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MINIMUM CONTRADICTIONS THEORY OF EVERYTHING

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Abstract

The purpose of this lecture is to show that no theory can be regarded as a complete one and this leads to indeterminism. This can be proved by means of a theorem which can show that the logical communication system, through which all theories are stated, is contradictory. According to the present point of view any uncertainty-incompleteness derives from the logical communication system itself and not from empirical principles which cannot be proven as valid. On this basis can be proved that space time is matter itself, Schrödinger's relativistic equation is valid and that the Ψ wave function is a complex space time function described in a Hypothetical Measuring Field (HMF). Thus, a Space Time Quantum Mechanics-Quantum Gravity can be stated and this is a Minimum Contradictions Theory of Everything. According to the present point of view, new phenomena and technologies, related to free energy and reactionless propulsion, can be explained and this reinforces the credibility of this theory.

1. LOGIC ANALYSIS

If we denote by Λ a logic consisting of the Classical Logic denoted as P_I and the Sufficient Reason Principle regarded as a Complete Proof Principle denoted as P_{II}, we will have [1, 2]:

$$\Lambda \equiv P_I \cdot P_{II}$$

where P_{II} is defined as:

Complete Proof Principle **-** P_{II}: *"No statement is valid if there is not a complete logical proof of the statement, through valid statements different from it.*

"On this basis Theorem I can be derived i.e.:

Theorem I: *"Any system that includes logic* Λ *and a statement that is not theorem of logic* Λ *leads to contradiction"* [1, 2].

We name '0' the state before our communication and '1', '2', '3' the sequent states of this communication. '0' corresponds to the non-existence of any communication symbol while '1' to some symbol existence. From the non-existence of something cannot derive logically its existence. Working in the same way we have that a "posterior" does not derive logically from its "anterior". Therefore *the Anterior – Posterior Axiom is not theorem of* Λ. Applying Theorem I we obtain Statement I [1, 2]:

Statement I: *"Any system that includes logic* Λ *and the anterior-posterior axiom leads to contradiction."*

where the anterior – posterior axiom is stated as follows:

Anterior – Posterior Axiom (**A-P Axiom)**: *"There is anterior-posterior everywhere in communication."*

Kantian 3rd antinomy has similarities with statement I [3, 4]; Kant's 3rd antinomy proof requires that a transcendental causality can exist a priori in contrast to theorem I.

Gödel and Rosser theorems [5, 6] could lead to same results as theorem I [1, 2]; however as H. Putnam [7] in his critic to R. Penrose's *Shadows of the Mind* [8] has noticed, they are restricted to computational well defined processes, not to logical communication in general [9].

Based on Theorem I and Statement I, we conclude that a system including the principles of logical communication leads to contradiction. This leads to the silence, and therefore when communicating logically it means that we decide to break the silence by avoiding contradictions on purpose [1, 2].

Despite statement I, we do communicate in a way we consider logical avoiding contradictions on purpose. Since contradictions are never vanished, we try to understand things through minimum possible contradictions. On this basis we can state [1, 2]:

***Statement II - The Claim for Minimum Contradictions**: "What includes the minimum possible contradictions is accepted as valid."*

According to this claim we obtain a logical and an illogical dimension. In fact, through this claim we try to approach logic (minimum possible contradictions), but at the same time we expect something illogical since the contradictions cannot be vanished.

All axioms mentioned, the claim for minimum contradictions included, constitute the principles of the active logical language; when we speak we persist in logic despite of the existing contradictions.

Every theory includes at least the axioms mentioned; therefore no theory can be complete since it includes contradictions. On this basis, a *minimum contradictions physics* can derive where *the physical laws are the principles of the active logical language;* this physics is a stochastic matter-space-time QM implying a quantum gravity [1].

COMMON ROOTS OF RELATIVITY THEORY AND QM

2.1 General

According to theorem I, further axioms beyond the ones of logical communication must be avoided since they can cause further contradictions. The systems of axioms we use in physics include the logical communication and, therefore, their contradictions are minimized when they are reduced to the logical communication itself.

At first sight, for a minimum contradictions physics we can make the following statement [1, 2]:

Statement III: *In a minimum contradictions physics everything is described in anterior–posterior terms.*

If there is space-time then there is anterior posterior so that space-time can be measured and denoted through the communication (language). Inversely, if there is anterior-posterior in communication then there is space-time. In fact, in order to write something we need space; also we need time since we cannot write in a simultaneous way. Thus, because of Statement III we can state the following [1, 2]:

Statement IV: *In a minimum contradictions physics everything is described in space-time terms.*

Since everywhere there is space-time and not something else, space-time can be regarded as the matter itself. A matter system, in general, has differences within its various areas. This means that a matter system, in general, is characterized by different rates of anterior - posterior (space-time) within its various points. This means that time can be regarded as a 4^{th} dimension which is compatible to Lorentz' transformations and in extension to a relativistic theory [1].

2.2. Hypothetical Measuring Field

Basic tool of this work is the Hypothetical Measuring Field (HMF) [1].

As Hypothetical Measuring Field (HMF) is defined a hypothetical field, which consists of a Euclidean reference space-time, in which at each *point A_0 the real characteristics of the corresponding, through the transformations of deformity, point A of the real field exist.*

In the HMF, it is defined as a relative space time magnitude sr, the ratio of the real infinitesimal space time magnitude ds to the corresponding, through the deformity transformations, infinitesimal magnitude ds_0 of the reference space- time, i.e. $sr = ds / ds_0$. This can apply to relative time $tr = dt / dt_0$, to relative length in a direction **n** $lr_n = dl_n / dl_{n0}$ and to a relative volume $vr = dv / dv_0$.

2.3 Equivalence of Energy and Time

In a space-time description we don't know a priori what energy is. We define energy dE of an infinitesimal space-time element its 'ability to exist'. We may notice that an infinitesimal space-time element with energy dE exists on condition that some corresponding 'anterior-posterior' exist too. With respect to the HMF a space-time element is observed during a time dt that is different from the time dt_0 of the corresponding reference space-time element. Various space-time elements in the HMF have different dt for the same dt_0. Thus, dt measures the duration *i.e.* the ability of a space-time element to exist; this ability, by definition is energy; when $dt = dt_0$, this ability is dE_0. Thus, we can write [1]:

$$dE \sim dt \quad \text{and} \quad dE / dE_0 = dt / dt_0 \tag{1}$$

which is a relativistic relation. *Relations (1) show the equivalence of a space-time element energy to the time flow rate within this element.* When dt is a constant period of time in the HMF, then Eq. (1) can be written in the form:

$$dE / dE_0 = dt / dt_0 = (f / v)/(f / v_0) = v_0 / v \tag{2}$$

where v is the frequency of a periodic phenomenon of comparison and f an arbitrarily constant factor through which we can change the scale of time.

If $v = 1$, v_0 must be different in various points $(\mathbf{r},t)$ of the HMF. If this is the case Eq. (2) can be written in the form:

$$dE / dE_0 = v_0(\mathbf{r},t) \tag{3}$$

Thus, for the same equation we have the following versions [1]:

$$dE / dE_0 = dt / dt_0 \text{ observation (relativity theory)} \tag{4}$$

$$dE / dE_0 = v_0(\mathbf{r},t) \text{ action (quantum mechanics)} \tag{5}$$

Thus, at a first sight relativity theory and QM have common roots [1]. On this basis, we can reach the basic De Broglie's principle for a particle energy; in fact for $E_0 = h$ we have (arithmetically) that:

$$E = hv \tag{6}$$

3. STOHASTIC SPACE TIME

3.1 General

At second sight, taking into account the above mentioned and applying the claim of the minimum contradictions, we conclude that matter-space-time has logical and contradictory behavior at the same time; this can be valid when space time exists and not exists at the same time (illogical behavior) while it implies the existence of logic. This can be approached by the aid of a hypothetical measuring field HMF. If this is the case we can say that space-time has a probability to exist and to correspond to an infinitesimal area around a point $(\mathbf{r},t)$ of the HMF. Thus we can state the following:

Statement V: *Minimum Contradictions Physics can be described by Stochastic Space-Time.*

However physics describes any matter system i.e. matter, anti-matter, mass and charge. On this basis statement V has sense if there are various kinds of space time corresponding to the various forms of matter. Thus we can use signs +1, -1, +i, -i for various states. For the purposes of stochastic space-time description, the following definition is useful [1]:

In a HMF, we define as mean relative space time magnitude $\overline{sr}$ the ratio of the mean real infinitesimal space time magnitude $\overline{ds}$ to the corresponding infinitesimal magnitude ds_0 of the reference space- time i.e. $\overline{sr} = \overline{ds} / ds_0$. Thus, for mean relative time we obtain: $\overline{tr} = \overline{dt} / dt_0$. The relative space-time magnitudes mentioned above, are denoted by SR, TR,.. when they refer to mean values of a particle space-time field.

4. EQUATIONS OF MINIMUM CONTRADICTIONS PHYSICS

The minimum contradictions equations are here mentioned in order that a general idea on the results of the minimum contradictions physics might be introduced.

The electromagnetic (*em*) space-time, is a space-time whose all magnitudes are considered imaginary and behave exactly like the gravitational (*g*) space-time. Electromagnetic (*em*) space-time is described by means of space-time wave functions such that [1]:

$$\Psi_{em}(\mathbf{r}_{em}, t_{em}) = \Psi_{em}^{g}(\mathbf{r}, t) \qquad (7)$$

where Eq. (7) has meaning due to the coexistence of (*g*) and (*em*) space-time under a scale which appears to be equal to the fine structure constant α [1].

According to the spirit of this paper ***there is not potential acting at a distance*** **since space time is matter itself.** By the aid of Fourier analysis and *without any other physical principles* the following can be obtained [1]:

a. *Relative Space-Time Operators (Relative Time, Volume, Length in a direction* $\mathbf{n}$)

$$\hat{TR} = \frac{i}{2\pi}\frac{\partial}{\partial t}, \quad \hat{VR} = -2\pi i \frac{1}{\partial / \partial t}, \quad \hat{LR}_n = \left(1 - c^2 \frac{\partial^2 / \partial x_n^2}{\partial^2 / \partial t^2}\right)^{1/2} \frac{h}{m_0 c^2} \qquad (8)$$

b. *Particle Schrödinger's Relativistic Equation (Klein–Gordon) for (g) and for (em) Space-Time:*

$$\frac{\partial^2\Psi}{\partial t^2}(\mathbf{r},t) - c^2\nabla^2\Psi(\mathbf{r},t) = -(m_0 c/\hbar)^2\,\Psi(\mathbf{r},t) \tag{9}$$

c. *Many body Schrödinger's Relativistic Equations for (g) and for (em) space time:*

$$\frac{\partial}{\partial x_j}\frac{\Box\Psi_g(\mathbf{r},t)}{\Psi_g(\mathbf{r},t)} = 0 \qquad (j = 1,2,3,4) \tag{10}$$

$$\frac{\partial}{\partial x_j}\frac{\Box\Psi^g_{em}(\mathbf{r},t)}{\Psi^g_{em}(\mathbf{r},t)} = 0 \qquad (j = 1,2,3,4) \tag{11}$$

d. *Energy Conservation:*

$$\partial_t\left(\frac{\partial_t\Psi_g(\mathbf{r},t)}{\Psi_g(\mathbf{r},t)} + \frac{\partial_t\Psi^g_{em}(\mathbf{r},t)}{\Psi^g_{em}(\mathbf{r},t)}\right) = 0 \tag{12}$$

e. *Momentum Conservation:*

$$\partial_t\left(\frac{\nabla\Psi_g(\mathbf{r},t)}{\Psi_g(\mathbf{r},t)} + \alpha\frac{\nabla\Psi^g_{em}(\mathbf{r},t)}{\Psi^g_{em}(\mathbf{r},t)}\right) = 0 \tag{13}$$

f. *Geometry of (g) or (em) Space-Time i.e. Mean Relative Time and Mean Relative Length in a Direction* $\mathbf{n}$ *at a Point* $(\mathbf{r},t)$:

$$\overline{tr}(\mathbf{r},t)=\frac{ic}{2h}\frac{\partial_t\Psi}{(\Psi\Box\Psi)^{1/2}}\left(\Psi^*\partial_t\Psi-\Psi\partial_t\Psi^*\right) \tag{14}$$

$$\overline{lrn}(\mathbf{r},t)=-\frac{ih}{2}\frac{\Psi}{\Box\Psi}\left(1-c^2\frac{\partial^2\Psi/\partial x_n{}^2}{\partial^2\Psi/\partial t^2}\right)^{1/2}\left(\Psi^*\partial_t\Psi-\Psi\partial_t\Psi^*\right) \tag{15}$$

The above equations cannot describe a unified field on the basis of common boundary conditions; they need to be self determined [1]. This implies the existence of incompleteness-volition (Free Will) and Indeterminism. These equations imply a statistical interpretation and a distribution of matter space-time according to Schrödinger Relativistic Equation probability density

$$P\left(\mathbf{r},t\right)=(i\hbar/2m_0c^2)\left(\Psi^*\partial_t\Psi-\Psi\partial_t\Psi^*\right) \tag{16}$$

In this case, Ψ function locally is described by an equivalent local space-time particle field wave function Ψi, where this field is regarded as extended to infinity. This can occur when equations (10 to 13) have constant values of m_{0g} or m_{0em} only in the vicinity of various $(\mathbf{r},t)$ of the (HMF).

