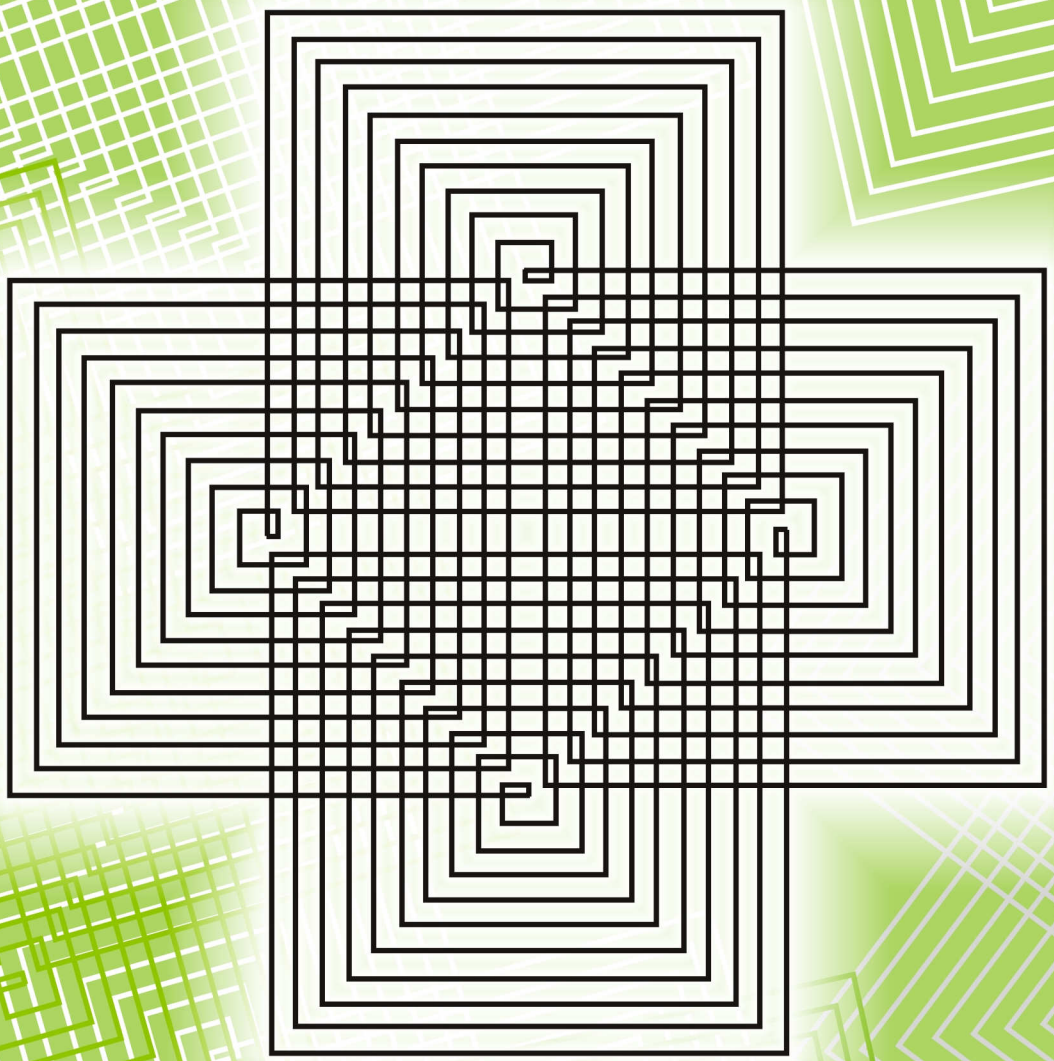
ISSN-0974-1623

ISSUE- 6 Jan. -2012-13

NUMTA BULLETIN

(A BULLETIN OF PURE AND APPLIED MATHEMATICS)



Published by

Nagpur University Mathematics Teachers' Association

(Regd. No. MAH/359/2001)

NUMTA BULLETIN, 6 (2012-13), 1-33

An Introduction to Santilli's Isodual Theory of Antimatter and the Open Problem of Detecting Antimatter Asteroids

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Abstract We review evidence suggesting that Earth has been devastated in the past by antimatter asteroids, as well as evidence of contemporary detection of antimatter asteroids and cosmic rays. Via the use of Santilli's isodual mathematics and related theory specifically built for the *classical* representation of *neutral* (or charged) antimatter as needed for the study of the gravity or detection of antimatter, we point out that antimatter asteroids are not necessarily visible with light originating from matter such as our Sun, thus constituting a threat for our planet. To stimulate the initiation of much needed research in antimatter, we outline proposed experiments to measure the gravity of antiparticles, since such a knowledge is a necessary prerequisite for serious studies in the detection of antimatter asteroids.

Keywords antimatter, antigravity, isodual theories.

§1. Historical Notes

As it is well known, Newton's equations, Galileo's relativity and Einstein's special and general relativities were conceived before the discovery of antimatter and therefore no classical representation of "neutral" antimatter could be generated since at that time only conjugation

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Figure 1: Newton, Galileo, and Einstein conceived their theories solely for the description of matter and not of antimatter that was discovered by Dirac in 1928, some fourteen years following the formulation of general relativity [18].

from matter to antimatter was the change of sign of the charge (see Figure 1 and monograph [18] for historical references).

This occurrence created one of the biggest scientific imbalance in the history because throughout the 20^{th} century matter was studied at all possible levels, from Newtonian mechanics to second quantization, while antimatter was solely studied at the particle level. In essence, the prevalent stand still adopted is that, since Einstein's special and general relativities do not provide a proper description, antimatter does not exist in the universe in any appreciable amount. The sole generally admitted exception is that of of man made antiparticles created in laboratory, since their existence cannot be denied.

The above scientific imbalance was identified, apparently for the first time, by the Italian-American scientist Ruggero Maria Santilli (see his CV at http://www.world-lecture-series.org/santilli-cv) who, during his graduate studies in the 1960s at the University of Torino, Italy, decided to ascertain whether a far away star or galaxy is made up of matter or of antimatter. In so doing, Santilli soon discovered that the entire body of mathematical, theoretical and experimental knowledge he was acquiring during his graduate studies was inapplicable to his aim because far away stars and galaxies have to be studied at the classical level and one has to necessarily assume it as being neutral because of their isolation in space.

In this way, Santilli initiated a solitary scientific journey reported in this paper on the achievement of: firstly, a mathematics suitable for the classical and operator representation of either neutral or charged antimatter; secondly, a reformulation of Newton's, Galileo's and Einstein's theories suitable for the study of neutral or charged antimatter at all possible levels; and, thirdly, the formulation of experiments to ascertain, in due time, whether far away stars and galaxies are made up of matter or of antimatter.

As it is well known, the literature in antimatter is rather vast indeed although, following an extensive search, the authors could identify no *classical* studies on *neutral* antimatter other than those by Santilli and his associates [1-33]. Therefore, to avoid a prohibitive length, the authors apologize for delaying a general historical review at some later time, and for restricting the content of this paper to mathematical, theoretical and experimental aspects specifically

suited for the open problem of detecting antimatter asteroids. However, the authors would gratefully appreciate the indication of references not quoted in this paper specifically dealing with the classical treatment of neutral antimatter.

§2. Evidence of Antimatter in the Universe

The above indicated simplistic 20^{th} century position on antimatter implied the rather general belief that antimatter galaxies do not exist. This stringent stand eliminated altogether the problem of detecting antimatter asteroids on grounds that they do not exist due to the absence of the antimatter galaxies and related antimatter supernovas needed for their origination.

This position was evidently based in the unspoken intent of maintaining the validity of Einstein's theories for all of the universe via the denial of the existence of antimatter galaxies, despite it being disproved by evidence since our Earth has indeed been hit in the past by devastating antimatter asteroids, and similar asteroids have been recently detected by various observatories.

In fact, the catastrophic 1908 Tunguska explosion in Siberia with the power of one thousand Hiroshima nuclear bombs can be solely interpreted in a scientific way as being due to an antimatter asteroid annihilating in our atmosphere (see Ref. [34] for a first treatment and monograph [18] for a comprehensive study). This is due to various reasons, such as the complete absence of debris, let alone of a crater, in the ground.

Figure 2: Pictures of the Siberia area hit by the 1908 Tunguska explosion showing lack debris in the ground.

Most importantly, the Tunguska explosion excited the entire Earth's atmosphere for days, to such an extent that two days following the explosion, people could read newspapers in Sydney, Australia, at midnight without artificial light; and other reasons. Such a large excitation of the atomic and molecular constituents of our atmosphere can only be scientifically (i.e., quantitatively) represented as being due to huge radiations that, in turn, can only originate from the annihilation of an antimatter asteroid with our matter atmosphere.

The widely accepted "interpretation" of the Tunguska explosion as being due to a (matter) comet has no scientific credibility due to the impossibility of such an origination to excite the

entire Earth's atmosphere for days, and occur with the absence of debris in the grounds, let alone with the absence of a crater.

Quite recently, NASA has reported explosions in our upper atmosphere that can only be due to small antimatter asteroids because annihilating at the time of contact with the upper portion of our matter atmosphere.

Similarly, both astronauts and cosmonauts have observed flashes in our upper atmosphere when on the dark side with respect to our Sun, these flashes can be best interpreted as being due to antimatter cosmic rays that annihilate in our atmosphere, because the only cosmic rays that can reach us at sea level being those due to matter cosmic rays.

As indicated above as well as earlier by Santilli and others, the existence of antimatter stars and galaxies is imperative and should not be ignored. As a representative example out of many, we recall antimatter is thought to exist in the Oort Cloud in view of a possible explanation for gamma ray bursts [35]. In fact, these phenomena can be explained by the annihilation of matter and antimatter asteroids or small comets. The explosion would create powerful gamma ray bursts and accelerate matter.

