. The first row in the table, for example, says that a bradyon projectile interacting with a bradyon target can produce a bradyon under the Product-1 constraint. If the Product-2 constraint applies, a bradyon projectile interacting with a bradyon target can produce either a bradyon or a tachyon. The final product would depend on the properties of the target and projectile. Allowed interactions would also have to account for four-momentum constraints that represent four-momentum conservation.

Table 1. Possible Mass State Transitions

Target Q_b	**Projectile** q_a	**Product-1** $q_\alpha = q_a + Q_b$	**Product-2** $q_\alpha = q_a - Q_b$
B	B	B	B T
B	T	B T	T
T	B	B T	B
T	T	T	B T

According to Table 1, tachyons may be created when two particles interact, even if the two interacting particles are bradyons. Futhermore, an existing tachyon may be annihilated in interactions with bradyons or tachyons. Thus, a bradyon interaction may produce a tachyon that exists for a short time and then is annihilated to produce more bradyons. These observations are still valid when four-momentum conservation is added to the kinematics of mass state transitions.

Equation (16.11) is the one feature of parameterized relativistic dynamics which makes the formulation different from other tachyon kinematic systems [Fanchi, 1990]. It serves to constrain the types of physically allowable processes. The additional constraint arises from an additional conservation law that is fundamental to parameterized relativistic dynamics. A legitimate question to ask is whether the new mass constraint violates existing experimental data.

Two types of experiments have been frequently performed in high-energy physics: scattering of a particle beam by a thin foil; and counter-circulating beam experiments. If tachyons exist, why have they not been observed in these experiments?

The constraints imposed on the allowed physical properties of tachyons by our simple model are quite restrictive. This implies that an experimental search must be very specific. Some possible experiments are suggested by the above calculations. If the theory is correct, a projectile bradyon interacting with a thin foil of target bradyons should produce tachyons if the projectile energy and momentum are properly set and a nonzero coupling constant for a parameter-dependent potential exists between the projectile and target. In this regard the

question of detectability becomes relevant since we have seen that tachyons may be annihilated as well as created. Thus, the observable signature of the produced tachyon may not be tachyonic, but bradyonic. Specific predictions will depend on the particular physical experiment being considered.

The addition of a new constraint does not contradict corresponding experimental results. There are mass relations in the literature that are used to compute the mass of a product particle as the sum of the masses of its constituents. These relations are usually accompanied by corrections for binding energy and other interactions. Examples include the quark model and Weizacker's semi-empirical mass formula. The system considered here is a relatively simple system. At this level of complexity, the new mass constraint is not in violation of known experimental data.

Perhaps the most significant physical point of this presentation is that tachyon creation within the context of PRD does not require continuous acceleration of a classical time-like entity through the light cone. Instead, tachyons are created or annihilated by quantum transitions. This is an important conceptual advantage of PRD. The true test of the theory is in the realm of experiment.

The possibility that tachyons may be produced from bradyon interactions may impact calculations of the rate of expansion of the universe immediately after the big bang. Tachyon production could be a physical mechanism for cosmological inflation. This topic should be studied further.

17. Conclusions

Parameterized relativistic dynamics (PRD) is a non-local, manifestly covariant quantum theory with invariant evolution parameter. The theory appears to have great potential as a practical formalism for working with relativistic bound state problems, as illustrated by solutions of the harmonic oscillator problem and the hydrogen problem. Many other problems representing a range of applications in a variety of areas have already been solved, including classical and quantum statistical mechanics, general relativity and cosmology, electrodynamics, scattering, the Klein paradox, quantum field theory, string theory, etc. For more detailed discussion of these applications or for more references, see Fanchi [1993b], Trump and Schieve [1999], and Pavsić [2001].

Mathematically, PRD is a straightforward extension of the conventional paradigm: PRDcontains an additional temporal parameter, and expectation values are defined over all space-time (or energy-momenta). As a consequence of these extensions, the theory is an unconstrained theory in the sense that the probability amplitude receives contributions from on-shell mass terms and off-shell mass terms.

The formulation of PRD that uses a Stueckelberg-like equation can be viewed as a non-local theory because a relativistic quantum potential can be defined. Furthermore, if parameter dependent interactions are allowed, PRD provides a physically consistent description of tachyons, including a mechanism for crossing the light cone.

PRD contains a combination of assumptions that yields significant differences with the conventional paradigm in our understanding of such fundamental concepts as mass and time. The differences manifest themselves in experimentally testable predictions. Some of the predictions discussed here were interference in time, and mass state transitions represented by physically realistic and observable systems such as K-meson stability and neutrino oscillation. The next step in a search for objective reality in nature is to test the paradigms.

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Appendix: Notation and Metric

The following notation is used throughout the article unless indicated otherwise.

Nonzero elements of the fundamental metric tensor are

$$g_{00} = 1 = -g_{11} = -g_{22} = -g_{33} \tag{A.1}$$

Greek indices run from 0 to 3 and Latin indices run from 1 to 3. Index 0 denotes the temporal component, while indices 1 to 3 denote spatial components.

The contravariant four-vector A^{μ} has the elements

$$\left\{A^{\mu}\right\} = \left(A^{0}, \vec{A}\right) \tag{A.2}$$

and the covariant four-vector has the elements

$$\left\{A_{\mu}\right\} = \left(A^{0}, -\vec{A}\right) \tag{A.3}$$

A space-time position four-vector x^{μ} may be written as

$$\left\{x^{\mu}\right\} = \left(x^{0}, \vec{x}\right) = \left(ct, x, y, z\right), \left\{x_{\mu}\right\} = \left(x^{0}, -\vec{x}\right) = \left(ct, -x, -y, -z\right) \tag{A.4}$$

in terms of Cartesian coordinates $\{x, y, z\}$ in three space dimensions.

Scalar product is defined by

$$A_{\mu} B^{\mu} = A^{0} B^{0} - \vec{A} \cdot \vec{B} = A^{\mu} B_{\mu} \tag{A.5}$$

Partial differential operators are

$$\partial^{\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\nabla\right) = \frac{\partial}{\partial x_{\mu}}, \partial_{\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right) = \frac{\partial}{\partial x^{\mu}} \tag{A.6}$$

and

$$\partial_{\mu}\partial^{\mu} = \frac{\partial^2}{\partial\left(x^0\right)^2} - \nabla^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \tag{A.7}$$

where ∇^2 is the Laplacian operator.

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