5. QUANTUM GRAVITY

The gravitational acceleration $\mathbf{g}(\mathbf{r},t)$ represents the force that must be applied to a unit of mass at every point $(\mathbf{r},t)$ in order that energy will be distributed according to the probability density function $P(\mathbf{r},t)$. It can be proven that [1]:

$$\mathbf{g}(\mathbf{r},t)=\frac{c^2}{P(\mathbf{r},t)}\nabla P(\mathbf{r},t)=\frac{c^2}{\overline{tr}(\mathbf{r},t)}\nabla\overline{tr}(\mathbf{r},t) \tag{17}$$

From equation (17) for a particle field, because of equation (16) it holds:

$$\mathbf{g}(\mathbf{r},t)=\frac{c^2\nabla\left(\Psi^*\partial_t\Psi-\Psi\partial_t\Psi^*\right)}{\left(\Psi^*\partial_t\Psi-\Psi\partial_t\Psi^*\right)} \tag{18}$$

Equations (17, 18) describe a unified relationship which is valid everywhere. Under certain simplifications it can be proven that equation (17) is compatible to Newton and to Coulomb law as well as to the relativistic formula for gravity [1]. This implies a convergence of a Deterministic with an Indeterministic point of view.

6. POSSIBILITY TO VERIFICATION OF (G) AND (EM) INTERACTION

6.1 General

According to the present point of view new phenomena and technologies, related to free energy and reactionless propulsion, can be explained and this reinforces its credibility. There have been made many devices to produce energy or to create propulsion through the vacuum [15-24]. In the present paper the effects - devices we use to possibly verify the (g) and (em) interaction are the following:

6.2 Santilli's Etherino

The *neutron* was conceived by H. Rutherford as a "compressed hydrogen atom" in the core of a star. Don Borghi claimed the laboratory synthesis of the neutron from protons and electrons; this experiment remained unverified for decades due to the lack of theoretical understanding of the results. R.M. Santilli has verified and theoretically explained this experiment by the aid of a particle process which he called as *etherino* (from ether) on the basis of Hadronic Mechanics [10, 11]. The *Etherino Process* has been found to be compatible to 'Minimum Contradictions Physics' which implies that space time is matter itself and consists of gravitational (g) and electromagnetic (em) space time which are interconnected and communicate through photons (particles with zero rest mass). A basic consequence of this, is the *Statement: "During the approach of an electron to a proton there is absorption of gravitational space time energy"* [1, 12].

6.3 U.S. Patent No. 8,952,773

The device of the U.S. Patent No. 8,952,773 [13] is illustrated in fig. 1; this device consists of a superconducting nozzle connected at its narrow end with a permanent magnet, which can create propulsion without any external energy source but only in the direction South to North. Experimental verification was carried out both at the Technological Institute of Thessaly (now the University of Thessaly) in the Laboratory of Renewable Energy and in the Solid State Physics Laboratory of the National Kapodistrian University of Athens.

6.4 PCT/GR2020/000040-Priority GR 20190100373 patent application

The device of the PCT/GR2020/000040 patent application [14] is illustrated in fig. 2 and fig.3 and consists of a soft iron core, of constant or changing axis of symmetry and constant or changing cross sectional area, which is surrounded by a REBCO tape solenoid producing DC or AC magnetic field and a magnetic shield which does not permit the magnetic field to penetrate it; this device can create a propulsive force upwards.

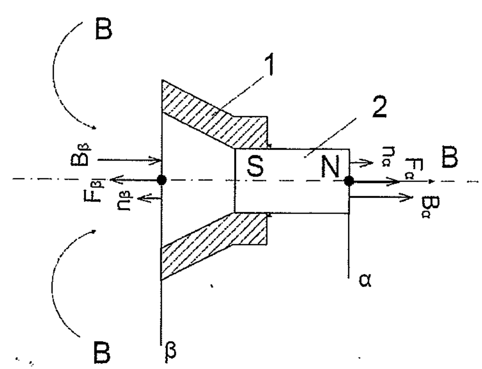

Figure1

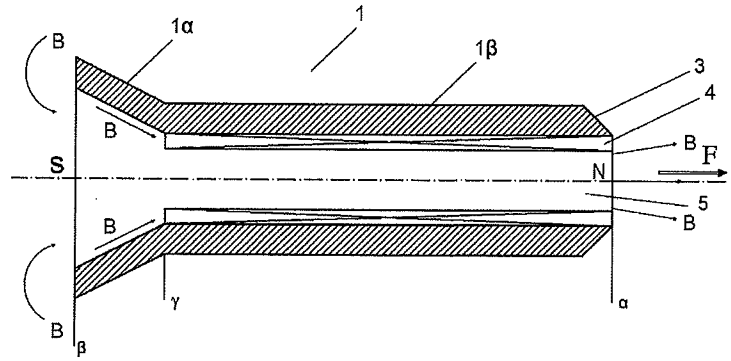

Figure 2

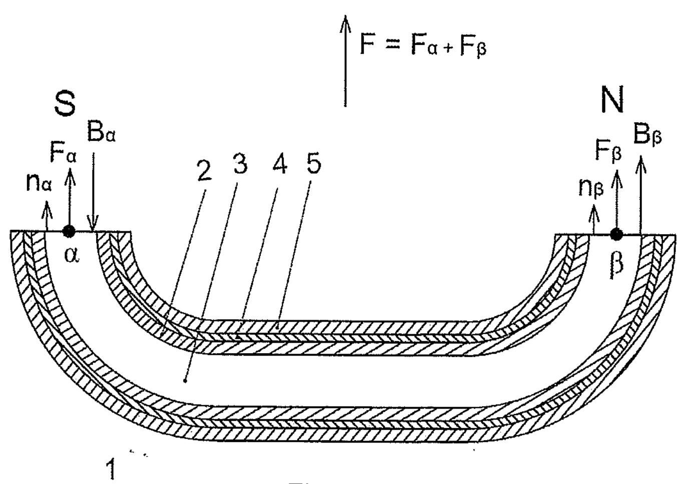

Figure 3

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A PROPOSED PHYSICAL BASIS FOR QUANTUM UNCERTAINTY EFFECTS

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Abstract

Quantum scale "uncertainty" effects limiting measurement accuracy appear to reflect the actual properties of quantum particles as has been well substantiated in numerous experimental examples. However, the concept of uncertainty appears to lack any clear physical basis and stands as an effects descriptor, not as a causal description of actual particulate physical properties. The famous EPR paradox is examined, assessed and placed into current perspective then new theory is presented defining the functional causal basis of observed uncertainty effects. Lastly, experimental evidence will be presented in support of this new model.

"When we see probability we do not see causality, we see the limits placed upon our ability to observe overcome by way of an ingenious guess at the result. In this clever approach where cause is neglected *for the prediction of outcome*; we must not forget it is we who can not see. *Physical systems are not guessing at themselves."*

—R.N.

Introduction.

To many the whole notion of uncertainty, as discussed in quantum mechanics, comes as something of anathema; the whole idea seems to contradict commonsense. It would appear, therefore, sensible to examine this apparent basis for much of modern physics again and in some detail. At the same time it would seem appropriate to examine other uncertainty relations which come into modern physics. Amongst these must be the idea of uncertainty relations in thermodynamics and it is an examination of these which could lead to an understanding of the entire issue, including possibly a further consideration of the question of the completeness of quantum mechanics as a theory and, therefore, of the validity of the claims of Einstein, Podolsky and Rosen[1]. There are no uncertainty relations in classical thermodynamics where, almost by definition, all physical quantities are taken to have quite definite values. However, for example when systems composed of a large number of particles are to be investigated, statistical methods have to be employed since it is impossible, at least at present, to evaluate exactly the behavior of each and every individual particle. Hence, the subject 'statistical mechanics' came into being. By its reliance on statistical methods and, therefore, the idea of probability, the outcome of investigations becomes less definite and uncertainties creep in. This is the source of the so-called thermodynamic uncertainty relations which are considered in many texts[2]. Note though that these uncertainty relations arise out of the introduction of uncertainty into the theory by investigators; they do not appear purely as a result of the physical situation being discussed. Hence, such relations and any deductions made using them must be viewed with a degree of skepticism and treated accordingly since it is not at all obvious that any such deductions are physically realistic. It might be wondered if the same could be true of the uncertainty relations of quantum mechanics. In his seminal book[3], Heisenberg first introduces his relations via a quite simple but definitely approximate method using a wave picture. He then proceeds to derive them also without explicit use of the wave picture but then obtains from them *the mathematical scheme of quantum theory and its physical interpretation.* However, at the basis of much of the mathematics associated with quantum theory is the wave equation with the so-called wave function associated with a probability. Once probability comes into things, uncertainties in measured quantities must necessarily follow. Hence, the question must be raised as to whether, or not, the uncertainties associated with quantum theory are real physical uncertainties or uncertainties introduced surreptitiously by theoreticians, just as occurs in statistical thermodynamics?

All the work that follows is really an extension of earlier work which appeared in the Hadronic Journal and is available online[4]. Any reader of the current work is encouraged to read the full work mentioned here first in order to grasp more easily that which follows.

Some Preliminaries.

To begin with, the original paper by Einstein, Podolsky and Rosen[1] should be examined. It may be noted that several important points concerning the thought experiment are proposed:

" ...every element of the physical reality must have a counterpart in the physical theory."

"The elements of the physical reality cannot be determined by a priori philosophical considerations, but must be found by an appeal to results of experiments and measurements."

"More generally, it is shown in quantum mechanics that, if the operators corresponding to two physical quantities, say A and B, do not commute, that is, if $AB \neq BA$, then the precise knowledge of one of them precludes such a knowledge of the other. Furthermore, any attempt to determine the latter experimentally will alter the state of the system in such a way as to destroy the knowledge of the first. From this follows that either (1) the quantum mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality. For if both of them had simultaneous reality-and thus definite values-these values would enter into the complete description, according to the condition of completeness."

"Thus, by measuring either A or B we are in a position to predict with certainty, and without in any way disturbing the second system, either the value of the quantity P (that is p_k) or the value of the quantity Q (that is q_r)."

"Previously we proved that either (1) the quantum-mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality. Starting then with the assumption that the wave function does give a complete description of the physical reality, we arrived at the conclusion that two physical quantities, with noncommuting operators, can have simultaneous reality. Thus the negation of (1) leads to the negation of the only other alternative (2). We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete."

Two primary elements of the EPR argument may now be noted separately:

1. It is possible to define both position and momentum of two previously interacting quantum particles/systems.
2. Measurement may not (non locally) disturb system two if system one is measured, unless a hidden variable not yet defined within the context of *wave function* is identified.

Point two is clearly implied from the last sentence in the paper:

"We believe, however, that such a theory is possible."

and the aforementioned sentence:

"...every element of the physical reality must have a counterpart in the physical theory."

It is important to note at this juncture, the concerns of Heisenberg regarding such fanciful methods of deduction and exploration as thought experiment and human imagining alone, which appear to closely parallel Einstein's views of the same, as already noted above.

From Heisenberg's book[3], p. 15, concerning the reality of uncertainty as per his equations in physical systems, he states that

"*In this connection one should particularly remember that the human language permits the construction of sentences which do not involve any consequence and which therefore have no content at all—in spite of the fact that these sentences produce some kind of picture in our imagination.*"

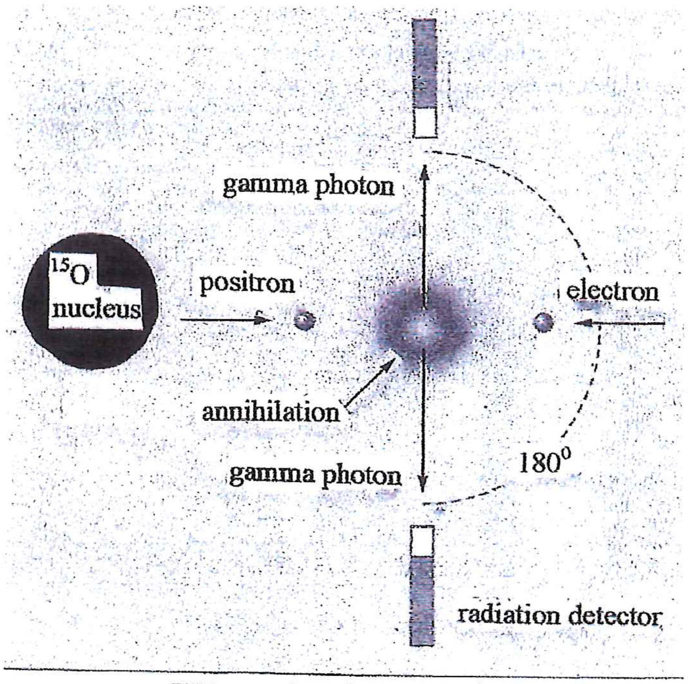
The reader of this present article should note this point as it is important in what follows.

Analysis of EPR feasibility.

If the notion of the EPR argument is sound, one would expect the scheme to be used in some sort of demonstrable way. If the idea is good and leads to accurate measurement, some practical usage must have been made of it after all these years. Entangled science aside, is the basic notion in point one above actually demonstrable?

Let us bring forward the usual interpretations of the EPR ideas, and imagine two quantum particles which have interacted, and are now moving directly away from each other at a 180 degree relation. This is the interpretation most used, that akin to the thinking of Kumar[5] which defines the EPR idea as "two particles, A and B, [that] interact briefly and then move off in opposite directions."

Is this scheme actually able to measure anything, and is it used? Seemingly yes. Positron Emission Tomography scanning (the PET scan) appears to use this idea to measure biological processes and define the locations thereof. A PET scanner is essentially a gamma ray detector. In PET scans, Blood Oxygenation Level-Dependent relations indicative of tissue oxygen metabolism are detected through positron/electron annihilations created by way of an injected radioactive oxygen tracer such as ^{15}O, which has a half life of 123 seconds. As the unstable nucleus of a ^{15}O atom decays having been absorbed by dynamic oxygen using tissues such as neurons, it emits a positron. The positron annihilates when brought in contact with an electron, emitting 2 (gamma) annihilation photons which travel in exactly opposite directions, a 180 degree relation of two quantum particles moving at a constant mutual speed, allowing accurate measurement of the location of the source interaction in space, and also, inference could easily be drawn from one particle measurement to the values of the other.