Besides antimatter asteroids, it is possible that Earth has been hit in the past by antimatter comets as indicated by the old observations, since the biblical times, not only of excessive brilliance but also of trajectories in our atmosphere that cannot be interpreted as being due to matter comets, e.g., because of slow penetration of the said objects in our atmosphere [36].

In conclusion, the evidence on the existence of antimatter asteroids as well as of antimatter comets and their possibility of hitting Earth again is sufficiently serious to warrant the writing of this paper and consequantly the initiation of systematic studies for their detection.

§3. A Guide to Santilli's Literature on Antimatter

As indicated earlier, scientific studies in the detection of antimatter asteroids requires mathematical and physical theories suitable for the *classical* treatment of *neutral* antimatter evidently because antimatter asteroids are too large to be treated via operator theories and they must be assumed as being neutral since they are isolated in space.

In this paper we shall only use Santilli's mathematical and theoretical studies in antimatter since they are the only ones the authors could identify capable of classical representations of neutral antimatter.

Since Santilli's literature in the field is too vast for a comprehensive review [1-25], it appears recommendable to provide a guide through said literature for interested readers to get initiated to undertake deeper studies. Virtually all papers and some of the monographs quoted in this work are available for free pdf download.

Santilli has repeatedly stated in his writings that: *A protracted lack of solution of physical problems is generally due to the use of insufficient or inadequate mathematics* [18]. Additionally, Santilli stated that: *There cannot exist a really new physical theory without a really new mathematics, and there cannot exist a really new mathematics without new numbers* [*loc. cit.*]. For this reason, Santilli had spent decades in purely mathematical research, firstly, to identify new numbers and, secondly, to develop new mathematics that would allow a classical treatment

of neutral or charged antimatter, because the entire body of applied mathematics is built on numbers.

Along these lines, the most fundamental and very first paper published by Santilli on antimatter is Ref. [1] of 1993 that introduced for the first time new numbers called "isodual" where the prefix "iso" was introduced in the Greek sense of indicating the preservation of conventional axioms used for matter and the term "dual" stands to indicate the map from matter to antimatter. The role of Santilli isodual numbers is such that his entire theory of antimatter that is called "isodual" precisely because of the main character of the new basic numbers.

It should be noted that Santilli discovered the new isodual number in Ref. [1] (written when he was visiting the *Joint Institute of nuclear Research* in Dubna, Russia) as a particular case of much more general numbers he called "isonumbers" and "genonumbers" and their isoduals but we refrain ourselves to review them to avoid and excessive length. Readers interested in these broader numbers are advised to read reference [1].

Following the discovery of new numbers, Santilli constructed in Ref. [2] also of 1993 (see also Ref. [4]) the isodualities of the Euclidean and Minkowski spaces which were evidently needed for any possible physical applications. He then proceeded in Ref. [3] to the construction of the isodual image of Lie's theory because it is evidently necessary for the construction of basic symmetries for antimatter, viz. the isodual images of the rotation, Galileo and Lorentz symmetries.

Finally, in the mathematical memoir [5] Santilli made a second fundamental mathematical discovery, a new formulation of the ordinary differential calculus that resulted in being crucial for the achievement of the first known formulation of Newton's equations for neutral or charged antiparticles. The complete formulation of the novel isodual mathematics was first presented in monograph [10] of 1994 and then updated in monographs [18] of 2001.

Following the achievement of structural consistency of the new isodual mathematics, and only thereafter, Santilli initiated his physical studies with paper [6] of 1993 written on his original aim of the 1960s as a graduate student, how to detect possible antimatter stars and galaxies.

Subsequently, Santilli wrote: a paper [7] of 1994 on the all important classical representation of neutral antimatter; the submission of experiment in the paper [8] of 1994 to test the gravity of positrons (see also papers [11,16,25]); and a paper [9] also of 1994 on the causal space-time machine, i.e., the capability of moving as desired in space and time without the violation of causality which is an inevitable consequence of gravitational repulsion between matter and antimatter (although such a causal space-time motion is not possible for ordinary matter or antimatter, but for a special combination of both called "isoselfdual" reviewed later with a comprehensive treatment provided in monograph [18]).

Ref. [12] of 1996 is the first independent contribution in the field by the experimentalist A. P. Mills establishing that Santilli's gravity experiment [8] is "resolutory" because the displacement due to gravity of very low energy positrons on a scintillator at the end of a flight in a super-cooled super-vacuum tube is visible to the naked eye.

The first isodualities of Galileo's and Einstein's relativities were presented in paper [13] of

Figure 3: We here present pictures of flashes in the upper portion of our atmosphere that can only be scientifically interpreted as being due to the annihilation of antimatter cosmic rays. In fact, the ordinary matter cosmic rays rapidly penetrate in our atmosphere and, in the event there is a collision with an atmospheric nucleus, we have the typical production of a shower without any possible production of a ball of light. In any case, when traveling on the dark side of Earth with respect to the Sun, astronauts and cosmonauts have routinely detected flashes of light in the upper atmosphere that can solely be due to the annihilation of antimatter cosmic rays.

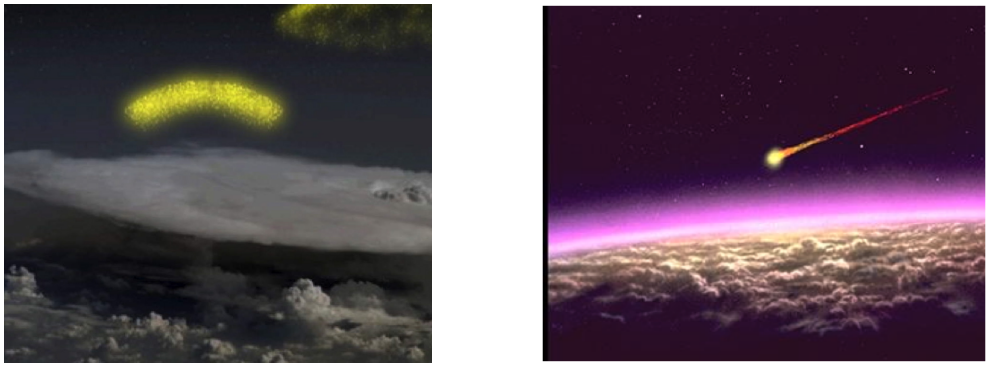

Figure 4: The existence of antimatter throughout the universe continues to be indicated by reliable scientific institutions, despite dismissal by physicists not admitting limitations of Einstein's theories. In the left, we show a picture released by FERMILAB as an apparent detection of antimatter in the universe. In the right we show a picture released by NASA as representing an apparent, rather small antimatter asteroid that annihilated at the contact with the upper portion of our atmosphere.

1997; another basic physical discovery has been discussed in paper [14] of 1997 via the prediction that light emitted by antimatter is repelled by a matter gravitational field. This prediction is an inevitable consequence of the main feature of the studies herein reviewed, namely, the classical conjugation from neutral matter to neutral antimatter that evidently also applied to light.

Except for updates, Santilli's subsequent studies were centered around the possible use of the latter prediction to ascertain whether a far away star or galaxy is made up of matter or of antimatter.

The paper [15] of 1998 is one of the best scientific papers written by Santilli in the field because it achieves the first known representation of the gravitational field of antimatter, that additionally serves in achieving the universal "invariance" (in lieu of the popularly accepted "covariance") for both the gravitational field of matter and antimatter, and sets the foundation for the first known grand unification of electroweak and gravitational interactions which is inclusive of antigravity subsequently developed in details in Ref. [18] of 2001.

The paper [17] of 1999 is written by the theoretical physicist J. Dunning-Davies and provides the first quantitative study in scientific records on the thermodynamics of antimatter bodies.

The memoir [19] of 2006 is the most advanced paper written by Santilli in the field because it treats matter and antimatter in their general conditions of irreversibility over time at the classical and operator levels.

In paper [20] of 2011, Santilli is finally in a position to address his main objective of the 1960s, how to identify experimentally antimatter stars and galaxies and, consequently, how to detect antimatter asteroids.

In paper [21] of 2011, the experimentalist V. de Haan confirms Mills's results [12] that Santilli's experiment [8] on the gravity of positrons in horizontal flight on Earth is indeed resolutory.

Santilli's paper [22] of 2011 with the mathematicians B. Davvaz and T. Vougiouklis is conceptually, mathematically and physically the most advanced paper in the field because it establishes that the universe is "multi-valued" rather than "multi-dimensional" since matter and antimatter coexist in physically distinct space-times, thus implying multi-valuedness.

Monograph [24] of 2011 by theoretical physicists I. Gandzha and J. V. Kadeisvili provides an independent general review of Santilli's studies on antimatter at three successive levels of study, including single-valued reversible, single-valued irreversible, and multi-valued irreversible conditions,

Reference [32] of 2012 by C. Corda is an excellent review and elaboration of the fundamental entities of the isodul theory of antimatter, Santilli's isodual numbers.

Reference [33] of 2013 by A. A. Bhalekar is an introductory account of the basic mathematics, with minimum abstract contents, behind the subject matter of this paper which is best suited for the beginners.

4. Santilli's Isodual Mathematics for Antimatter

It is evident that the problem of detecting possible antimatter asteroids is of such a magnitude that it cannot be left unaddressed just to maintain the validity of Einstein's theories for antimatter. In an event North America is hit by an antimatter asteroid even with the size of a football, all North American communications will be disrupted, while the Military will be inoperative for days, due to extreme radiations absorbed and re-emitted by Earth's atmosphere.

The same holds in the unfortunate event an antimatter asteroids hits India, Russia, China or other regions. Consequently, the problem of possible antimatter asteroids requires attention not only by the people at large, but also by the scientific and military communities.

Figure 5: A View of the Science Center of Harvard University housing the Department of Mathematics s where Santilli made his main mathematical discoveries .

The multitude of open problems created by the detection of antimatter have been studied for decades by Santilli who has provided scientific arguments establishing that the threat to Earth caused by antimatter asteroids is more serious than what popularly believed in contemporary academia.

When he was at the Department of Mathematics of Harvard University in the early 1980s, Santilli spent years in the local libraries to search for a mathematics suitable for the classical representation of classical neutral antimatter and he found none.

Therefore, he had to confront the problem of *identifying a mathematical conjugation (also called map or duality) capable of performing the transition from matter to antimatter at the purely classical Newtonian level, irrespectively of whether matter and antimatter are neutral or charged, under the condition that such a map recovers charge conjugation at the quantum level for the sake of consistency, which is evidently needed.*

Following a decade of unpublished trials and errors, Santilli selected the following main assumption for the construction of the needed new mathematics . Recall that the conventional charge conjugation is defined on a Hilbert space $\mathcal{H}$ with states $\psi(r)$ over the field of complex numbers $\mathcal{C}$ and can be characterized by a conjugation of the type applied to the quantum representation of matter

$$C\ \psi(r) = -\ \psi^\dagger(r), \tag{1}$$

where r is the coordinate of the Euclidean representation space.

Consequently, Santilli introduced an anti-Hermitean conjugation called *isoduality* and denoted with the upper index ^{d} that, by central condition, has to be applied to all physical quantities, to all their units and to all their operations, and can be written as,

$$Q(t, r, v, ...) \to Q^d(t^d, r^d, v^d, ...) = -Q^\dagger(-t^\dagger, -r^\dagger, -v^\dagger, ...) \tag{2}$$

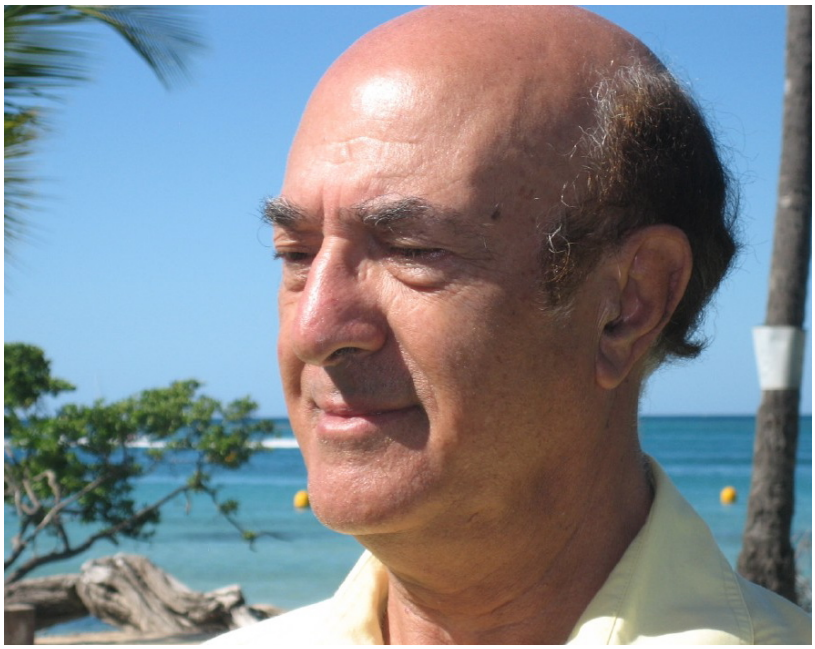

Figure 6: A picture of Prof. Ruggero Maria Santilli in 2009 when 74 years old.

where Q denotes a generic quantity depending on time t, coordinates r, velocities v, and any other needed variables.

For the trivial case of real numbers, isoduality reduces to the mere change of the sign of all quantities, all their units and the related operations. In the event a given body is charged, the isoduality evidently also applies by changing the sign of the charge. Hence, Santilli's isoduality applies irrespective of whether the body is charged or not.

The main difference between conventional charge conjugation and Santilli's isoduality is that the former solely applies at the quantum level, while the latter applies, by central conception, at the classical Newtonian level, as well as at all subsequent levels of study, including the quantum level in which charge conjugation and isoduality are equivalent [18].

In terms as simple as permitted by the advanced nature of the topic, the classical isodual map from neutral matter to the corresponding neutral antimatter requires that all physical quantities change their sign. Consequentially, under the isodual map, time, energy, linear momentum, entropy, and other positive-definite physical quantities, become negative.

There have been a number of proposals that antimatter must move backward in time as a condition to admit annihilation into light. This rather natural assumption has been dismissed due to the violation of causality when motion backward in time is treated with the mathematics used for matter (see Ref. [18] for historical notes and references).

However, for Santilli's isodual mathematics, antimatter's motion backward in time $t^d = -t < 0$, when referred to a negative unit of time (e.g., $\mathrm{s}^d = -1$ s), is as causal as matter moving forward in our time $t > 0$ when referred to the usual positive units of time (e.g., $+1$ s).

Similarly, it is known since Dirac's time that negative energies also violate causality laws.

However, Santilli has shown that negative-definite isodual energies $E^d = -E < 0$ referred to negative units (e.g., $\text{erg}^d = -1$ erg) are as physical as conventional positive energies $E > 0$ referred to positive units (e.g., $+1$ erg). Inconsistencies emerges only under crossovers of the two worlds, such as when positive energies are measured with negative units and vice versa.

In going deeper into the problem, Santilli discovered that the correct formulation of antimatter requires an entire new mathematics, today known as *Santilli's isodual mathematics,* which can be defined as the anti-Hermitean image of the entire mathematics used for matter, thus including *isodual numbers, isodual functional analysis, isodual differential calculus, isodual Lie's theory, etc.* In fact, any mix-up, even minute, of the mathematics for matter and that for antimatter leads to catastrophic inconsistencies that are generally not realized by non-experts in the field.

Santilli's research in antimatter was delayed for years by the classification of all "numbers" (namely, sets verifying the axioms of a numeric field) into real, complex and quaternion numbers that had been achieved by historic masters such as Gauss, Cayley, Hamilton and others.

Finally, in 1993 Santilli re-inspected this historical classification and discovered that the axioms of a numeric field do not require *the basic multiplicative unit to be necessarily positive definite,* since said unit can be negative definite as well, provided that the conventional associative product of numbers $n \times m$ is redefined in such a way to admit the newly assumed negative unit.

We have in this way *Santilli's isodual fields* $F^d(n^d, \times^d, 1^d)$ consisting of *isodual real, isodual complex and isodual quaternion numbers* with negative-definite *isodual multiplicative unit* and related *isodual (associative) multiplication* [1].

$$F^d(n^d, \times^d, 1^d): \quad n^d = -n^\dagger, \quad n^d \times^d m^d = n^d \times (1^d)^{-1} \times m^d, \quad 1^d = -1^\dagger, \tag{3}$$

under which 1^d is indeed the basic multiplicative unit at all levels (the additive unit 0 of a field remains evidently unchanged under isoduality because $0^d \equiv 0$ [1]),

$$1^d \times^d n^d \equiv n^d \times^d 1^d \ \forall \ \ n^d \in F^d. \tag{4}$$

Note, to prevent insidious misinterpretations, that is *for the evident mathematical consistency*, all real numerical values in isodual mathematics must be elements of Santilli's isodual fields and, as such, must be given by ordinary numbers multiplied by the isodual unit.

For years Santilli did not publish any physics paper in antimatter because he thought that applications would be "too strange" due to the unusual implications of his isodual mathematics.

For instance, the isodual mathematics requires that the basic unit of the theory must be the negative unit $1^d = -1$ (meaning in quantum mechanics that the basic unit must be $\hbar^d = -\hbar$); the product is redefined in such a way that "$2 \times 3 = -6$"; the positive and negative value in a bank checking account for matter are reversed in the transition to antimatter according to which, in the latter case, a positive \$1M > 0 balance in the account implies that we are bankrupt, since the unit of the account is now the "negative dollar $\$^d = -1\$$; and so on.

Yet, Santilli continued his solitary research on isodualities. The discovery of the new fields F^d as being evidently anti-isomorphic to conventional fields F that reassured Santilli on the existence of the isodual mathematics he was looking for to represent antimatter, because it is a known fact that applied mathematics is build step by step on the base field.

Figure 7: Some of the honors received by Prof. Santilli from the top left: 1) The 1981 Gold medal o the Université D'Orleans, France, for the initiation of the Lie-admissible treatment of irreversible processes and the organization at that University of the first international conference in the field; 2) The nomination in 1990 by the Estonia Academy of Sciences (under communist regime) of Prof. Santilli among the most illustrious applied mathematicians of all times, such as Newton, Gauss, Hamilton, etc. for his discoveries in mathematics; 3) The prestigious 2007 Mediterranean Prize for being one of the few scientists in history to have achieved important discoveries in mathematics, physics and chemistry, a prize also granted to Hillary Clinton, Price Albert of Monaco , France President Nicolas Sarkozy, Juan Carlos King of Spain, international architect Renzo Piano, and other famous people.; 4) The reception in 2011 of the prestigious Gran Cross of the Sant'Agata Order , the highest honor granted by the Republic of San Marino, for his studies on antimatter and the organization at that Republic of the first international meeting on "antimatter astrophysics."