PET scan schematic representation.

It may be concluded that the basic notion is in fact quite functional as a system of measurement when used in a general way. It is clear also that scientific observers could easily infer the position and momentum of one particle from measurement of the other, which travels in mirror opposite, both at a known speed.

It seems the EPR scheme does allow actual measurement as it should in reality, and is not just a fanciful idea one may draw up to form a picture in one's head, and so, answers in this one aspect at least, Heisenberg's and also Einstein's standards of a workable theory as represented in good science.

Next, we move to the nonlocal aspects of the EPR theory and assess the outcome of experiments. Local realism insists that measurement of one separated system part could not ever superluminally affect the other separated parts of the system (presumably unless some missing, hidden variable is in play). Recall that, in the Copenhagen interpretation of QM, the wave function is entirely a probabilistic entity! However, it is found that nonlocal measurement effects moving well in excess of light speed are evidenced and those results then repeated in experiments involving entanglement.

In an article by Yin, et al.[6], it may be read

> "In the well-known EPR paper, Einstein et al. called the nonlocal correlation in quantum entanglement as `spooky action at a distance'. If the spooky action does exist, what is its speed? All previous experiments along this direction have locality and freedom-of-choice loopholes. Here, we strictly closed the loopholes by observing a 12-hour continuous violation of Bell inequality and concluded that the lower bound speed of `spooky action' was four orders of magnitude of the speed of light if the Earth's speed in any inertial reference frame was less than 10^{-3} times of the speed of light."

Here, the new theories come good and the matter may be resolved in favor of a hidden variable: the scalar wave within the aether. See reference 4. Of course, in any modern discussion of the EPR paradox, it must never be forgotten that a resolution was presented in 1998 by Ruggero Santilli[7] and this has, as far as is known, never been discredited. Hence, it appears that, when the whole question of the EPR paradox comes under discussion, reference should be made to this work.

Cause of quantum uncertainty effects.

Again the reader should remember of the cautionary words of Heisenberg:

> "In this connection one should particularly remember that the human language permits the construction of sentences which do not involve any consequence and which therefore have no content at all—in spite of the fact that these sentences produce some kind of picture in our imagination."

It might be postulated that the notion of "uncertainty" itself is exactly such an error as Heisenberg himself cautions against! This property is particulate anthropomorphism...we assign a human quality, *uncertainty*, a kind of affective and logical confusion, to a physical particle. Yes; humans can form this idea, an idea of a particle which is somehow confused as they are, but that is a human idea, not a physical idea. Although it may be pictured, it has no actual physical content.

What could actually be causing the observed measurement results of quantum experiments? If not uncertainty, what is the physical cause of the measurement problem and seeming duality between particle and wave? Duality is always the mark of confused thinking, as are most if not all paradoxes. What could be causing the plainly available "uncertain" experimental effects. It must be a real physical object, and not some confused human imagining!

In truth, Heisenberg's uncertainty relation

$$\Delta x \Delta p_x \geq h$$

describes effects, not causes. There seems to be no physics in this! What actual object could cause these measurement and other "uncertain" quantum effects?

There is a hidden variable; that is, the aether and the longitudinal pressure waves (scalar waves) which form up "force carrier," entangled and gravitational effects. See reference 4.

Now recall boundary layer theory as applied to the aether; that is, the boundary between the aether itself and any body passing through it or over which it passes. Details of the theory of the boundary layer, due initially to Prandtl[8], may be found in most books on fluid mechanics such as that by Cole[9].

Imagine an aetherial boundary layer around a particle. The original uncertainty equation is missing the basis - there is no basis to the physics - it describes only effects. The boundary layer as a *particle-surrounding scalar wave* accounts for the causal mechanism of uncertainty effects, (as well as, possibly, nuclear decay and fusion as will be discussed in future work) - the measurement uncertainty is then caused by an actual wave surrounding the actual particle; not a wave-like particle duality. Physics has left out the aether and, hence, the wave around each quantum particle. The "uncertain" momentum and x component of velocity in the Heisenberg equations are themselves caused by this wave obscuring those aspects of the particle. The overall change as diffusion then refers to the heat within the scalar wave and hence its initial (quantum) size, *delta in the Heisenberg equations then referring to the amount of change in temperature above absolute zero*, in a causal analysis and proper treatment[3]. That wave is the source of "diffusion" effects. Note how in the paper, *Entropy of a column of gas under gravity*[10], heat first added to the system creates gravitational potential (in part) and not only increase in temperature. That gravitational potential is, by our present theories, the creation of the scalar waves which create a gravitational field. See reference 4.

If this is so, and our theory correct, a violation of measurement "uncertainty" should be observed in experiments if the scalar waves around the particles are deprived of heat. Indeed, this is exactly what is seen in experiments. The back action limit, the quantum limit on measurement precision bounded by uncertainty, is violated, and now, just as might be expected, absolute zero may be approached *arbitrarily close* to deprive the actual source of uncertainty effects of the heat needed to create them. As Clark and colleagues have pointed out recently[11]:

> "Here we propose and experimentally demonstrate that squeezed light can be used to cool the motion of a macroscopic mechanical object below the quantum backaction limit. We first cool a microwave cavity optomechanical system using a coherent state of light to within 15 per cent of this limit. We then cool the system to more than two decibels below the quantum backaction limit using a squeezed microwave field generated by a Josephson parametric amplifier."

Uncertainty is experimentally demonstrable as a function of heat instantiated within the boundary scalar wave surrounding the particle. It appears likely that, as heat is further reduced as absolute zero is approached more closely, *the cause of quantum uncertainty and fluctuation which is the omnidirectional motion of aether particles within the particle boundary scalar wave* is then reduced, perhaps by way of energy reduction of the aether particle itself and/or alignment of said omnidirectional particle motions, leading to the absence of any wave-forming particulate energy value at absolute zero temperature.

Quantum fluctuation effects and related uncertainty are caused by omnidirectional aether particle motion. Uncertainty itself within quantum particulate measurement dynamics is actually caused by the boundary wave, surrounding a quantum particle as a function of heat.

Uncertainty effects emerge as a function of quantum scale, as the aether particle size is more closely approached.

Lastly, new experiments are seen where, as might be expected, heat is reduced to permit the proliferation of related condensate and EPR effects to emerge. Note, for example the paper by Fadel, et al[12] in which it is stated that

> "While spin-squeezed and other nonclassical states of atomic ensembles were used to enhance measurement precision in quantum metrology, the notion of entanglement in these systems remained controversial because the correlations between the indistinguishable atoms were witnessed by collective measurements only. Here we use highresolution imaging to directly measure the spin correlations between spatially separated parts of a spin-squeezed Bose-Einstein condensate. We observe entanglement that is strong enough for Einstein-Podolsky-Rosen steering: we can predict measurement outcomes for non-commuting observables in one spatial region based on a corresponding measurement in another region with an inferred uncertainty product below the Heisenberg relation."

Uncertainty within internal and external dynamical systems.

Clearly the ideas within this brief work refer only to uncertainty effects within the external dynamical problem, that of particulate interactions and not to the internal dynamical problem of hadronic construction which is that of non-potential contact interactions, meaning non-Hamiltonian systems (that is, variationally nonself-adjoint systems not representable with a Hamiltonian). Those hadronic and other like systems then, may be rightly understood without erroneous reference to uncertainty by way of the mathematics of Santilli.

These topics are discussed in detail at:
http://www.galileoprincipia.org/santilli-confirmation-of-the-epr-argument.php

Briefly, as derived from the web reference above:
Extended and hyperdense protons and neutrons in conditions of partial mutual penetration as occurring in a nuclear structure demonstrate

nonHamiltonian forces. The assumption of the exact validity of Heisenberg's uncertainty in the interior of a nucleus is non-scientific. The hadronic isomomentum is uniquely defined by

$$p' * \psi'(t', r') = - i \partial' \psi'(t', r') = - i U \partial \psi'(t', r') \qquad (1)$$

It is then plain that isolinear momenta isocommute on isospace over isofields by therefore confirming the principle of isotopies

$$[p'_i, p'_j]' = p'_i * p'_j - p'_j * p'_i = 0 \qquad (2)$$

This occurs because the isotopic element T of the isoproduct "*" , cancels out with its inverse, the isounit U =1/T. However, isomomenta no longer commute in our spacetime,

$$[p'_i, p'_j] = p'_i \, p'_j - p'_j \, p'_i \neq 0 \qquad (3)$$

because, in the absence of the isotopic product, the derivative does act non-trivially on the isounit U due to its general dependence on local coordinates, and this eliminates Heisenberg's uncertainty principle for the study of interior problems and actually replaces it with a much more general principle.

Some Final Thoughts on the Aether.

Before concluding, it might be appropriate to reflect on the demise of the aether theories over the last hundred years and more. In the intervening time, several people have doggedly pursued investigations into theories involving the aether concept, often at personal cost. Among those was Kenneth Thornhill and it might benefit many to read his work which is readily available on the internet.[13] In the cited article, he starts by showing

that Planck's energy distribution for a black body radiation field may be derived for a gas-like aether with Maxwellian statistics. The gas consists of an infinite variety of particles whose masses are integral multiples of the mass of the unit particle. Also the frequency of electromagnetic waves correlates with the energy per unit mass of the particles, not with their energy, thus differing from Planck's quantum hypothesis. Identifying the special wave-speed, usually called the speed of light, with the wave-speed in the 2.7^0K background radiation field, leads to a mass of 0.5×10^{-39}kg for the unit aether particle. Interestingly, in this article he also shows that the speed of light should vary with the square root of the background temperature. It is not without interest to note that this suggestion by Thornhill would obviate any need for introducing theories of inflation to protect the Big Bang notion. More may be found on the whole question of the constancy, or otherwise, of the speed of light in the article by Farrell and Dunning-Davies[14].
Also, before ending this section, attention should be drawn to a companion paper by Thornhill[15] in which he discusses in detail the fact that, in a gas-like aether, the duality between the oscillating electric and magnetic fields, which are transverse to the direction of propagation of electromagnetic waves, becomes a triality with the longitudinal oscillations of the motion of the aether if electric field, magnetic field and motion are coexistent and mutually perpendicular. He points out that it must be shown that, if electromagnetic waves also comprise longitudinal condensational oscillations of a gas-like aether, analogous to sound waves in a material gas, then all three aspects of such waves must propagate together along identical wave fronts. This he shows to be the case. Further he finds that the equations governing the motion and the electric and magnetic field strengths in such an aether, together with their common characteristic hyperconoid, are all invariant under Galilean transformation.

Conclusion.

The notion of "uncertainty" within physical systems is only an anthropomorphic effects descriptor, not a causal description of physics. Fluctuation effects in quantum systems and uncertain measurement effects are in fact caused by a real object and not probability: the aether and the scalar waves within it. Quantum mechanics as interpreted by the Copenhagen interpretation is in point of fact: incomplete. The wave function must be augmented in its interpretation to represent aetherial and scalar wave dynamics, at which point the adjusted theory would in fact satisfy Einstein's highest standards as a physical theory.

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POSSIBILITY OF GEOMETRICAL INTERPRETATION OF QUANTUM MECHANICS AND GEOMETRICAL MEANING OF "HIDDEN VARIABLES"

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Abstract

Interpretation of wave function for free particle is suggested as a description of microscopic distortion of the space-time geometry, namely, as some closed topological 4-manifold. Such geometrical object looks in three-dimensional Euclidean space as its topological defect having stochastic and wave-corpuscular properties of quantum particle. All possible deformations (homeomorphisms) of closed topological manifold play the role of "hidden variables", responsible for statistical character of the theory.

1.Introduction Interpretation of quantum mechanics means here an explanation of strange features of its mathematical formalism ("copenhagen" interpretation) with the help of notions from everyday life (physical model). Attempts to find such explanation started just after the creation of quantum mechanics and this problem is still considered by many physicians as actual. For example, V.Ginsburg considered interpretation of quantum mechanics as the one of three great problems of modern physics (as the problem of appearance of life and the problem of irreversibility of time) [1]. The problem of interpretation of quantum mechanics was investigated for many years by t'Hooft [2] (here is a detailed list of references on the problem). . But why any interpretation is needed for mathematical formalism if it is in a good agreement with experiment? One of reasons is the fact that new physical models open new opportunities for development of theories. For example, many attempts (Einstein Weyl, Calutza and others) have been made for this reason to find geometrical interpretation of classical electrodynamics, although it is in a very good agreement with experiment [3,4]. In addition, the quantum theory cannot be considered as the final one. Another, more concrete, reason—the contradiction between Bohr and Einstein regarding the completeness of quantum mechanics which did not resolved until now [5,6]. In contrast to Bohr, Einstein thought that the quantum mechanics is not a complete theory because it says nothing about physical reality, responsible for statistical character of the theory (so called "hidden variables" [2,7,8]), and the answer to this question is, may be, the main result of this work. As for physical models, author knows two interpretations of quantum mechanics where mathematical formalism of quantum mechanics is not questioned. One is the Everett's "Many Universes Interpretation", where statistical character of quantum theory is explained by existence of infinite number of Universes, corresponding to various realizations of reality [9], This interpretation has its supporters in spite of exotic character and serious criticism [10]. Another interpretation is the 't Hooft's " The Cellular Automaton Interpretation of Quantum Mechanics", where a very special set of mutually orthogonal sates in Hilbert space is considered [2]. This approach is now under development. Among the works where the apparatus of quantum physics is undergoing serious changes we can mention the string theory (see, e.g. [11]) and Santlli's investigations [12]. The possibility is shown in this work to interpret the quantum mechanical wave function for free particle as a description of microscopic distortion of the space-time geometry. Some characteristics of this geometrical object play the role of "hidden variables" responsible for stochastic behavior of quantum particle, and these characteristics are the physical reality that exists before measurement. Other characteristics explain wave-corpuscular properties of the particle. It may be said that quantum mechanics within suggested interpretation satisfies the completeness criterion formulated by Einstein. Preliminary results see [13-17].

2. Quantum particle as the microscopic distortion of the space-tome geometry

Let's consider the free neutral particle with mass m and spin 0. It will be shown that wave function of such particle can be interpreted as a mathematical description of some geometrical object. This scalar wave function is the solution of the Klein-Fock-Gordon equation, and it has the form [18,19]

$$\Psi = const \cdot exp\left(-\frac{i}{\hbar}(Et - \boldsymbol{pr})\right). \qquad (1)$$

This function describes within existing interpretation the particle's state with definite energy E and definite momentum $\boldsymbol{p}$. The particle's position before measurements is unknown – it may be observed in any point with equal probability. This fact reflects statistical character of quantum mechanics – unusual property within classical representations. Another unusual property – wave-corpuscular dualism of quantum particles that is defined by phase of the wave function and by wavelength and frequency, connecting with the particle's energy and momentum by known relations [18,19]

$$\lambda_i = \frac{\hbar}{p_i},\ \omega = \frac{E}{\hbar},\ i = x, y, z. \qquad (2)$$

Substituting (2) in (1), we have

$$\Psi = const \cdot exp(-i\omega t + i\boldsymbol{kr}),\ k_i = 2\pi\lambda_i \qquad (3)$$

This type of functions (plane wave) is often used in classical physics (for example, for description of plane running sound wave). Within existing interpretation of quantum mechanics, the origin of periodical dependence of wave function is not discussing.

Let us rewrite the function (2) not with space-time coordinates x, y, z, t, but with only space coordinates $x^1, x^2, x^3, x^4$of the space of events of the special theory of relativity – four dimensional pseudo Euclidean space of index 1(the Minkowski space [20]). Time, multiplied by light velocity, plays in this space the role of fourth space coordinate ($ct = x^4$). Let us rewrite (3), using relativistic designations where $\hbar = \mathrm{c} = 1$

$$\Psi = const \cdot exp(-ix^\mu p_\mu). \qquad (4)$$

Here p_μ – the particle's 4-momentum ($p_1 = E, p_{2,3,4} = p_{x,y,z}$). Summation over repeating indexes is suggested in (4) with signature ($+ - - -$). In relativistic case [18.19]

$$p_1^2 - p_2^2 - p_3^2 - p_4^2 = m^2 \qquad (5)$$

where m – the particle's mass. Let's write down (4) in such a way that it contains only values with dimensionality of length

$$\Psi = const \cdot exp(-2\pi i x^{\mu} \lambda_{\mu}^{-1}), \tag{6}$$

where

$$\lambda_1^{-2} - \lambda_2^{-2} - \lambda_3^{-2} - \lambda_4^{-2} = \lambda_m^{-2}, \quad \lambda_{\mu} = 2\pi p_{\mu}^{-1}, \quad \lambda_m = 2\pi m^{-1}. \tag{7}$$

In contrast to (1,3) function (6) does not look as a plane wave - it represent periodical function of four space coordinates in the Minkowski space.

Function (6) may be considered as a function realizing representation of the group whose elements are discrete translations along four coordinates axes in the Minkowski space. Indeed, function (6) goes into itself at translations

$$x^{\mu} \to x^{\mu'} + n_{\mu} \lambda_{\mu}, \tag{8}$$

where n_{μ} – integers ($\mu = 1{,}2.3.4$). This group is isomorphic to the group $\mathbb{Z}^4$, whose elements are products of integers n_{μ} In turn, the group $\mathbb{Z}^4$ is isomorphic to the fundamental group of closed 4-manifold that is homeomorphic to the foir dimensional torus T^4 [21,22]. Now we nay formulate the main hypostasis: quantum particle, described by the wave function (6), can be considered as a closed space-time manifold that is homeomorphic to the four dimensional torus imbedded into five dimensional pseudo Euclidean space of index 1. Relation (7) imposes a metric restriction on the acceptable under deformations path lengths λ_i ($i = 1{,}2{,}3{,}4$). Thus, the relation (7) defines also the geometrical interpretation of the particle's mass and 4-momentum. It will be shown in the next Section that such geometrical object looks in three dimensional Euclidian space as moving topological defect of this space having stochastic and wave-corpuscular properties of quantum particle.

Representation of particle as a closed manifold means that this particle before measurement may be considered as a "mixture" of its all possible geometrical representations (homeomorphisms), and only interaction with device fixes one of them. This means that wave function describes not an individual particle, but statistical ensemble of all its possible geometrical representations, and this explains statistical character of quantum mechanics. Thus, ensemble of all possible homeomorphisms plays the role of "hidden variables," responsible for stochastic behavior of particles.

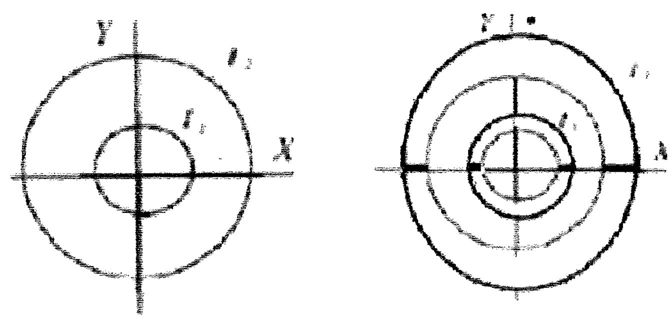
3. Quantum particle as a topological defect of Euclidean space.

Let's proceed to decoding of the representation of quantum particle as a closed 4-manifold, that is let's show how such object looks from the point of view of the observer in Euclidian space. But the important notice should be made before going to this problem. The geometry of four-dimensional closed manifolds is now under development: the full recognition algorithm is not now known even for three dimensional closed manifolds [22]. Therefore the only way to establish what the representation of quantum particle as a closed 4-manifold means from the point of view of the observer in Euclidtan space is to use low dimensional analogies. Having this in mind let's consider closed manifold homeomorphic to the two dimensional torus embedded into three dimensional pseudo Euclidean space of index 1. To obtain concrete result only one of infinite number of possible homeomorphisms of this manifold will be considered, namely usual two dimensional torus $T^2 = S^1 \times S^1$, where S^1 – a circle. Such torus may be considered in three-dimensional Euclidean space as a surface obtained by rotation of a circle around vertical axis lying in the plain of this circle (Fig.1a). In pseudo Euclidean three-dimensional space this circle is located in pseudo Euclidean plane and it looks on Euclidean plane of Fig1b as a isosceles hyperbola [23]. That is two dimensional torus, representing particle, looks in three dimensional Euclidean space as a hyperboloid (Fig.1b). Within considered low dimensional analogy physical space-time (space of events) is a two dimensional pseudo Euclidean space, and the particle's positions in different moments of time in the Euclidean (one dimensional) space are defined by points of intersection with this space of the projections of the hyperboloid's temporary cross-sections. These cross-sections look as expanding circles in two-dimensional Euclidean plane XY (Fig.2a).

a b

Fig 1.. Two-dimensional torus embedded into three-dimensional Euclidean and pseudo Euclidean spaces.

These circles can be considered as moving topological defect of one dimensional physical space. It is the fact that intersection point belongs to topological defect that distinguishes this point at Fig.2a from neighboring points of one dimensional Euclidean space, turning it into a physical "material point".



a b

Fig.2. Topological defect of one dimensional Euclidean space (X-axis).

The particle's positions in Euclidean (one dimensional) space are defined by points of its intersection with the circle, corresponding to the only one of the torus possible homeomorphisms. Accounting for all possible homeomorphisms leads, obviously, to "blurring" of this circle and so leads to transformation of the one intersection point in finite region of Euclidean space (this region is indicated at Fig.2b by a bold line segment on X-axis). This region has at every moment of time a finite size because the range of all possible homeomorphisms is limited by metric condition (7) that restrict the maximum possible dimensions of closed manifold. As a result, the observer in Euclidean space will detect the particle with equal probability in one of points of above mentioned region. This means that wave function describes not a position of separate particle but the ensemble of its possible positions, and this explains statistical character of quantum mechanics. It is obvious that all possible homeomorphisms of the closed manifold, representing this particle, play the role of "hidden variables", responsible for the particle's stochastic behavior: each homeomorphism corresponds to the one particle's possible position in Euclidean space. The points of the intersection region have different velocities. This means that the intersection region at Fig. 2b are moving expanding, and finally it will fill all Euclidean (one dimensional) space. In result the probability to observe the particle in any point of space will be the same, as it should be according to laws of quantum mechanics for free particle, described by wave function (1).

The fact that the particle can be represented in physical Euclidean space as a part of topological defect allows to explain the particle's wave properties. It is sufficient for this to suppose that the defect's position in the external five dimensional Euclidean space relative to the three dimensional space changes according to periodical low described by wave function (1) (a rigorous proof of this assumption is not possible within the framework of low dimensional analogy). It can be said that the phase of the defect's periodical movement is an additional degree of freedom on which the effect of the particle on the device depends. The particle's corpuscular properties (4-momentum) are defined through parameters of above periodical movement of defect by relations

$$p_\mu = 2\pi\lambda_{\mu.} \qquad (9)$$

These relations are identical to the definition (2) of the particle's wave length through its momentum within existing interpretation [19] ,but now they have the "reverse" meaning of definition of momentum through the wave length, as it should be in the consistent theory where less general concepts (classical momentum) are defined through more fundamental ones (wave length of the defect's periodical movement).

Conclusion. The wave function plays a dual role within suggested interpretation. First, it is a function, realizing the representation of the fundamental group for a closed 4-manifold, representing a free particle. Second, this function describes periodical movement of topological defect in the external space, and intersection of this defect with physical space defines the possible particle's positions. These properties of the wave function make it possible to explain the stochastic behavior of the particle and its wave-corpuscular dualism. The role of "hidden variables", responsible for the particle's stochastic behavior, is played by all possible homeomorphisms of the closed 4-manifold, representing the particle. Notice in conclusion that relation (7) defines geometrical interpretation of the particle's mass as a characteristic of some fundamental length $\lambda_{m.}$ Geometrical interpretation of elementary electrical charge and the particle's spin will be considered in subsequent publications.

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A NEW CONCEPTION OF LIVING ORGANISMS AND ITS REPRESENTATION VIA LIE-ADMISSIBLE H_v-HYPERSTRUCTURES

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Abstract

Recent studies have confirmed Einstein's 1935 legacy implying that quantum mechanics and chemistry are "incomplete" theories in the sense of being excellent for the description of systems composed by point-like constituents under potential interactions (such as the atomic structure), but said theories are "incomplete" for the description of complex time-irreversible systems of extended constituents with internal non-potential interactions (as expected in a cell). Sadi verifications were achieved thanks to the prior "completion" over the past half a century of quantum theories into the covering hadronic mechanics and chemistry with a time irreversible Lie-admissible structure. In this paper we present, apparently for the first time, a new conception of living organisms, solely permitted by the verifications of Einstein's legacy, composed by a very large number of extended wavepackets in conditions of continuous mutual penetration/entanglement and, therefore, of continuous communications via contact non-potential interactions. Due to the extremely large number of constituents and the extreme complexity of the multi-valued internal communications, in this paper we introduce, also apparently for the first time, the representation of the indicated new conception of living organisms via two hyperbimodular, Lie-admissible H_v-hyperstructures, the first with all hyperoperations ('hope') ordered to the right and the second with all hopes ordered to the left. The irreversibility of living organisms is represented by the inequivalence of the left and right hopes. The extremely large number of internal communications is represented by the extremely large number of solutions of the indicated hopes. We close the paper with the indication that new medical diagnostics and treatments are expected in the transition from the current quantum chemical conception of living organisms as collections of isolated point-like constituents to the indicated new conception.

1. INTRODUCTION ON HYPERSTRUCTURES

The largest class of hyperstructures are called H_v*-structures* and introduced in 1990 [33], [34]. These satisfy the *weak axioms* where the non-empty intersection replaces the equality. Some basic definitions are the following:
In a set H equipped with a hyperoperation (abbreviation *hyperoperation*=***hope***)

$$\cdot: H\times H \to P(H)-\{\varnothing\},$$

we abbreviate by *WASS* the *weak associativity*: $(xy)z\cap x(yz) \neq \varnothing,\ \forall x,y,z\in H$ and by *COW* the *weak commutativity*: $xy\cap yx \neq \varnothing,\ \forall x,y\in H$.

The hyperstructure (H,·) is called ***H_v-semigroup*** if it is *WASS*, it is called ***H_v-group*** if it is reproductive H_v-semigroup, i.e. $xH = Hx = H,\ \forall x\in H$.
Motivation. I the classical theory the quotient of a group with respect to an invariant subgroup is a group. F. Marty from 1934, states that, the quotient of a group with respect to any subgroup is a hypergroup. Finally, the quotient of a group with respect to any partition is an H_v-group [34].

The *powers* of an element $h\in H$ are: $h^1=\{h\}$, $h^2=h\cdot h$, ..., $h^n=h\,^\circ\ldots\,^\circ h$, where (°) is the *n-ary circle hope*: the union of hyperproducts, n times, with all patterns of parentheses put on them. An H_v-semigroup (H,·) is a *cyclic of period s*, if there is a *generator* g, and a natural n, such that $H=h^1\cup\ldots\cup h^s$. If there is an h and s, such that $H=h^s$, then (H,·) is called *single-power cyclic of period s*.