Consequently, Santilli undertook to the construction of isodual functional analysis and isodual metric spaces, such as the *Euclid-Santilli isodual space* with line element [2]

$$E^d(r^d, \delta^d, I^d) : r^d = r \times I^d,\ \delta^d = Diag(-1,-1,-1) = \delta \times I^d,\ I^d = Diag.(-1,-1,-1), \quad (5a)$$

$$r^{d2d} = (r^{di} \times^d \delta^d_{ij} \times^d r^{dj}) \times I^d \equiv r^2 = (r^i \times \delta_{ij} \times r^j) \times I^d, \quad (5b)$$

where one should note the necessity of multiplying the line element by the isodual unit as a condition to be an **isodual scalar,** that is, an element of the isodual field. It then follows that *the line element of the Euclid-Santilli isodual space coincides with that of the conventional Euclidean space,* Eq. (5b), and this explains the reason for the lack of detection of the isodual spaces for centuries.

Note that the study of the isodualities of the Euclidean space was a necessary pre-requisite to reach the yet unknown (in 1993) formulation of Newton's equation for neutral or charged antimatter (see next section).

Additionally, Santilli constructed the isoduality of Lie's theories, Minkowskian and Riemannian geometries and of virtually all mathematics used for the study of matter [10].

Thanks to his keen self-criticism, the emerging new mathematics continued to have hidden inconsistencies whose solution required additional years of study. Finally, in 1995, Santilli had the courage to re-inspect another pillar of 20^{th} century applied mathematics, the ordinary differential calculus, by discovering that, contrary to popular belief since Newton's time, *the differential calculus does indeed depend on the assumed basic unit and related field.*

We reach in this way the discovery of *Santilli's isodual differential calculus* with *isodual differentials* and *isodual derivatives* that, for the case of a real-valued new unit acquires the simple form [5]

$$d^d r^d = 1^d dr^d = 1^d d(r1^d) \equiv dr, \quad (6a)$$

$$\partial^d F^d(r^d)/^d\partial^d r^d = -\partial F/\partial r, \quad (6b)$$

where one can see that (again for the case of a real-valued new unit) *Santilli's isodual differential coincides with the conventional differential,* and this explains the reason the isodual differential calculus remained unidentified for centuries.

Following the discovery of the isodual differential calculus, Santilli completed the construction of the isodual mathematics with a rigorous structural consistency, and passed only thereafter to physical applications.

5. Isodual Newton, Galileo, Einstein and Quantum Theories

By using the isodual mathematics specifically built for the representation of antimatter, Santilli constructed the isodual image of all main theoretical formulations used in the 20^{th} century for matter, including:

Figure 8: Some of the additional honors received by Prof. Santilli from the top left: 1) The reception of the 2011 Scientific Award granted by the Sons of Italy, Tampa, Florida; 2) The 2011 Nepal Scientific Prize for is Lie-admissible studies of irreversible processes and the organization at the University of Kathmandu of the third International Conference in the field; 3) The reception of the 2011 Fellowship of the European Society of Computational Methods in Science and Engineering, Alkidiki, Greece; 4) A ten stories high picture of Prof. Santilli in Times Square, New York, at his ringing of the bell of the stock exchange NASDAQ on October 18, 2012.

1. The *Newton-Santilli isodual equations* defined on an isodual Euclidean space $E^d(r^d, \delta^d, I^d)$ over the field of isodual real numbers $\mathcal{R}^d(n^d, \times^d, I^d)$

$$m^d \times^d \frac{d^d v^d}{d^d t^d} = F^d(t^d, r^d, v^d, ...), \tag{7}$$

which are and remain to this day *the only known, classical, mathematically and physically consistent differential equations for the representation of neutral or charged antiparticles at the primitive Newtonian level.* It should be recalled that Newton had to invent the differential calculus as a condition to formulate his celebrated equations. Exactly on the similar lines, Santilli had to discover first a new form of differential calculus, anti-isomorphic to Newton's version, as a condition to achieve a consistent classical representation of antiparticles.

2. The *Galileo-Santilli isodual relativity,* not reviewed here for the sake of brevity (see Ref. [10,18]), that also is and remains to this day *the only known consistent theory of relativity for the classical non relativistic description of antimatter, by keeping in mind that, prior to Santilli's advances, there was none.* This second achievement was permitted, not only by the isodualities of numbers, differential calculus and the Euclidean space, but also by the isoduality of Lie's theory and its application to the fundamental Galilean symmetry [10], today known as the *Galileo-Santilli isodual symmetry.* It should be also noted that the Galileo-Santilli isodual relativity in its classical and operator forms (see below) are the fundamental formulations for antimatter due to the impossibility for our current technological knowledge of producing and maintaining antiparticles at relativistic speeds for significant time duration.

3. The *Einstein-Santilli isodual special and isodual general relativities,* also has not been reviewed here for the sake of brevity (see Ref. [18]). These isodual relativities were developed thanks to the additional isoduality of the Minkowskian and Riemannian geometries. Throughout the 20^{th} century, antiparticles were believed to exist in our space-time, while the Minkowski-Santilli isodual geometry established that antiparticles exist in a space-time completely distinct from our own, yet coexisting with the latter, thus leading to the multi-valued (rather than multi-dimensional) structure of the universe [22] indicated in the preceding section. Following decades of research, the isodualities of the Riemannian geometry finally permitted Santilli to achieve one of his primary goals: a mathematically consistent representation of the gravity of antimatter bodies, such as antimatter planets, stars and galaxies.

4. The *isodual quantum mechanics,* which constitutes one of the eight different branches of *hadronic mechanics* [10]. It is also given by the application of the anti-Hermitean transformation (2) to the totality of quantities, their units and their operations of conventional quantum mechanics. We merely recall here the fundamental unit of isodual quantum mechanics, the *isodual Plank's constant*

$$\hbar^d = -\hbar = -1; \tag{8}$$

the *Schrödinger-Santilli isodual eigenvalue equation*

$$H^d(r^d,p^d)\times^d|\psi^d\rangle = E^d\times^d|\psi^d\rangle \equiv E|\psi^d\rangle, \quad E^d<0, \quad E>0; \tag{9}$$

and the *isodual expectation value* of an observable A

$$\langle\psi^d|\times^d A^d\times^d|\psi^d\rangle I^d \equiv \langle\psi^d|\times A\times|\psi^d\rangle I, \tag{10}$$

and other properties (see monographs [10,18]).

An important feature of isodual quantum mechanics is that, as it was the case for line element (5b) and differential (6a), *the equations for isodual eigenvalue and expectation values coincide with conventional quantum equations.*

This simple mathematical feature has the physically important implication according to which, even though existing in a separate space-time with negative energies, antiparticles manifest themselves in our space-time with conventional positive energies. Additionally, identities (9)-(10) imply that the isoduality of quantum mechanics is a concrete realization of "hidden variables."

Recall that particles and antiparticles exist in the same space for quantum mechanics because they are treated via the same Hilbert space over the same field of complex numbers. By contrast, for Santilli's isodual theory, antiparticles exist in a space physically distinct from that of particles, because antiparticles are treated in a distinct space, the **Hilbert-Santilli isodual space** defined over the field of isodual complex numbers.

To understand the irreconcilable character of this dual representation of matter and antimatter, one should know that there exists no inner map in quantum mechanics (such as space and time inversions) capable of achieving Santilli's isodual map Eq. (2) which, as such, is a new transformation of space, time, Hilbert spaces and other quantum mechanical quantities.

A particularly intriguing implication of the above studies is a necessary re-interpretation of the celebrated Dirac's equation

$$[\gamma^\mu(p_\mu - eA_\mu/c) + im]\Psi(x) = 0, \tag{11a}$$

$$\gamma_k = \begin{pmatrix} 0 & -\sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad \gamma^4 = i\times\begin{pmatrix} I_{2\times2} & 0, \\ 0 & -I_{2\times2} \end{pmatrix}, \tag{11b}$$

$$\{\gamma_\mu,\tilde{\gamma}_\nu\} \;=\; 2\eta_{\mu\nu}, \quad \Psi = i\times\begin{pmatrix} \Phi \\ -\Phi^\dagger \end{pmatrix} \tag{11c}$$

As it is well known, $20^{\rm th}$ century physics interpreted Dirac's equation as representing "one spin 1/2 particle," the electron, since Dirac's equation implies negative energies for the related antiparticle, the positron, with consequential violation of causality and other physical laws that forced Dirac to construct the celebrated "hole theory" (see Ref. [18] for historical accounts).

Santilli's isodual theory of antimatter has changed rather dramatically the above scenario. First of all, Santilli pointed out a structural inconsistency of the $20^{\rm th}$ century interpretation of Dirac's equation consisting in the fact that [18] *there exists no irreducible or reducible four-dimensional representation for spin 1/2 of the SU(2)-spin symmetry with the structure of Dirac*

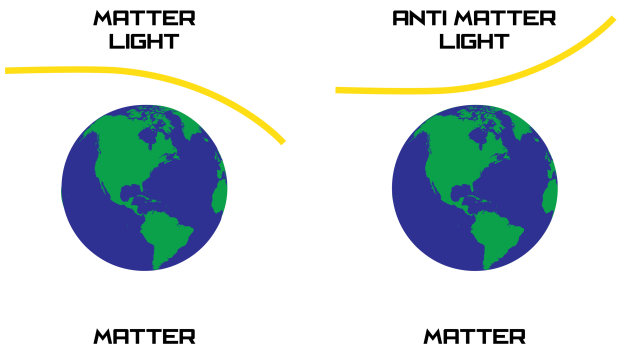

Figure 9: Since light has no mass, a main consequence of Santilli's isodual map for neutral antimatter is that light emitted by antimatter (called antimatter light or also isodual light) is physically distinct from light emitted by matter (called matter light) in an experimentally detectable way. Since all physical characteristics, thus including the energy, are subjected to isodual conjugation, it then follows that antimatter light is repelled by a matter gravitational field according to isodual laws that can be experimentally verified via sufficiently sensitive detectors, such as those in neutron interferometry and other detectors.

gamma matrices (11b). In this way, thanks to his advanced knowledge of Lie's theory, Santilli established the structural inconsistency of the 20^{th} century interpretation of Dirac's equation as representing "one spin 1/2 particle."