In a similar way more complicated hyperstructures can be defined:
$(R,+,\cdot)$ is ***H_v-ring*** if (+) and (·) are *WASS*, the reproduction axiom is valid for (+) and (·) is *weak distributive* with respect to (+):

$$x(y+z)\cap(xy+xz)\neq\varnothing,\quad (x+y)z\cap(xz+yz)\neq\varnothing,\quad \forall x,y,z\in R.$$

Let $(R,+,\cdot)$ H_v-ring, $(M,+)$ *COW* H_v-group and there exists an external hope

$$\cdot:\ R\times M \to P(M):\ (a,x) \to ax$$

such that, $\forall a,b\in R$ and $\forall x,y\in M$, we have

$$a(x+y)\cap(ax+ay) \neq \varnothing,\quad (a+b)x\cap(ax+bx) \neq \varnothing,\quad (ab)x\cap a(bx) \neq \varnothing,$$

then *M* is an H_v*-module* over *F*. In the case of an H_v-field *F* instead of an H_v-ring *R*, then the H_v*-vector space* is defined.

For more definitions and applications on H_v-structures one can see the books [2], [4], [5], [8], [31], [32], [34], [35], [38], [40], [45], [52].

Definition 1.1 The ***fundamental relations*** β^*, γ^* and ε^*, are defined, in H_v-groups, H_v-rings and H_v-vector spaces, respectively, as the smallest equivalences so that the quotient would be group, ring and vector spaces, respectively [33], [34], [35], [50], [51].

The way to find the fundamental classes is given by the following:

Theorems 1.2 Let $(H,\cdot)$ be H_v-group and denote by U the set of finite products of elements of H. We define the relation β in H by setting $x\beta y$ iff $\{x,y\}\subset u$ where $u\in U$. Then β^* is the transitive closure of β.

Let $(R,+,\cdot)$ be H_v-ring. Denote by U the set of finite polynomials of elements of R. We define the relation γ in R as follows: $x\gamma y$ *iff* $\{x,y\}\subset u$ where $u\in U$. Then the relation γ^* is the transitive closure of the relation γ.

An element is called *single* if its fundamental class is singleton.

Fundamental relations are used for general definitions. Thus, an H_v-ring $(R,+,\cdot)$ is called ***H_v-field*** if R/γ^* is a field.

Let $(H,\cdot)$, $(H,*)$ be H_v-semigroups defined on the same set H. $(\cdot)$ is called ***smaller*** than $(*)$, and $(*)$ ***greater*** than $(\cdot)$, iff there exists an

$$f\in Aut(H,*) \quad \text{such that} \quad xy\subset f(x*y),\ \forall x,y\in H.$$

Then we say that *$(H,*)$ contains $(H,\cdot)$*. If $(H,\cdot)$ is a structure then it is called *basic structure* and $(H,*)$ is called *H_b-structure*.

Theorem 1.3 (The Little Theorem). Greater hopes than the ones which are *WASS* or *COW*, are also *WASS* or *COW*, respectively.

This Theorem leads to a partial order on H_v-structures.

A very interesting class of H_v-structures, is the following [9], [32]:

An H_v-structure is called ***very thin*** iff all hopes are operations except one, which has all hyperproducts singletons except only one, which is a subset of cardinality more than one.

A large class of H_v-structures is the following [9], [41]:

Let $(G,\cdot)$ be groupoid (resp., hypergroupoid) and $f:G\to G$ be a map. We define a hope (∂), called *theta-hope,* we write ***∂-hope***, on G as follows

$$x\partial y = \{f(x)\cdot y,\ x\cdot f(y)\},\ \forall x,y\in G.\ \ (resp.\ x\partial y= (f(x)\cdot y)\cup(x\cdot f(y),\ \forall x,y\in G)$$

If $(\cdot)$ is commutative then ∂ is commutative. If $(\cdot)$ is *COW*, then ∂ is *COW*.

Let $(G,\cdot)$ be groupoid (or hypergroupoid) and $f:G\to P(G)-\{\varnothing\}$ be multivalued map. We define the (∂), on G as follows $x\partial y = (f(x)\cdot y)\cup(x\cdot f(y)),\ \forall x,y\in G.$

Motivation for the ∂-hope is the map *derivative* where only the multiplication of functions can be used. Basic property: if $(G,\cdot)$ is a semigroup then $\forall f$, the (∂) is *WASS*.

Another well known and large class of hopes is given as follows [9], [31], [47]:

Let $(G,\cdot)$ be groupoid, then $\forall P\subset G$, $P\neq\varnothing$, we define the following hopes called *P-hopes*: $\forall x,y\in G$

$$\underline{P}: x\underline{P}y=(xP)y\cup x(Py),\quad \underline{P}_r: x\underline{P}_ry=(xy)P\cup x(yP),\quad \underline{P}_l: x\underline{P}_ly=(Px)y\cup P(xy).$$

The $(G,\underline{P})$, $(G,\underline{P}_r)$ and $(G,\underline{P}_l)$ are called *P-hyperstructures*. If $(G,\cdot)$ is semigroup, then $x\underline{P}y=(xP)y\cup x(Py)=xPy$ and $(G,\underline{P})$ is a semihypergroup but we do not know about $(G,\underline{P}_r)$ and $(G,\underline{P}_l)$. In some cases, depending on the choice of P, the $(G,\underline{P}_r)$ and $(G,\underline{P}_l)$ can be associative or *WASS*.

A generalization of P-hopes is the following [6], [9]:

Construction 1.4 Let $(G,\cdot)$ be an abelian group and P any subset of G. We define the hope $\times_P$ as follows:

$$\begin{cases} x\times_P y = x\cdot P\cdot y = \{x\cdot h\cdot y \,/\, h\in P\} & \text{if } x\neq e \text{ and } c\neq e \\ x\cdot y & \text{if } x=e \text{ or } y=e \end{cases}$$

we call this hope P_e-*hope*. The hyperstructure $(G,\times_P)$ is an abelian H_v-group.

H_v-structures are used in Representation Theory of H_v-groups which can be achieved by generalized permutations or by H_v-matrices [34], [38], [49]. *H_v-matrix* is called a matrix if has entries from an H_v-ring. The hyperproduct of H_v-matrices is defined in a usual manner. The problem of the H_v-matrix representations is the following:

Definition 1.5 Let $(H,\cdot)$ be an H_v-group, find an H_v-ring $(R,+,\cdot)$, a set $M_R=\{(a_{ij})\,/\,a_{ij}\in R\}$ and a map

$$T: H\to M_R: h\mapsto T(h) \text{ such that } T(h_1h_2)\cap T(h_1)T(h_2)\neq\varnothing,\ \forall h_1,h_2\in H.$$

Then T is *H_v-matrix representation*. If $T(h_1h_2)\subset T(h_1)(h_2)$, $\forall h_1,h_2\in H$ is valid, then T is an *inclusion representation*. If $T(h_1h_2) = T(h_1)(h_2) = \{T(h)\,/\,h\in h_1h_2\}$, $\forall h_1,h_2\in H$, then T is a *good representation*.

Hopes on any type of ordinary matrices can be defined [8], [49], [53] they are called *helix hopes*.

Definition 1.6 Let $A=(a_{ij})\in M_{m\times n}$ be matrix and $s,t\in N$, *with* $1\leq s\leq m$, $1\leq t\leq n$. The helix-projection is a map $\underline{st}: M_{m\times n}\to M_{s\times t}: A\to A\underline{st}=(\underline{a}_{ij})$, where $A\underline{st}$ has entries

$$\underline{a}_{ij} = \{ a_{i+\kappa s, j+\lambda t} \,/\, 1\leq i\leq s,\ 1\leq j\leq t \text{ and } \kappa,\lambda\in N,\ i+\kappa s\leq m,\ j+\lambda t\leq n \}$$

Let $A=(a_{ij})\in M_{m\times n}$, $B=(b_{ij})\in M_{u\times v}$ be matrices and $s=min(m,u)$, $t=min(n,v)$. We define a hyper-addition, called ***helix-sum***, by

$$\oplus : M_{m\times n}\times M_{u\times v}\to P(M_{s\times t}):(A,B)\to A\oplus B=A\underline{st}+B\underline{st}=(\underline{a}_{ij})+(\underline{b}_{ij})\subset M_{s\times t}$$

where $(\underline{a}_{ij})+(\underline{b}_{ij})=\{(c_{ij})=(a_{ij}+b_{ij}) \,/\, a_{ij}\in\underline{a}_{ij} \text{ and } b_{ij}\in\underline{b}_{ij}\}$.
Let $A=(a_{ij})\in M_{m\times n}$, $B=(b_{ij})\in M_{u\times v}$ and $s=min(n,u)$. Define the ***helix-product***, by

$$\otimes: M_{m\times n}\times M_{u\times v}\to P(M_{m\times v}):\ (A,B)\to A\otimes B=A\underline{ms}\cdot B\underline{sv}=(\underline{a}_{ij})\cdot(\underline{b}_{ij})\subset M_{m\times v}$$

where $(\underline{a}_{ij})\cdot(\underline{b}_{ij})=\{(c_{ij})=(\sum a_{it}b_{tj}) \,/\, a_{ij}\in\underline{a}_{ij} \text{ and } b_{ij}\in\underline{b}_{ij}\}$.

The helix-sum is commutative, *WASS*, not associative. The helix-product is *WASS*, not associative and not distributive to the helix-addition.

Using several classes of H_v-structures one can face several representations [48].

Definition 1.7 Let $M=M_{m\times n}$ be module of $m\times n$ matrices over a ring R and $P=\{P_i: i\in I\}\subseteq M$. We define, a kind of, a P-hope $\underline{P}$ on M as follows

$$\underline{P} : M\times M \to P(M):\ (A,B) \to A\underline{P}B = \{ AP^t_iB:\ i\in I \}\subseteq M$$

where P^t denotes the transpose of the matrix P.

We present a proof for the fundamental relation analogous to Theorem 1.2 in the case of an H_v-module:

Theorem 1.8 Let $(M,+)$ be H_v-module over R. Denote U the set of expressions of finite hopes either on R and M or the external hope applied on finite sets of elements of R and M. We define the relation ε in M by: $x\varepsilon y$ *iff* $\{x,y\}\subset u$, $u\in U$. Then the relation ε^* is the transitive closure of the relation ε.

Proof. Let $\underline{\varepsilon}$ be the transitive closure of ε, and denote by $\underline{\varepsilon}(x)$ the class of the element x. First, we prove that the quotient set $M/\underline{\varepsilon}$ is a module over R/γ^*.
In $M/\underline{\varepsilon}$ the sum ($\oplus$) and the external product ($\otimes$), using the γ^* classes in R, are defined in the usual manner:

$$\underline{\varepsilon}(x)\oplus\underline{\varepsilon}(y) = \{\underline{\varepsilon}(z):\ z\in \underline{\varepsilon}(x)+\underline{\varepsilon}(y)\},$$

$$\gamma^*(a)\otimes\underline{\varepsilon}(x) = \{\underline{\varepsilon}(z):\ z\in \gamma^*(a)\cdot\underline{\varepsilon}(x)\},\quad \forall a\in R,\ x,y\in M.$$

Take $x'\in\underline{\varepsilon}(x)$, $y'\in\underline{\varepsilon}(y)$. Then $x'\underline{\varepsilon}x$ *iff* $\exists x_1,\dots,x_{m+1}$ with $x_1=x'$, $x_{m+1}=x$, $u_1,\dots,u_m\in U$ such that $\{x_i, x_{i+1}\}\subset u_i$, $i=1,\dots,m$, and $y'\underline{\varepsilon}y$ *iff* $\exists y_1,\dots,y_{n+1}$ with $y_1=y'$, $y_{n+1}=y$ and $v_1,\dots,v_n\in U$ such that $\{y_j,y_{j+1}\}\subset v_j$, $j=1,\dots,n$. From the above we obtain

$$\{x_i, x_{i+1}\}+ y_1 \subset u_1+v_1,\ \ i=1,\dots,m-1,\ \ x_{m+1} +\{y_j, y_{j+1}\} \subset u_m+v_j,\ \ j=1,\dots,n.$$

The $u_i+v_1=t_i$, $i=1,..,m-1$, $u_m+v_j=t_{m+j-1}$, $j=1,..,n \in U$, so $t_k \in U$, $\forall k \in \{1,..,m+n-1\}$. Take $z_1,...,z_{m+n}$ with $z_i \in x_i+y_1$, $i=1,...,n$ and $z_{m+j} \in x_{m+1}+y_{j+1}$, $j=1,...,n$, thus, $\{z_k,z_{k+1}\} \subset t_k$, $k=1,...,m+n-1$. Therefore, $\forall z_1 \in x_1+y_1=x'+y'$ is $\underline{\varepsilon}$ equivalent to $z_{m+n} \in x_{m+1}+y_{n+1}=x+y$. Thus, $\underline{\varepsilon}(x)\oplus\underline{\varepsilon}(y)$ is a singleton so we can write $\underline{\varepsilon}(x)\oplus\underline{\varepsilon}(y)=\underline{\varepsilon}(z)$, $\forall z \in \underline{\varepsilon}(x)+\underline{\varepsilon}(y)$. Similarly, using the properties of γ^* in R, we prove that $\gamma^*(a)\otimes\underline{\varepsilon}(x)=\underline{\varepsilon}(z)$, $\forall z \in \gamma^*(a)\cdot\underline{\varepsilon}(x)$.

The *WASS* and the weak distributivity on R and M guarantee that the associativities and the distributivity are valid for $M/\underline{\varepsilon}$ over R/γ^*. Therefore, $M/\underline{\varepsilon}$ is a module over R/γ^*.