Following that, and thanks to the newly discovered isodual mathematics, Santilli pointed out that [18] *Dirac's gamma matrices are the Kronecker product of one two-dimensional spin 1/2 fundamental representation of the $SU(2)$-spin symmetry, given by Pauli's matrices σ_k, $k = e, 1, 2, 3$, multiplied by its two-dimensional isodual image, that is, the fundamental representation of the isodual $SU^d(2^d)$-spin symmetry given by the* **Pauli-Santilli isodual matrices** $\sigma_k^d = -\sigma_k^\dagger$, $k = e, 1, 2, 3,$.

The use of the entire body of isodual formulations then permitted the following "identical" reformulation of Eq. (11), today known as the *Dirac-Santilli isodual equation* (see Sect. 2.3.6, pages 122-118 of Ref. [18])

$$[\tilde{\gamma}^\mu(p_\mu - e \times A_\mu/c) + im]\tilde{\Psi}(x) = 0, \tag{12a}$$

$$\tilde{\gamma}_k = \begin{pmatrix} 0 & \sigma_k^d \\ \sigma_k & 0 \end{pmatrix}, \ \tilde{\gamma}^4 = i \times \begin{pmatrix} I_{2\times 2}^d & 0, \\ 0 & I_{2\times 2}^d \end{pmatrix}, \tag{12b}$$

$$\{\tilde{\gamma}_\mu, \tilde{\gamma}_\nu\} = 2^d \times {}^d\eta_{\mu\nu}^d, \ \ \tilde{\Psi} = -\tilde{\gamma}_4\Psi = i \begin{pmatrix} \Phi \\ \Phi^d \end{pmatrix} \tag{12c}$$

In particular, as visible from the diagonal values of Dirac's forth gamma matrix, the representation of the electrons has the unit $+I_{2\times 2}$ while that for the positrons has the unit $-I_{2\times 2}$, thus having a remarkable compatibility with the isodual theory. It then followed that the

former characterizes particles with positive energy referred to positive units, while the latter characterizes antiparticles with negative energies referred to negative units.

In this way, Santilli resolved Dirac's historical problem of the unphysical negative energies. The solution was visible in the very structure of the gamma matrices. It is unfortunate that Dirac did not know the very deep isodual mathematical meaning of his forth gamma matrix

$$\gamma^4 = i \begin{pmatrix} I_{2\times 2} & 0, \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{13}$$

As a result of these studies, the fundamental 20^{th} century equation for particle physics is now interpreted as representing the Kronecker product of "one spin 1/2 particle" and its antiparticle, each existing in its own space-time, the Minkowski space-time for the former and the Minkowski-Santilli isodual space-time for the latter.

The usual objection against this interpretation (that antiparticles solely exist at the level of second quantization) is readily dismissed because Santilli's isodual theory provides a consistent representation of antiparticle at the purely classical Newtonian level, let alone that in first quantization here referred to.

Finally, Santilli applied all the preceding studies for the construction of the only known axiomatically consistent *grand unification of electroweak and gravitational interactions* that includes, for the first time a full treatment of antimatter at the particles as well as at the gravitational level, and the use of a universal "symmetry" of gravitation for matter and its isodual for antimatter locally isomorphic to the corresponding symmetries for electroweak interactions [18].

The latter symmetries assure the prediction of the same numerical values under the same conditions at different times. By contrast, the 20^{th} century "covariance" of gravitation causes serious insufficiencies due to the evident impossibility (by the very definition of covariance) of having time invariant numerical predictions.

It should be also noted that this important advancement in grand unifications was due not only to the isodual theory of antimatter, but also to the achievement of the universal symmetry of all Riemannian line elements, today known as the *Poincaré-Santilli isosymmetry for matter and its isodual for antimatter* [10].

It should be finally indicated that Santilli constructed his isodual theories as particular cases of six much broader generalizations of 20^{th} century mathematical and physical theories, today known as *isotopic, genotopic and hyperstructural formulations for matter and their isoduals for antimatter*.

These broader formulations were constructed for particles and antiparticles in corresponding conditions of increasing complexities, such as: reversible, single-valued, non-linear, non-local and non-Hamiltonian interactions; irreversible, single-valued, non-linear, non-local and non-Hamiltonian interactions; and irreversible, multi-valued, non-linear, non-local and non-Hamiltonian interactions.

It is evident we cannot possibly review the latter studies due to their advanced mathematical and physical nature, and have to refer the interested reader to monographs [10].

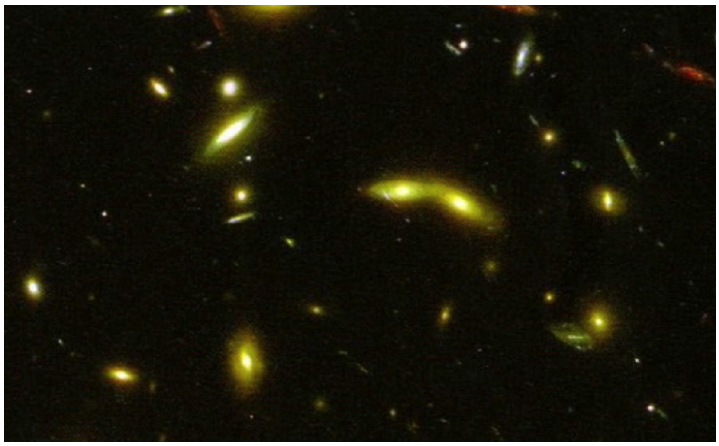

Figure 10: As indicated in Section 1, when he was a graduate student at the University of Torino, Italy, in the 1960s, Santilli became interested in astrophysics and decided to ascertain whether a far away star or galaxy is made up of matter or of antimatter. However, he soon discovered that the mathematical and physical theories he had learned at the graduate school were inapplicable to his objective since said body of knowledge was solely applicable to charged quantum particles, while far away stars and galaxies must be studied at the classical level and be assumed as being neutral. As a result of these insufficiencies, Santilli initiated a fifty years old scientific journey reviewed in this paper.

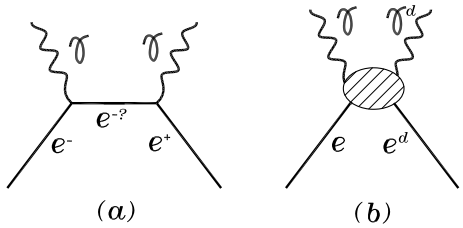

Figure 11: In going deeper into the study of antimatter, Santilli discovered that, as it was the case for all other 20^{th} century physical theories, Feynman's diagrams have serious flaws when applied to antimatter, since when applied to particle-antiparticle annihilation Feynman's diagrams violate Santilli's new "isoselfdual invariance" (defined as the invariance of a matter-antimatter system under isoduality). In fact, Feynman's diagrams would require that the electron-positron annihilation is mediated by the exchange of a particle as shown in the left of this picture. However, this creates a clear breaking of the full democracy between particles and antiparticles achieved by Santilli because set forth in nature. Additionally, annihilation occurs at the very "contact" of particle and antiparticles, thus prohibiting the very notion of intermediate particle exchanges, as shown in the right of the figure. This limitation of Feynman's diagrams prevents a theoretical resolution of the attraction or repulsion of antimatter light by a matter gravitational field, thus requiring basically new experiments, most importantly, Santilli gravity experiment [8,12,21,25].

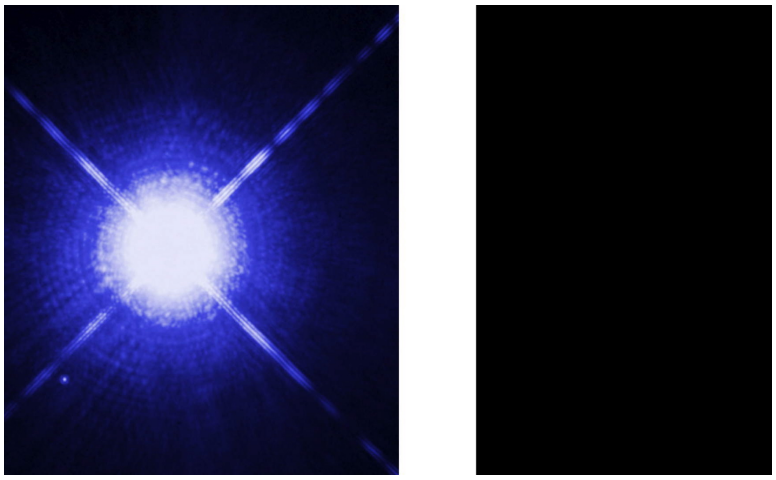

Figure 12: As a result of decades of studies, Santilli indicates that, in the current state of our knowledge on antimatter, the repulsion of antimatter light by our matter planet, even if confirmed, is not sufficient to ascertain whether a far away star is made up of matter or of antimatter because of the unknown behavior of antimatter light at the contact with our matter instruments. Similarly, it is not possible to establish at this writing whether the celebrated Crab nebula is composed of the inert matter or antimatter, since the latter may prevent the passage of matter light in its background.

6. Verifications of the Isodual Theory of Antimatter

It is important to note that Santilli's isodual theory is verified by all known experimental evidences on antimatter available to date on antiparticles.