Now let σ equivalence relation in M such that M/σ is module on R/γ^*. Denote $\sigma(x)$ the class of x. Then $\sigma(x)\oplus\sigma(y)$ and $\gamma^*(a)\otimes\sigma(x)$ are singletons $\forall a \in R$ and $x,y \in M$, i.e.

$$\sigma(x)\oplus\sigma(y)=\sigma(z),\ \ \forall z \in \sigma(x)+\sigma(y),\quad \gamma^*(a)\otimes\sigma(x)=\sigma(z),\ \ \forall z \in \gamma^*(a)\cdot\sigma(x).$$

Thus we write, $\forall a \in R$, $x,y \in M$ and $A \subset \gamma^*(a)$, $X \subset \sigma(x)$, $Y \subset \sigma(x)$

$$\sigma(x)\oplus\sigma(y)=\sigma(x+y)=\sigma(X+Y),\quad \gamma^*(a)\otimes\sigma(x)=\sigma(ax)=\sigma(A\cdot X).$$

By induction, extend these relations on finite sums and external products. Thus, $\forall u \in U$, we have $\sigma(x)=\sigma(u)$, $\forall x \in u$. Consequently $x' \in \varepsilon(x)$ implies $x' \in \sigma(x)$, $\forall x \in M$.

But σ is transitively closed, so we obtain: $x' \in \underline{\varepsilon}(x)$ implies $x' \in \sigma(x)$.

Thus, $\underline{\varepsilon}$ is the smallest equivalence on M such that $M/\underline{\varepsilon}$ is a module on R/γ^*, i.e. $\underline{\varepsilon}=\varepsilon^*$. ■

The general definition of an H_v-Lie algebra was given as follows [30], [44]:

Definition 1.9 Let $(L,+)$ H_v-vector space on $(F,+,\cdot)$, $\varphi:F\to F/\gamma^*$ canonical, $\omega_F=\{x \in F:\varphi(x)=0\}$, where 0 is zero of F/γ^*. Let ω_L the core of $\varphi':L\to L/\varepsilon^*$ and denote 0 the zero of L/ε^*. Consider the *bracket (commutator) hope*:

$$[\ ,\ ]: L\times L \to P(L): (x,y) \to [x,y]$$

then L is an *H_v-Lie algebra* over F if the following axioms are satisfied:

(L1) The bracket hope is bilinear, i.e.

$$[\lambda_1x_1+\lambda_2x_2,y]\cap(\lambda_1[x_1,y]+\lambda_2[x_2,y]) \neq \varnothing$$

$$[x,\lambda_1y_1+\lambda_2y_2]\cap(\lambda_1[x,y_1]+\lambda_2[x,y_2]) \neq \varnothing,\quad \forall x,x_1,x_2,y,y_1,y_2 \in L,\ \forall \lambda_1,\lambda_2 \in F$$

(L2) $[x,x]\cap\omega_L \neq \varnothing$, $\forall x \in L$

(L3) $([x,[y,z]]+[y,[z,x]]+[z,[x,y]])\cap\omega_L \neq \varnothing$, $\forall x,y \in L$

The enlargement or reduction of hyperstructures are examined in the sense that an extra element appears in one result or we take out an element. In both directions most useful in representation theory, are those H_v-structures with the same fundamental structure [36], [37]:

Let $(H,\cdot)$ be H_v-semigroup and $v\notin H$. Extend $(\cdot)$ into the $\underline{H}=H\cup\{v\}$ as follows: $x\cdot v=v\cdot x=v$, $\forall x\in H$, and $v\cdot v=H$. The $(\underline{H},\cdot)$ is an h/v-group where $(\underline{H},\cdot)/\beta^*\cong Z_2$ and v is a single element. We call $(\underline{H},\cdot)$ the *attach h/v-group* of $(H,\cdot)$.

Let $(G,\cdot)$ be semigroup and $v\notin G$ be an element appearing in a product ab, where $a,b\in G$, thus the result becomes a hyperproduct $a\otimes b=\{ab,v\}$. Then the minimal hope $(\otimes)$ extended in $G'=G\cup\{v\}$ such that $(\otimes)$ contains $(\cdot)$ in the restriction on G, and such that $(G',\otimes)$ is a minimal H_v-semigroup which has fundamental structure isomorphic to $(G,\cdot)$, is defined as follows:

$$a\otimes b=\{ab,v\},\quad x\otimes y=xy,\quad \forall(x,y)\in G^2-\{(a,b)\}$$

$$v\otimes v=abab,\quad x\otimes v=xab \text{ and } v\otimes x=abx,\quad \forall x\in G.$$

$(G',\otimes)$ is very thin H_v-semigroup. If $(G,\cdot)$ is commutative then the $(G',\otimes)$ is strongly commutative.

Let $(H,\cdot)$ be hypergroupoid. We say that *remove* $h\in H$, if we consider the restriction of $(\cdot)$ on $H-\{h\}$. We say $\underline{h}\in H$ *absorbs* $h\in H$ if we replace h by $\underline{h}$. We say $\underline{h}\in H$ *merges* with $h\in H$, if we take as product of $x\in H$ by $\underline{h}$, the union of the results of x with both h, $\underline{h}$ and consider h and $\underline{h}$ as one class.

The ***uniting elements*** method, introduced by Corsini & Vougiouklis [3], is the following: Let G be algebraic structure and let d be a property, which is not valid and it is described by a set of equations; then, consider the partition in G for which it is put together, in the same class, every pair that causes the non-validity of d. The quotient G/d is an H_v-structure. Then, quotient out the G/d by β^*, a stricter structure $(G/d)/\beta^*$ for which the property d is valid, is obtained.

An application of the uniting elements is when more than one property is desired. The following Theorem is valid [3], [34].

Theorem 1.10 Let $(G,\cdot)$ be a groupoid, $F=\{f_1,\dots,f_m,f_{m+1},\dots,f_{m+n}\}$ be system of equations on G consisting of two subsystems $F_m=\{f_1,\dots,f_m\}$, $F_n=\{f_{m+1},\dots,f_{m+n}\}$. Let σ, σ_m the equivalence relations defined by the uniting elements procedure using the systems F and F_m, and let σ_n be the equivalence relation defined using the induced equations of F_n on the grupoid $G_m=(G/\sigma_m)/\beta^*$. Then

$$(G/\sigma)/\beta^*\cong(G_m/\sigma_n)/\beta^*.$$

In the paper [42], there is a first description on how Santilli's theories effect in hyperstructures and how new theories in Mathematics can be appeared by Santilli's pioneer research.

Hyperstructures have applications in mathematics and in other sciences. These applications range from biomathematics -conchology, inheritance- and hadronic physics or on leptons, in the Santilli's iso-theory, to mention but a few. The hyperstructure theory is closely related to fuzzy theory; consequently, can be widely applicable in linguistic, in sociology, in industry and production, too. For these applications the largest class of the hyperstructures, the class *H_v-structures*, is used, they satisfy the *weak axioms* where the non-empty intersection replaces the equality. The main tools of this theory are the fundamental relations which connect, by quotients, the H_v-structures with the corresponding classical ones. These relations are used to define hyperstructures as H_v-fields, H_v-vector spaces and so on, as well. The definition of the general hyperfield was not possible without the H_v-structures and their fundamental relations. *Hypernumbers or H_v-numbers* are called the elements of H_v-fields and they are important for the representation theory [6], [7], [29], [30], [39], [46].

The problem of enumeration and classification of hyperstructures, was started from the beginning, it is complicate in H_v-structures because we have very great numbers. The number of H_v-groups with three elements, up to isomorphism, is 1.026.462. There are 7.926 abelian; the 1.013.598 are cyclic. The partial order in H_v-structures and the Little Theorem, transfers and restrict the problem in finding the minimal, *up to isomorphisms*, H_v-structures.

2. LIE-SANTILLI ADMISSIBILITY IN HYPERSTRUCTURES

The *isofields* needed in the theory of *isotopies* correspond into the hyperstructures were introduced by Santilli & Vougiouklis in 1999 [6], [7], [29] and they are called *e-hyperfields*. The H_v-fields can give e-hyperfields which can be used in the isotopy theory in applications as in physics or biology. We present in the following the main definitions and results restricted in the H_v-structures.

Definitions 2.1 A hyperstructure $(H,\cdot)$ which contain a unique scalar unit e, is called e-hyperstructure. In an e-hyperstructure, we assume that for every element x, there exists an inverse x^{-1}, *i.e.* $e\in x\cdot x^{-1}\cap x^{-1}\cdot x$. Remark that the inverses are not necessarily unique.

A hyperstructure $(F,+,\cdot)$, where $(+)$ is an operation and $(\cdot)$ is a hope, is called *e-hyperfield* if the following axioms are valid:

1. $(F,+)$ is an abelian group with the additive unit 0,

2. $(\cdot)$ is *WASS*,
3. $(\cdot)$ is weak distributive with respect to $(+)$,
4. 0 is absorbing element: $0\cdot x = x\cdot 0 = 0,\ \forall x\in F$,
5. exist a multiplicative scalar unit 1, i.e. $1\cdot x = x\cdot 1 = x,\ \forall x\in F$,
6. for every $x\in F$ there exists a unique inverse x^{-1}, such that $1\in x\cdot x^{-1}\cap x^{-1}\cdot x$.

The elements of an e-hyperfield are called *e-hypernumbers*. If the the relation: $1=x\cdot x^{-1}=x^{-1}\cdot x$, is valid, then we say that we have a *strong e-hyperfield.*

Definition 2.2 [6], [7], [43]. *The Main e-Construction.* Given a group $(G,\cdot)$, where e is the unit, then we define in G, a large number of hopes $(\otimes)$ as follows:

$$x\otimes y = \{xy,\ g_1,\ g_2,\dots\},\ \forall x,y\in G\text{-}\{e\},\ \ g_1,\ g_2,\dots\in G\text{-}\{e\}$$

$g_1,\ g_2,\dots$ are not the same for each pair (x,y). Then $(G,\otimes)$ becomes an H_v-group, because it contains the $(G,\cdot)$. The H_v-group $(G,\otimes)$ is an e-hypergroup. Moreover, if for each x,y such that $xy=e$, so we have $x\otimes y=xy$, then $(G,\otimes)$ becomes a strong e-hypergroup.

Another important new field in hypermathematics comes straightforward from Santilli's Admissibility. We can transfer Santilli's theory in admissibility for representations in two ways: using either, the ordinary matrices and a hope on them, or using hypermatrices and ordinary operations on them [13], [15], [42], [43], [44], [47], [48].

The general definition is the following:

Definition 2.3 Let L be H_v-vector space over the H_v-field $(F,+,\cdot)$, $\varphi:F\to F/\gamma^*$, the canonical map and $\omega_F=\{x\in F:\varphi(x)=0\}$, where 0 is the zero of the fundamental field F/γ^*. Let ω_L be the core of the canonical map $\varphi':L\to L/\varepsilon^*$ and denote by the same symbol 0 the zero of L/ε^*. Take two subsets $R,\ S\subseteq L$ then a ***Lie-Santilli admissible hyperalgebra*** is obtained by taking the Lie bracket, which is a hope:

$$[\ ,\ ]_{RS}: L\times L\to P(L):\ [x,y]_{RS}= xRy-ySx= \{xry-ysx\ /r\in R,\ s\in S\}$$

Special cases, but not degenerate, are the 'small' and 'strict' ones:

(a) When only S is considered, then $[x,y]_S= xy-ySx= \{xy-ysx\ /s\in S\}$

(b) When only R is considered, then $[x,y]_R= xRy-yx= \{xry-yx\ /r\in R\}$

(c) When $R=\{r_1,r_2\}$ and $S=\{s_1,s_2\}$ then

$$[x,y]_{RS}= xRy-ySx= \{xr_1y-ys_1x,\ xr_1y-ys_2x,\ xr_2y-ys_1x,\ xr_2y-ys_2x\}.$$

(d) When $S=\{s_1,s_2\}$ then $[x,y]_S= xy-ySx= \{xy-ys_1x,\ xy-ys_2x\}$.

(e) When $R=\{r_1,r_2\}$ then $[x,y]_R= xRy-yx= \{xr_1y-yx,\ xr_2y-yx\}$.

(f) We have one case which is 'like' P-hope for any subset $S\subseteq L$:

$$[x,y]_S = \{xsy-ysx \,/\, s\in S\}.$$

On non square matrices we can define admissibility, as well:

Construction 2.4 Let $(L=M_{m\times n},+)$ be H_v-vector space of m×n hyper-matrices on the H_v-field $(F,+,\cdot)$, $\varphi:F\to F/\gamma^*$, canonical map and $\omega_F=\{x\in F:\varphi(x)=0\}$, where 0 is the zero of the field F/γ^*. Similarly, let ω_L be the core of $\varphi':L\to L/\varepsilon^*$ and denote by the same symbol 0 the zero of L/ε^*. Take any two subsets $R,S\subseteq L$ then a *Santilli's Lie-admissible hyperalgebra* is obtained by taking the Lie bracket, which is a hope:

$$[\ ,\ ]_{RS}: L\times L\to P(L):\ [x,y]_{RS}=xR^ty-yS^tx.$$

Notice that $[x,y]_{RS}=xR^ty-yS^tx=\{xr^ty-ys^tx \,/ r\in R$ and $s\in S\}$.