At the classical level, we have the *Coulomb-Santilli isodual laws* for charged particles-antiparticles in our world ,

$$F = k \times \frac{q_1 \times q_2^d}{r^2}. \tag{14'}$$

and in the isodual world

$$F^d = k^d \times^d \frac{q_1 \times^d q_2^d}{r^{d2d}} \tag{15}$$

as well as the corresponding laws for charged antiparticles-antiparticles in our world

$$F = k \times \frac{q_1^d \times q_2^d}{r^2}. \tag{916)}$$

and in the isodual world

$$F^d = k^d \times^d \frac{q_1^d \times^d q_2^d}{r^{d2d}}. \tag{17}$$

As one can verify, the above laws correctly represent all known classical experimental behavior of antiparticles, provided the isodual mathematics [18] is used properly.

At the operator level, charge conjugation (1) and isoduality (2) are equivalent by conception and construction. Consequently, the isodual theory of antimatter verifies all known experimental data on antiparticle including that at the quantum level (see Section 2.3.7, pages 122-125 of monograph [18] for a details).

7. New Predictions of the Isodual theory of Antimatter

A fundamental prediction of the isodual theory of antimatter is that *light emitted by antimatter (antimatter light) is physically different than light emitted by matter (matter light) in a number of experimentally verifiable ways* [14]. This important prediction was presented by Santilli at the *International Conference on Antimatter* held in Sepino, Italy, on June 1996.

Recall that light has no charge. But the isodual theory has been constructed to provide a differentiation between neutral matter and antimatter. Therefore, the physical distinction between matter and antimatter light is a direct and unavoidable consequence of the physical distinction between neutral matter and antimatter, as reflected in the distinction between their representation.

Besides, the physical differentiations at advanced level we cannot review here, the most visible difference is *the prediction by the isodual theory of antimatter that antimatter light is repelled by a matter gravitational field* (see Figure 9).

The simplest way to illustrate this prediction is that at the primitive Newtonian level since all subsequent levels of study are merely consequential. Let us recall that Santilli has formulated Newtonian gravitation in a truly "universal" way via the "identical" representation of the historical equation in terms of "energy," rather than mass [10,20]

$$F = g\frac{m_1 m_2}{r^2} \equiv G\frac{E_1 E_2}{r^2}, \quad G = \frac{g}{c^4}. \tag{18}$$

In fact, the origin of gravitation is indeed the energy, as established rigorously by the Riemannian geometry, while the mass is a mere expression of our ignorance on the physical origin and nature of inertia.

Santilli's reformulation (19) of Newton's gravitation has permitted the first known representation of the attraction of matter light by a matter gravitational field (the so-called "bending of light"), via the mere interpretation of E_1 in Eq. (18) as representing the energy of a body such as Earth, and $E_2 = h\nu$ as representing the energy of light.

In this way, Newtonian gravitation becomes indeed "universal", because it includes the attraction of light, but also the historical consequence is that *the bending of matter light by a matter body is primarily a "Newtonian" effect without any need for curvature* [18].

By following the above representation of the bending of light, Santilli then proceeded to the prediction of gravitational repulsion of antimatter light by a matter body, and vice versa, via the corresponding simple extension of Eq. (18)

$$F = G\frac{E_1 E_2^d}{r^2}, \tag{19a}$$

$$F^d = G^d \times^d \frac{E_1^d \times^d E_2}{r^{d2d}}. \tag{19b}$$

Recall that the original and main motivation for the studies reported herein was Santilli's 1960s desire to study whether a far away star or galaxy is made up of matter or of antimatter (Figure 10). The prediction of the repulsion of antimatter light by matter gravitational fields provides evident hope for the achievement of this decades old objective.

Figure 13: Santilli also indicates that it is not possible to establish at this writing whether the celebrated Crab nebula is composed of the inert matter or antimatter, since the latter may prevent the passage of matter light originating from its background.

However, the actual capability to ascertain whether a far away star or galaxy is made up of matter or of antimatter can solely be resolved via new, basic resolutory experiments, such as Santilli's gravitation experiment reviewed in the next section because no theoretical resolution is possible on grounds of our current theoretical knowledge.

To understand the impossibility of a theoretical resolution of the attraction or repulsion of antimatter light by a matter field, we have to recall the additional prediction of the isodual theory of antimatter according to which, contrary to popular beliefs, *Feynman's diagrams are inapplicable to matter-antimatter annihilation* (rather than being "violated" because not originally conceived for that) [20].

In turn, to understand the latter unavoidable prediction, let us recall the new fundamental symmetry of nature discovered by Santilli and called *IsoSelfDuality,* (ISD) [18], namely, the invariance of classical or operator formulations under isoduality, Eq. (2). Mathematically, isoselfduality is verified by the imaginary unit,

$$i \to i^d \equiv i, \tag{20}$$

and, physically, isoselfduality is verified by Dirac's equation when reformulated according to Santilli's Eq. (12).

The reader should keep in mind that, in the event the universe is made up of 50% matter and 50% antimatter, Santilli's isoselfduality may well emerge as one of the most fundamental symmetries of nature.

Consider now the well known electrons and positrons annihilation,

$$e^- + e^+ \ \to 2\gamma. \tag{21}$$

The insufficiency of Feynman's diagrams here considered is caused by the central assumption of mediating all interactions via the exchange of a particle. This basic assumption is indeed physically consistent for the originally represented Coulomb interactions. However, the same assumption violates Santilli's isoselfduality when applied to annihilation (22), trivially, because

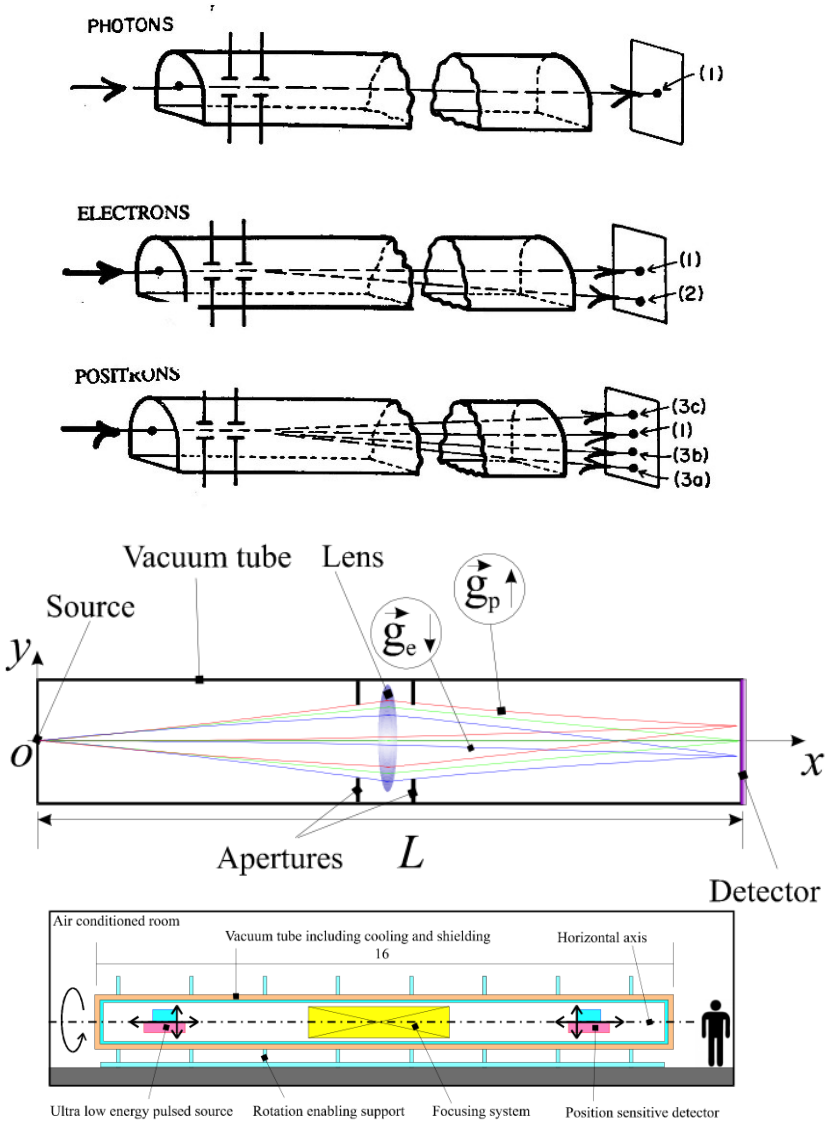

Figure 14: The original three renderings (from the top) by Santilli's of the 1994 proposal [8] to test the gravity of positrons in horizontal flight on a ten meter long super-vacuum and super-cooled tube. The top view shows the point of no gravity on a scintillator at the end of the tube obtained via the use of collimated photons (for which the deviation due to gravity after a flint of 10 m is evidently ignorable). The middle view shows the gravitational attraction on a collimated beam of electrons that, when having a very low energy of the order of meV, is of the order of 1 cm following a flight of 10 m, thus being visible to the naked eye. The lower view indicates the possible alternatives for a collimated beam of positrons. Santilli's isodual theory of antimatter predicts gravitational repulsion (antigravity) at all its levels for positrons in a horizontal flight on Earth that, for very low energy of the order of meV, is of the order of 1 cm following a 10 m flight, thus being visible to the naked eye on the scintillator at the end of the tube. For that reason, Santilli's proposed experiment has been stated to be "resolutory" by experimentalists in the field [12,21]. The lower two renderings are from the technical realization of the test [25] by the R. M. Santilli Foundation on the technical realization of proposal [8] (forth view from the top) and and illustration of its size compared to a person (bottom view).