Special cases, but not degenerate, are the 'small' and 'strict' ones:

(a) $R=\{e\}$ then $[x,y]_{RS} = xy-yS^tx = \{xy-ys^tx \,/ s\in S\}$

(b) $S=\{e\}$ then $[x,y]_{RS} = xR^ty-yx = \{xr^ty-yx \,/ r\in R\}$

(c) $R=\{r_1,r_2\}$ and $S=\{s_1,s_2\}$ then

$$[x,y]_{RS}=xR^ty-yS^tx=\{xr_1^ty-ys_1^tx, xr_1^ty-ys_2^tx, xr_2^ty-ys_1^tx, xr_2^ty-ys_2^tx\}$$

According to Santilli's iso-theory [9], [11], [13], [15], [22], [24], [25], [26], [27], [28], [39], [42], [46], [50], on a field $F=(F,+,\cdot)$, *a general isofield* $\hat{F}=\hat{F}(\hat{a},\hat{+},\hat{\times})$ is defined to be a field with elements $\hat{a}=a\times\hat{1}$, called *isonumbers*, where $a\in F$, and $\hat{1}$ is a positive-defined element generally outside F, equipped with two operations $\hat{+}$ and $\hat{\times}$ where $\hat{+}$ is the sum with the conventional additive unit 0, and $\hat{\times}$ is a new multiplication

$$\hat{a}\,\hat{\times}\,\hat{b} := \hat{a}\times\hat{T}\times\hat{b},\quad \text{with } \hat{1}=\hat{T}^{-1},\ \ \forall\hat{a},\hat{b}\in\hat{F} \qquad \text{(i)}$$

called *iso-multiplication*, for which $\hat{1}$ is the left and right unit of F,

$$\hat{1}\,\hat{\times}\,\hat{a} = \hat{a}\times\hat{1} = \hat{a},\ \forall\hat{a}\in\hat{F} \qquad \text{(ii)}$$

called *iso-unit*. The rest properties of a field are reformulated analogously.

In order to transfer this theory into the hyperstructure case we generalize only the new multiplication $\hat{\times}$ from (i), by replacing with a hope including the old one. We introduce two general constructions on this direction as follows:

Construction 2.5 *General enlargement.* On a field $F=(F,+,\cdot)$ and on the isofield $\hat{F}=\hat{F}(\hat{a},\hat{+},\hat{\times})$ we replace in the results of the iso-product

$$\hat{a}\,\hat{\times}\,\hat{b}=\ \hat{a}\times\hat{T}\times\hat{b},\quad \text{with } \hat{1}=\hat{T}^{-1}$$

of the element $\hat{T}$ by a set of elements $\hat{H}_{ab}=\{\hat{T},\hat{x}_1,\hat{x}_2,\ldots\}$ where $\hat{x}_1,\hat{x}_2,\ldots\in\hat{F}$, containing $\hat{T}$, for all $\hat{a}\,\hat{\times}\,\hat{b}$ for which $\hat{a},\hat{b}\notin\{\hat{0},\hat{1}\}$ and $\hat{x}_1,\hat{x}_2,\ldots\in\hat{F}\text{-}\{\hat{0},\hat{1}\}$. If one of $\hat{a}$, $\hat{b}$, or both, is equal to $\hat{0}$ or $\hat{1}$, then $\hat{H}_{ab}=\{\hat{T}\}$. Thus, the new iso-hope is

$$\hat{a}\,\hat{\times}\,\hat{b}=\hat{a}\times\hat{H}_{ab}\times\hat{b}=\hat{a}\times\{\hat{T},\hat{x}_1,\hat{x}_2,\ldots\}\times\hat{b},\ \forall\hat{a},\hat{b}\in\hat{F}\qquad\text{(iii)}$$

$\hat{F}=\hat{F}(\hat{a},\hat{+},\hat{\times})$ becomes *isoH_v-field.* The elements of $\hat{F}$ are called *isoH_v-numbers* or *isonumbers*.

Remarks 2.6 More important hopes, of the above construction, are the ones where only for few ordered pairs $(\hat{a},\hat{b})$ the result is enlarged, even more, the extra elements $\hat{x}_i$, are only few, preferable exactly one. Thus, this special case is if there exists only one pair $(\hat{a},\hat{b})$ for which

$$\hat{a}\,\hat{\times}\,\hat{b}=\ \hat{a}\times\{\hat{T},\hat{x}\}\times\hat{b},\quad \forall\hat{a},\hat{b}\in\hat{F}$$

and the rest are ordinary results, then we have a hyperstructure called *very thin isoH_v-field.*

The assumption that $\hat{H}_{ab}=\{\hat{T},\hat{x}_1,\hat{x}_2,\ldots\}$, $\hat{a}$ or $\hat{b}$, is equal to $\hat{0}$ or $\hat{1}$, with that $\hat{x}_i$, are not $\hat{0}$ or $\hat{1}$, give that the isoH_v-field has one scalar absorbing $\hat{0}$, one scalar $\hat{1}$, and $\forall\hat{a}\in\hat{F}$, has one inverse.

Construction 2.7 *The P-hope.* Consider an isofield $\hat{F}=\hat{F}(\hat{a},\hat{+},\hat{\times})$ with $\hat{a}=a\times\hat{1}$, the isonumbers, where $a\in F$, and $\hat{1}$ is a positive-defined element generally outside F, with two operations $\hat{+}$ and $\hat{\times}$, where $\hat{+}$ is the sum with the conventional unit 0, and $\hat{\times}$ is the iso-multiplication

$$\hat{a}\,\hat{\times}\,\hat{b}:=\ \hat{a}\times\hat{T}\times\hat{b},\quad \text{with } \hat{1}=\hat{T}^{-1},\ \forall\hat{a},\hat{b}\in\hat{F}.$$

Take a set $\hat{P}=\{\hat{T},\hat{p}_1,\ldots,\hat{p}_s\}$, with $\hat{p}_1,\ldots,\hat{p}_s\in\hat{F}\text{-}\{\hat{0},\hat{1}\}$, define the ***isoP-$H_v$-field***, $\hat{F}=\hat{F}(\hat{a},\hat{+},\hat{\times}_P)$, where the hope $\hat{\times}_P$ as follows:

$$\hat{a}\,\hat{\times}_P\,\hat{b}:=\begin{cases}\hat{a}\times\hat{P}^\wedge\times\hat{b}=\{\hat{a}\times\hat{h}^\wedge\times\hat{b}\,/\,\hat{h}^\wedge\in\hat{P}^\wedge\} & \text{if } \hat{a}\neq\hat{1} \text{ and } \hat{b}\neq\hat{1}\\ \hat{a}\times\hat{T}^\wedge\times\hat{b} & \text{if } \hat{a}=\hat{1} \text{ or } \hat{b}=\hat{1}\end{cases}\qquad\text{(iv)}$$

The elements of $\hat{F}$ are called *isoP-H_v-numbers*.

Remark. If $\hat{P} = \{\hat{T},\hat{p}\}$, that is that $\hat{P}$ contains only one $\hat{p}$ except $\hat{T}$. The inverses in isoP-H_v-fields, are not necessarily unique.

Example 2.8 In order to define a generalized P-hope on $\hat{Z}_7 = \hat{Z}_7(\underline{\hat{a}},\hat{+},\hat{\times})$, where we take $\hat{P}=\{\underline{\hat{1}},\underline{\hat{5}}\}$, the weak associative multiplicative hope is described by the table:

$\hat{\times}$	$\underline{\hat{0}}$	$\underline{\hat{1}}$	$\underline{\hat{2}}$	$\underline{\hat{3}}$	$\underline{\hat{4}}$	$\underline{\hat{5}}$	$\underline{\hat{6}}$
$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$
$\underline{\hat{1}}$	$\underline{\hat{0}}$	$\underline{\hat{1}}$	$\underline{\hat{2}}$	$\underline{\hat{3}}$	$\underline{\hat{4}}$	$\underline{\hat{5}}$	$\underline{\hat{6}}$
$\underline{\hat{2}}$	$\underline{\hat{0}}$	$\underline{\hat{2}}$	$\underline{\hat{4},\hat{6}}$	$\underline{\hat{6},\hat{2}}$	$\underline{\hat{1},\hat{5}}$	$\underline{\hat{3},\hat{1}}$	$\underline{\hat{5},\hat{3}}$
$\underline{\hat{3}}$	$\underline{\hat{0}}$	$\underline{\hat{3}}$	$\underline{\hat{6},\hat{2}}$	$\underline{\hat{2},\hat{3}}$	$\underline{\hat{5},\hat{4}}$	$\underline{\hat{1},\hat{5}}$	$\underline{\hat{4},\hat{6}}$
$\underline{\hat{4}}$	$\underline{\hat{0}}$	$\underline{\hat{4}}$	$\underline{\hat{1},\hat{5}}$	$\underline{\hat{5},\hat{4}}$	$\underline{\hat{2},\hat{3}}$	$\underline{\hat{6},\hat{2}}$	$\underline{\hat{3},\hat{1}}$
$\underline{\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{5}}$	$\underline{\hat{3},\hat{1}}$	$\underline{\hat{1},\hat{5}}$	$\underline{\hat{6},\hat{2}}$	$\underline{\hat{4},\hat{6}}$	$\underline{\hat{2},\hat{3}}$
$\underline{\hat{6}}$	$\underline{\hat{0}}$	$\underline{\hat{6}}$	$\underline{\hat{5},\hat{3}}$	$\underline{\hat{4},\hat{6}}$	$\underline{\hat{3},\hat{1}}$	$\underline{\hat{2},\hat{3}}$	$\underline{\hat{1},\hat{5}}$

The hyperstructure $\hat{Z}_7= Z_7(\underline{\hat{a}},\hat{+},\hat{\times})$, is commutative and associative on the multiplication hope.

Consruction 2.9 The generalized P-construction can be applied on rings to obtain H_v-fields. Thus for, $\hat{Z}_{10} = Z_{10}(\underline{\hat{a}},\hat{+},\hat{\times})$, and if we take $\hat{P}=\{\underline{\hat{2}},\underline{\hat{7}}\}$, then we have the table

$\hat{\times}$	$\underline{\hat{0}}$	$\underline{\hat{1}}$	$\underline{\hat{2}}$	$\underline{\hat{3}}$	$\underline{\hat{4}}$	$\underline{\hat{5}}$	$\underline{\hat{6}}$	$\underline{\hat{7}}$	$\underline{\hat{8}}$	$\underline{\hat{9}}$
$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$
$\underline{\hat{1}}$	$\underline{\hat{0}}$	$\underline{\hat{1}}$	$\underline{\hat{2}}$	$\underline{\hat{3}}$	$\underline{\hat{4}}$	$\underline{\hat{5}}$	$\underline{\hat{6}}$	$\underline{\hat{7}}$	$\underline{\hat{8}}$	$\underline{\hat{9}}$
$\underline{\hat{2}}$	$\underline{\hat{0}}$	$\underline{\hat{2}}$	$\underline{\hat{8}}$	$\underline{\hat{2}}$	$\underline{\hat{6}}$	$\underline{\hat{0}}$	$\underline{\hat{4}}$	$\underline{\hat{8}}$	$\underline{\hat{2}}$	$\underline{\hat{6}}$
$\underline{\hat{3}}$	$\underline{\hat{0}}$	$\underline{\hat{3}}$	$\underline{\hat{2}}$	$\underline{\hat{3},\hat{8}}$	$\underline{\hat{4}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{6}}$	$\underline{\hat{2},\hat{7}}$	$\underline{\hat{8}}$	$\underline{\hat{4},\hat{9}}$
$\underline{\hat{4}}$	$\underline{\hat{0}}$	$\underline{\hat{4}}$	$\underline{\hat{6}}$	$\underline{\hat{4}}$	$\underline{\hat{2}}$	$\underline{\hat{0}}$	$\underline{\hat{8}}$	$\underline{\hat{6}}$	$\underline{\hat{4}}$	$\underline{\hat{2}}$
$\underline{\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{0},\hat{5}}$
$\underline{\hat{6}}$	$\underline{\hat{0}}$	$\underline{\hat{6}}$	$\underline{\hat{4}}$	$\underline{\hat{6}}$	$\underline{\hat{8}}$	$\underline{\hat{0}}$	$\underline{\hat{2}}$	$\underline{\hat{4}}$	$\underline{\hat{6}}$	$\underline{\hat{8}}$
$\underline{\hat{7}}$	$\underline{\hat{0}}$	$\underline{\hat{7}}$	$\underline{\hat{8}}$	$\underline{\hat{2},\hat{7}}$	$\underline{\hat{6}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{4}}$	$\underline{\hat{3},\hat{8}}$	$\underline{\hat{2}}$	$\underline{\hat{1},\hat{6}}$
$\underline{\hat{8}}$	$\underline{\hat{0}}$	$\underline{\hat{8}}$	$\underline{\hat{2}}$	$\underline{\hat{8}}$	$\underline{\hat{4}}$	$\underline{\hat{0}}$	$\underline{\hat{6}}$	$\underline{\hat{2}}$	$\underline{\hat{8}}$	$\underline{\hat{4}}$
$\underline{\hat{9}}$	$\underline{\hat{0}}$	$\underline{\hat{9}}$	$\underline{\hat{6}}$	$\underline{\hat{4},\hat{9}}$	$\underline{\hat{2}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{8}}$	$\underline{\hat{1},\hat{6}}$	$\underline{\hat{4}}$	$\underline{\hat{2},\hat{7}}$

Then the fundamental classes are

$(0)=\{\underline{\hat{0}},\underline{\hat{5}}\}$, $(1)=\{\underline{\hat{1}},\underline{\hat{6}}\}$, $(2)=\{\underline{\hat{2}},\underline{\hat{7}}\}$, $(3)=\{\underline{\hat{3}},\underline{\hat{8}}\}$, $(4)=\{\underline{\hat{4}},\underline{\hat{9}}\}$,

and the multiplicative table is the following

×	**(0)**	**(1)**	**(2)**	**(3)**	**(4)**
(0)	(0)	(0)	(0)	(0)	(0)
(1)	(0)	(1),(2)	(2),(4)	(3),(1)	(4),(3)
(2)	(0)	(2),(4)	(3)	(2)	(1)
(3)	(0)	(3),(1)	(2)	(3)	(4)
(4)	(0)	(4),(3)	(1)	(4)	(2)

Consequently, $\hat{Z}_{10} = Z_{10}(\underline{\hat{a}}, \hat{+}, \hat{\times})$, is an H_v-field.