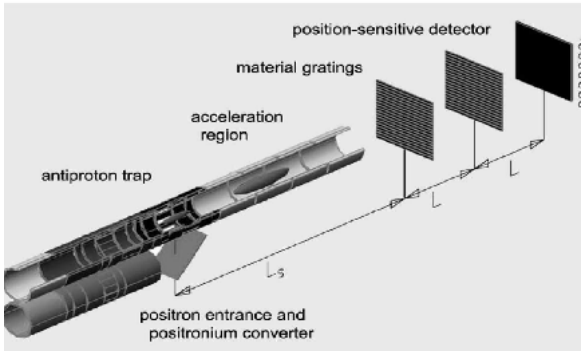

Figure 15: An illustration from ref. [29] of the proposed AEGIS experiment at the CERN Antiproton Decelerator intended to test of the Weak Equivalence Principle and CPT symmetry. The gravity of antihydrogen atoms is tested their via the fall in our gravitational field. The above picture illustrates the parallel Penn ing-Malmberg traps. The depicted gratings have the dimension $L_s = 50$ cm, $L = 30$ cm, transverse dimensions 20 cm^2.

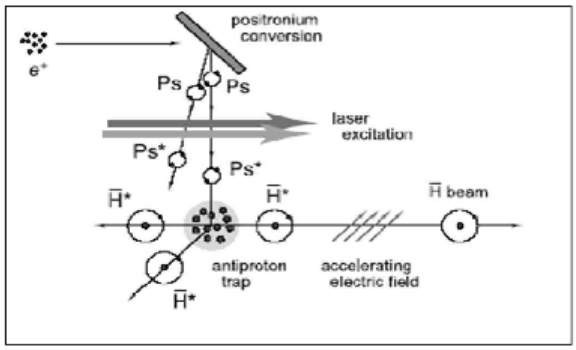

Figure 16: A second illustration from the proposed test [29]. The antiproton beam fall is measured with a Moir deflectometer equipped with a position sensitive microstrip detector. Antiprotons originate from the CERN Antiproton Decelerator delivering about 3×10^7 particles every 100 s at 6 MeV that are decelerated to about 100 mK with about 50 m/s. Antihydrogen atoms are produced via the injection of 20 ns positron bunch with the assistance of 10 ns laser pulses. These data should be compared with those of Santilli's gravity experiment [8,25].

the "particle-antiparticle" annihilation is mediated by the exchange of a "particle" or, alternatively, of an "antiparticle." (see Figure 11).

Feynman's diagrams for particle-antiparticle annihilations have additionally for the r.h.s. of Eq. (1) because based on the experimentally unverified belief that there exist only one type of photons in nature, while the representation of neutral antimatter mandates the existence of two photons, according to which Eq. (21) should be written under full verification of isoselfduality

$$e^- + e^+ \ \rightarrow \gamma + \gamma^d. \tag{22}$$

The moment Feynman's diagrams were understood as being inapplicable to matter-antimatter

annihilation, thus for matter-antimatter interactions at large, Santilli was unable to reach any theoretical resolution on the attraction or repulsion of antimatter light by a matter gravitational field, thus mandating basically new tests.

8. Proposed Experiments on the Gravity of Antimatter

The gravitational repulsion (antigravity) between matter and antimatter was suspected immediately following the discovery of antimatter, although without any possible theoretical treatment due to the absence of a theory capable of representing the gravitational field of neutral antimatter (for brevity, see Ref. [18] for historical accounts).

The above insufficiency has been resolved by Santilli's works on antimatter [1-25]. In fact, *the isodual theory of antimatter predicts in a consistent and systematic way at all levels of study, from Newtonian mechanics to the Riemannian geometry, that matter and antimatter must experience gravitational repulsion* [8,18].

For a comprehensive study of the prediction of antigravity at all levels, we suggest the reader to study Chapter 4, pages 252-285 of monograph [18] (that also contains the identification of the far reaching implications of antigravity we cannot review for brevity, such as the "causal space-time machine,.").

We can conceptually say that antigravity between matter and antimatter is a necessary consequence of the very existence of a "classical" gravitational representation of "neutral" antimatter because, since the charge is null, such a representation requires the sign conjugation of all physical quantities, thus including the sign of the gravitational force and, therefore,. of the curvature tensor.

On quantitative grounds, we refer to monograph [18] for the gravitational representation of antigravity via the Riemannian geometry for matter and its isodual for antimatter. For the limited scope of this paper, it is sufficient to recall the most primitive prediction of antigravity, that in Newtonian mechanics, since all subsequent levels of study are evidently compatible to such a primitive one.

In fact, the *Newton-Santilli isodual equation* clearly predict gravitational repulsion between matter and antimatter both in our space as well as in the isodual space, according to the respective the laws,

$$F = g\frac{m_1 m_2^d}{r^2} < 0, \tag{23a}$$

$$F^d = g^d \times^d \frac{m_1^d \times^d m_2}{r^{d2d}} > 0, \tag{23b}$$

where in our world we have a repulsion because the gravitational force is negative, $F < 0$, and referred to a positive unit of force, while in the isodual world we equally have a repulsion because the gravitational force is positive, $F^d > 0$, but it is referred to a negative unit of force.

The first experimental test of the gravity of positrons was formulated by W. E. Fairbanks and E. C. Witteborn at SLAC in 1967 [26] via the use of low energy positrons in vertical upward flight in a vacuum and cooled tube. Regrettably, the experiment could not be completed due to the unavailability at that time of detectors with the extreme sensitivity needed for meaningful measurements.

Numerous additional experiments have been proposed to test the gravity of positrons in vertical flights, either upwards or downwards, such as the tests of Refs. [27,28] and others. However, the gravitational force on particles is notoriously very weak, as a consequence of that the measurements with the most sophistical neutron interferometric or other techniques are expected to remain ambiguous.

Thus, the class of proposed experiments to measure the gravity of positrons in vertical flight cannot possibly be as resolutory as necessary for the safety of our planet and, consequently, this class of experiments will not be considered herein.

In view of the indicated limitations of testing the gravity of positrons in a vertical flight, Santilli proposed in paper [8] of 1994 *the experimental verification or dismissal of the predicted gravitational repulsion between matter and antimatter via measurements of the comparative behavior of very low energy electrons and positrons moving in a* 10 *m long horizontal supercooled and super-vacuum tube* (Figure 14).

It is evident that Santilli's gravity experiment via positrons in horizontal flight is strikingly better than preceding proposed tests [26-28] via positrons in a vertical flight. While the measurements in the latter tests are expected to remain ambiguous due to the smallness of the effect, *in Santilli's experiment [8], for very low energy electrons and positrons of the order of meV in horizontal flight in a* 10 *m long supercooled and super-vacuum tube, the displacement due to gravity detected on a scintillator at the end of the tube is of the order of* 1 *cm, thus being visible to the naked eye.*

The preference of Santilli's test [8] over the tests of Refs. [26-28] is confirmed by a number of experimentalists in the field. For instance, during the *International Conference on Antimatter* held in Sepino, Italy, in June 1996, the experimentalist A. P. Mills declared Santilli's gravity experiment as being "resolutory" [12] and, therefore, is preferable over the others not equally resolutory experiments.

Similarly, during the *Third International Conference on the Lie-Admissible Treatment of Irreversible Processes,* held at the University of Kathmandu, Nepal, in January 2011, the experimentalist V. de Haan [21] confirmed Mills analysis and also declared Santilli's gravity experiment as being "resolutory."

Besides the above proposed experiments via the use of positrons, the only remaining proposed experiments are those based on anti-hydrogen atoms produced at CERN. Among the latter tests, we have pointed out the test proposed in Ref. [29] by the AEGIS Collaboration outlined in Figures 15 and 16, and the test proposed in Ref. [30] by the ALPHA Collaboration outlines in Figures 17 and 18.

By assuming a technical knowledge of these proposed experiments (that can only be obtained by studying the original papers [29,30]), we here limit ourselves to the following comments.

To begin, the tests of Refs. [29,30] have the same ambiguities in measurements as those of the tests with vertically moving positrons [26-28], since the former too deal with extremely small effects requiring extremely sensitive detectors under these conditions. The "experimental results" are inevitably prone to the approximations and/or manipulations that occurred in similar tests.

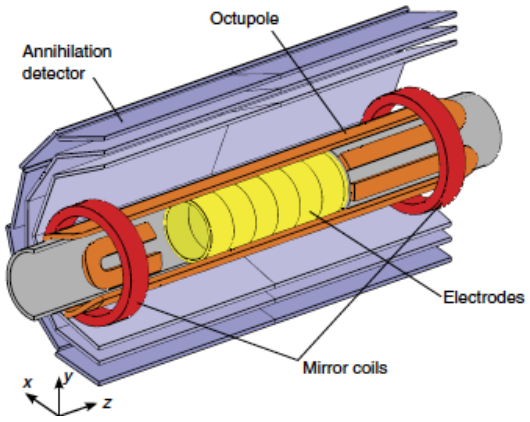

Figure 17: An illustration from Ref. [30] providing a cut-away diagram of the antihydrogen production and trapping of the ALPHA Collaboration, showing the relative positions of the cryogenically cooled Penning- Malmberg trap electrodes and other features.

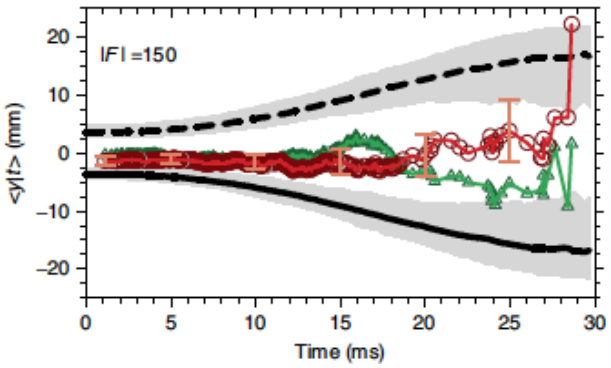

Figure 18: A second illustration from Ref. [30] on the proposed test of the gravity of antihydrogen atoms via their fall downward when released from the ALPHA antihydrogen trap of the preceding figures. The illustration depicts one of several simulated reverse cumulative average analysis. compared to the event data to the reverse cumulative average. The green-triangle line is the reverse cumulative average of the x annihilation positions of the event data, and is included as a comparison. The black solid line is the represents 900, 000 simulated antihydrogen atoms. The black dashed line mirrors the black-solid line, and is equivalent to a simulation study of antigravity. The grey bands separate the 90% confidence region. Again, the complexity of the apparatus and the high sensitivity of the detectors should be compared to corresponding data of Santilli's gravity test [8.25].

Besides that, the main problematic aspect of tests of Refs. [29,30] is the one identified by Santilli [18] according to which, despite a popular beliefs at CERN and elsewhere, *the "antiprotons" produced at CERN are not necessarily antiparticles, unless verified as such via annihilation processes, because at least in part, they can be anomalous protonic states created by the embedding of a singlet electron pairs inside ordinary protons.* These states are called by Santilli the *pseudoproton* and denoted with the symbol $\hat{p}^-$.

Consequently, no gravity experiment based on "antihydrogen atoms" produced at CERN can be considered as being resolutory under such a serious ambiguity. It is important for this paper to provide an outline of the above ambiguity.

Besides the study of antimatter, Santilli has dedicated decades of his research life also to the synthesis of neutrons inside a star according to Rutherford's historical conception that neutrons are synthesized by the "compression" of hydrogen atoms in the core of a star, nowadays represented with reaction

$$p^+ + e^- \ \rightarrow \ n + \nu. \tag{24}$$

It is well known that the energies needed to achieve the synthesis of the neutron are fully available at CERN.

In particular, Santilli has shown that neutrons can also be synthesized in laboratory from a hydrogen gas traverses by a DC arc, thus takin place at energies much smaller than those available at CERN (to avoid a prohibitive length, we refer the reader to review [31] and monograph [24] for a comprehensive perspective).

The experimental information important for the test of the gravity of antimatter obtained by Santilli is that Rutherford's compression is also achievable for an electron pair in singlet coupling (that occurs for valence electron pairs) resulting in the creation of pseudoproton according to the reaction

$$p^+ + (e^-_\uparrow + e^-_\downarrow) \ \rightarrow \ \hat{p}^- \tag{25}$$

where $\hat{p}^-$ is predicted to have a mean life essentially similar (if not longer) than that of the neutron due to the similarities of the two syntheses.

As a matter of fact, Santilli has shown that synthesis (25) is more probable than synthesis (24) for various reasons, such as: synthesis (25) does not require the emission of a neutrino for the conservation of the total angular momentum as necessary for synthesis (24); Rutherford's compression of a single electron pair inside the proton is statistically more probable than the compression of the electron due to spin zero of the electron pairs (thus requiring no special proton-pair coupling), compared to the need for a singlet proton-electron coupling for synthesis (24); and other reasons.

It should be stressed that quantum mechanics does not allow a quantitative representation of synthesis (24) because the rest energy of the neutron is bigger than the sum of the rest energies of the proton and the electron, thus requiring a "positive binding energy" which is anathema for quantum mechanics, since in this case the Schrödinger equation no longer admits physically meaningful solutions [10].

Thanks to its non-unitary invariant character, *hadronic mechanics has resolved these insufficiencies by achieving, for the first time to our knowledge, a numerically exact representation*

of "all" characteristics of the neutrons in synthesis (24) at both non-relativistic and relativistic levels [10,24,31].

In particular, the use of Santilli's non-unitary invariant methods that have permitted a representation of synthesis (24), when applied to synthesis (25), show that *the rest energy of the pseudoproton $\hat{p}^-$ can be close to that of the antiproton $\bar{p}^-$,* although expecting of exact numerical values are premature at this time since the sole experimentations to date have been conducted is by Santilli.

Therefore, Santilli stresses that *the distinction between the antiproton $\bar{p}^-$ and the pseudoproton $\hat{p}^-$ cannot be solely based on their charge and rest energy, their only resolutory distinction being that based on annihilation processes.* Needless to say, the antimatter nature of the "antiprotons" claimed at CERN cannot be denied. The point is that the antimatter character has to be proved beyond doubt prior to any true scientific claim.

Now, as it is well known, the production of "antiprotons" at CERN is based on hitting a target with the 26 GeV proton beam produced by the old Proton Synchrotron (PS). It is then evident to all that, *during the collision of protons with matter target, Santilli synthesis (25) is indeed possible, resulting in the synthesis of the pseudoproton $\tilde{p}^-$.* In fact, at the time of the impact, protons collide first with electrons clouds in general, including precisely the valence electron pairs of synthesis (25).

Once the pseudoproton has been synthesized, its capability to capture a positron in the anti-hydrogen trap is established by quantum mechanical laws, resulting in a neutral state $(\hat{p}^-, e^+)$ which is similar to, but not necessarily, the anti-hydrogen atom $(\bar{p}^-, e^+)$.

In summary, the mathematical, theoretical and experimental studies here referred to [10,18,24,31] illustrate Santilli's main objection against the test of the gravity of antimatter via "antihydrogen atoms" currently produced at CERN because of the lack of clear proof that they are indeed antimatter and the absence of experiments for the resolution of the ambiguities because, being necessarily beyond quantum mechanics [10,31], the said experiments are notoriously not even plausible at CERN under current control.

In conclusion, both classes of tests of the gravity of antimatter, those based on vertical motion of positrons and those based on the "anti-hydrogen atoms" produced at CERN, are not resolutory on grounds of our current knowledge. Consequently, Santilli's gravity test [8,12,21,25] is and remains the best measurement of the gravity of antimatter since it is the only experiment whose results would be visible to the naked eye.

The R, M, Santilli Foundation is promoting the funding and conduction of Santilli gravity experiment along the technical presentation of Ref. [25] and is seeking interested experimentalists as well as a laboratory equipped with super-cooled and super-vacuum facilities. Interested colleagues are welcome to contact the corresponding author.

9. Concluding Remarks

The most important consequence of Santilli's research in antimatter is that: *antimatter asteroids do indeed exist, have indeed caused devastation to our Earth in the past, they indeed can be detected by qualified laboratories and, as such, they constitute a threat to mankind due*

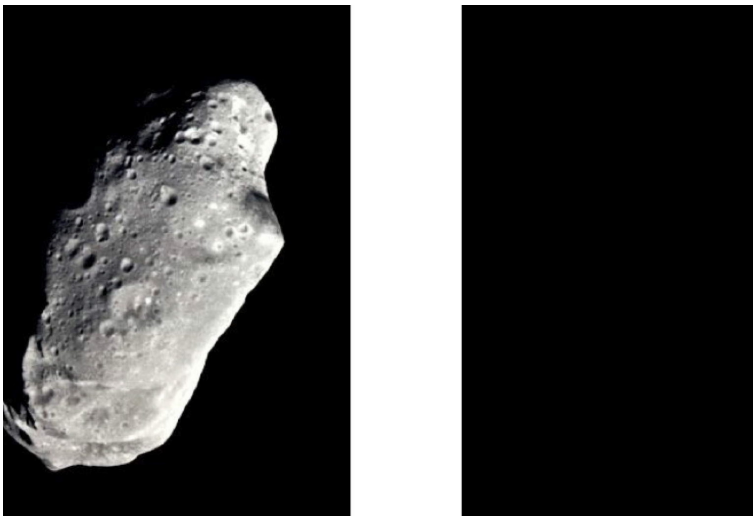

Figure 19: The main conclusion of this paper is that, following decades of research in antimatter, Santilli indicates our inability to ascertain in a final form on grounds of our current knowledge whether matter light emitted by our Sun is reflected by an antimatter asteroids, thus being visible to us (left view), or matter light from our Sun is absorbed by an antimatter asteroid or having other processes not allowing the detection via current telescopes [20] (right view). Therefore, antimatter asteroids do constitute indeed a serious threat to our planet, thus requiring collegial studies, with particular reference to new basic experiments.

to our inability at the current state of our knowledge to ascertain with certainty that they are detectable via the reflection of matter light from our Sun or matter radiations originating from Earth, such as laser beams.

The only possible way to ascertain scientifically the detection from Earth of antimatter asteroids is to conduct basically new experiments on antimatter with particular reference to the conduction of Santilli's gravity experiment for positrons in horizontal flight on Earth in a super-cooled and super-vacuum tube [8,12,21,25], plus additional particle physics experiments we could not review in this paper for brevity.

It then follows that possible obstructions at major physics laboratories against resolutory measurements of the gravity of positrons seems to be that it is expected to be beyond Einstein theories, besides sealing in history a clear scientific obscurantism and causing a misuse of trillions of dollars in public funds, and hence could constitute a threat to mankind.

Acknowledgments

Particular thanks are due to R. M. Santilli because, without his authorization for freely using the content of his publications, without his deep criticisms of our presentation, and without his continuous consultations, this paper would not have been possible. Additional thanks are due to all participants of the 2011 *San Marino Workshop on the Astrophysics of matter and Antimatter* and of the *2011 Nepal Third Conference on Lie-admissible Treatments*

of Irreversible Processes where the issues reviewed in this paper were subjected to a collegial inspection. Additional thanks are due to C. Corda because of an in depth critical review of an earlier version of the manuscript, as well as the indication of very important scientific references suggesting the existence of antimatter in the universe at a large scale level. Additional thanks are due to J. V. Kadeisvili for an in depth critical review of the manuscript and innumerable suggestions. Additional tans are due to P. Flaming for the linguistic control of the manuscript.

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