3. APPLICATIONS TO A NEW CONCEPTION OF LIVING ORGANISMS

3.1 Einstein's argument that 'quantum mechanics is not a complete theory.'

As it is well know, Einstein accepted the validity of quantum mechanics for the representation of the atomic structure and other systems, but never accepted quantum mechanics as being a final theory capable of representing all possible elements of reality.

For this reason, Einstein expressed the view in 1935, jointly with his students Boris Podolsky and Nathan Rosen, that *'Quantum mechanics is not a complete theory'* (EPR argument) [10], in the sense that quantum mechanics (and we add nowadays quantum chemistry) could admit suitable enlargements for the representations of more complex systems.

Additionally, Einstein never accepted the uncertainties of quantum mechanics as being final in the sense that they are indeed valid for point-particles in vacuum but there could exist conditions in the universe recovering classical determinism. For this reason, Einstein's made his famous quote: *'God does not play dice with the universe.'*

3.2 Verification of Einstein's legacy by irreversible systems.

The most evident illustration if the validity of the lack of 'completeness' of quantum mechanics (and, therefore, of quantum chemistry) is given by the fact that *quantum mechanics and chemistry can only represent systems of point-like particles that are invariant under time-reversal,* such as the atomic structure. This is due to the invariance under anti-Hermiticity of the quantum mechanical Lie product between Hermitean operators

$$[A,B] = AB - BA = -[A,B]^{\dagger},$$

where AB is the conventional classical associative product. In fact, the Lie product characterizes Heisenberg's time evolution of an observable A in terms of the Hamiltonian H,

$$idA/dt = [A,H] = AH - HA.$$

However, physical, chemical and biological processes such as nuclear fusion, combustion and living organisms, are irreversible over time.

The verification of Einstein's legacy via irreversible processes was first identified by R. M. Santilli during his Ph. D. Studies at the University of Torino, Italy, in the mid 1960s.

In fact, Santilli's Ph.D. thesis, published in the 1967 paper [13], provided the first known confirmation of the EPR argument (see also Ref. [13] of 1968) via the following Lie-admissible 'completion' of quantum mechanical Lie algebras for the representation of irreversible processes

$$(A,B) = ARB-BSA = (ATB-BTA)+(AJB+BJA), \ \ R=T-J, \ S=T+J \neq 0,$$

where the new product (A,B) is Lie-admissible according to A.A. Albert [1] when the attached antisymmetric product

$$[A,B]^{*} = (A,B) - (B,A) = ATB - BTA$$

verifies the Lie axioms whenever T is nowhere singular. Also, according to Albert [1] the product (A,B) is called *Jordan-admissible* when the attached symmetric product

$$\{A,B\}^{*} = (A,B) + (B,A) = AJB + BJA$$

verifies the axioms of Jordan algebras.

Santilli called *hadronic mechanics* [19] and *hadronic chemistry* [22] the 'completion' of quantum mechanics and chemistry, respectively, with a Lie-admissible structure for the representation of irreversible structures and processes.

3.3 Lie-admissible genomathematics

The mathematics underlying Lie-admissible formulations, collectively known as *genomathematics* [16], [17], [20], [23], can be summarized as follows. A generally tacit assumption of conventional, classical, numeric fields underlying Lie's theory is that *the multiplication of two numbers to the right n-->3 is equal to the multiplication of the same numbers to the left, 2<--3 = 2-->3.* Consequently, the indicated order of the multiplication is ignored in classical number theory, and we merely write $2\times3 = 6$.

In the transition from Lie theory to the covering Lie-admissible theory, the above ordering of the multiplication is no longer ignorable because the multiplication to the right $2>3 = 2S3$ is no longer equal to the multiplication to the left $2<3 = 2R3 \neq 2>3$. This occurrence has permitted the identification of two, classical, numeric fields underlying Lie-admissible formulations [16]:

1) The *forward genofields* $F^>(n^>,>,I^>)$ with *forward genounit* $I^> = 1/S$, *forward genonumbers* $n^> = nI^>$, and *forward genoproduct* $n^> > m^> = n^> S m^>$, where n, m represent ordinary numbers; and

2) The *backward genofields* $F^<(n^<,<,I^<)$ with *backward genounit* $I^< = 1/R$, *backward genonumbers* $n^< = I^< n$, and *backward genoproduct* $n^< < m^< = n^< R m^<$.

Recall that Lie algebras can be constructed via the universal enveloping associative algebras ξ with classical, associative, modular product AB. The indicated inequivalence of the multiplications to the right and to the left implies the existence for Lie-admissible theories of *two universal, enveloping, genoassociative genoalgebars,* that to the right $\xi^>$ (left $\xi^<$) with genoassociative genoproduct to the right $A>B$ (left $A<B$), resulting in a non-trivial bimodular formulation.

The indicated bimodular formulations characterize the *time-irreversible, Lie-admissible, Heisenberg-Santilli genoequation* [13], [19]

$$idA/dt = (A, H) = ARH - HSA = A < H - H > A.$$

Recall that, in quantum mechanics, the modular associative multiplication to the right of an operator H to a Hilbert stat, $H\psi(t,r) = E\psi(t,r)$ yields the same eigenvalues E for the modular associative multiplication to the left $\psi(t,r)H = \psi(t,r)E$.

The Lie-admissible 'completion' of the above Schrödinger's equation yields the non-trivial bimodular structure:

1) The modular genoassociative action to the right representing the time evolution forward in time via the *Schrödinger-Santilli genoequation to the right* [19]

$$H(r,p) > \psi^>(t^>,r^>) = H(r,p)S(\psi^>,\ldots)\psi(t,r) = E^> \psi^>(t^>,r^>)^>,$$

and

2) The modular genoassociative action to the left representing motion backward in time via *Schrödinger-Santilli genoequation to the left*

$$\psi^<(t^<,r^<) < H(r,p) = \psi^< R(\psi^<,\ldots)H(r,p) = \psi^<(t^<,r^<)E^<$$

where $E^> \neq E^<$.

The representation of irreversible processes from first axiomatic principles is then evident whenever $R \neq S$.

3.4 Verifications of Einstein's legacy.

Following the above mathematical studies, Santilli dedicated decades to experimental and industrial verifications of hadronic mechanics and chemistry. Following the achieved of such a mathematical and applied maturity, Santilli proved Einstein's legacy that 'quantum mechanics is not a complete theory' as well as the progressive recovering of Einstein's determinism in the interior of hadrons, nuclei and stars and its full recovering in the interior of gravitational collapse [21], [25], [26], [27], [28]. These results were achieved via the representation of the *extended and overlapping character of the constituents of irreversible systems in terms of the forward genotopic element* with realizations of the type

$$\hat{T} = \Pi_{k=1,...,N} Diag.(1/n_{1k}^2,\ 1/n_{2k}^2,\ 1/n_{3k}^2,\ 1/n_{4k}^2)e^{-\Gamma(\psi,\partial\psi,...)},$$

where n_{1k}^2, n_{2k}^2, n_{3k}^2, (called *characteristic quantities*) represent the deformable semi-axes of the k-particle normalized to the values $n_{\mu k}^2=1$, $\mu=1, 2, 3$ for the sphere; n_{4k}^2 represents the *density* of the k-particle considered normalized to the value $n_{4k} = 1$ for the vacuum; and $\Gamma(\psi,\partial\psi)$ represents non-linear, non-local and non-potential interactions caused by mutual overlapping/entanglement of the particles considered.

The aspects of studies [21], [25], [26], [27], [28] important for this paper are the following. Recall that particles originally in conditions of mutual overlapping/entanglement of their wave packets and then separated, have been experimentally proved to *instantly influence each other at a distance*, by therefore requiring superluminal communications that would violate special relativity. This is the very feature that prompted Einstein the argument that 'quantum mechanics is not a complete theory.' Santilli has achieved a quantitative representation of the indicated instantaneous communication at a distance via the representation of the extended character of the wavepacket of particles resulting in their continuous mutual penetration/entanglement at a distance of their center of mass, by therefore eliminating the need for superluminal communications. Above all, studies [21], [25], [26], [27], [28] have established that *the instantaneous communication of entangle particles at a distance occurs without any use of energy because the interaction are not derivable from a potential by basic assumptions.*

3.5 Application to a new conception of living organisms.

Note that all biological structures, including cells, viruses and large living organisms, are irreversible over time because they are born, grow and then die. Santilli introduced in monograph [18] of 1994 the representation of biological structures via *classical, multivalued, Lie-admissible formulations on a 3-dimensional Euclidean space,* namely, Lie-admissible formulations characterized by genounits, called *classical hyperunits*, with an ordered number of values all defined in the Euclidean space of our sensory perception

$$I^{>} = (I_1, I_2, ..., I_n) = 1/T = (1/T_1,\ 1/T_2, ...1/T_n) = 1/S.$$

Correspondently, the product of generic non-singular quantities a, b (such as numbers, functions, matrices, etc.), called *classical hyperproducts*, are equally multivalued, yet defined in our 3-dimensional Euclidean space

$$a > b = aSb = aT_1b + aT_2b + ... + aT_nb$$

in which all individual products are classical.

Correspondently, Ref. [18] introduced the notion of *classical hyperfields*, namely, sets of multi-valued elements, products and units which verify the axioms of numeric fields.

The transition from the classical, single-valued, Lie-admissible formulations outline in Section 3.3 to their multi-valued extension of was indicated in Ref. [18] as being necessary for the representation of the complexity of biological structures.

Note the fundamental character of the classical hyperunits and related hyperfields because the entire new formalism, including classical hyperalgebras, hyperspaces and hypertopology, are constructed via mere compatibility arguments with the base classical hyperfield.

In 1995, the Australian conchologist Chris Illert (see Part I of Ref. [12]) showed via computer simulations and direct calculations that the *growth of seashells over time cannot be consistently represented in a classical, 3-dimensional, single-valued Euclidean space $E(r,\delta,1)$* with classical coordinates $r=(x,y,z)$ metric $\delta=Diag.(1,1,1)$ and unit 1 over the field of real numbers $(R,n,\times,1)$, because, in said space, seashell grow irregularly and then crack. Illert then showed a consistent representation of seashell growth via the use of a *3-dimensional, two-valued Euclidean space* $(\hat{E},\ \hat{r},\ \delta,\ 1)$ where

$$\hat{r} = \{(x_1,x_2),\ (y_1,y_2),\ (z_1,z_2)\}$$

Santilli (see Part II of Ref. [18]) indicated that Illert's discovery confirms the need for hyperstructures in the representation of living organisms. In fact, the representation space used by Illert can be more accurately written as a *classical, 3-dimensional, two-valued, forward hyperspace* $(E^>, r^>, \delta^>, I^>)$, over the forward, classical hyperfield $(R^>, n^>, >, I^>)$ with classical forward hyperunit

$$I^> = \{(I_{1x}, I_{2x}), (I_{1y}, I_{2y}), (I_{1z}, I_{2z})\} = 1/T^> =$$
$$= \{(1/T_{1x}, 1/T_{2x}), (1/T_{1y}, 1/T_{2y}), (1/T_{1z}, 1/T_{2z})\}$$

and classical, 3-dimensional but two-valued products between arbitrary quantities *a, b* [18]

$$a > b = aT^>b = (a_xT_{1x}b_x + a_xT_{2x}b_x) + (a_yT_{1y}b_y + a_yT_{2y}b_y) + (a_zT_{1z}b_z + a_zT_{2z}b_z),$$

The Lie-admissible character of the representation and, therefore, its irreversibility, are assured when the backward hyperunit and, therefore, the hyperproducts, are different than the corresponding backward values.

A central notion of the above classical 2-valued, hyperstructural representation of seashells growth is the 3-dimensional character of the representation space, which is independent from the multi-valued character of each axis. Such a structure is necessary, on one side, to achieve compatibility of the mathematical representation with our sensory perceptions, while at the same time allowing an unlimited number if hidden degrees of freedom needed for a quantitative representation of the complexity of seashells. In fact, we inspect seashell growth with our three Eustachian tubes. Consequently, any *multi-dimensional representation, such as the use of a 6-dimensional space, would not be compatible with our sensory perception and, as such, not being experimentally verifiable.*

A major advance in the hyperstructural representation if biological structures was initiated by T. Vougiouklis in 1999 [39] with the lifting of the classical hyperstructures of 5 Ref. [18] to Vougiouklis H_v-structures (see also Ref. [42] and subsequent papers) which are formulated via hyperoperations (nicknamed 'hopes') including weak associativity (nicknamed '*WASS*'), weak commutativity (nicknamed '*COW*') and other hyperoperations.

The advantages of lifting the classical hyperstructures of Ref. [18] to Vougiouklis H_v-structures are several. The first advantage is a large increase of the representational capabilities which is necessary for a representation of biological structures such as the DNA, via a formulation that, at the abstract realization-free level, is compatible with the three-dimensional space of our sensory perception.

Other advantages are due to rather unique capabilities by Vougiouklis H_v-structures to characterize *bona fide* hyperfields on which the rest of the Lie-admissible formulation is expected to be built (see, e.g., Ref. [46]).

In this paper we introduce, apparently for the first time, a new conception of living organisms permitted by verifications [21], [25], [26], [27], [28] of the EPR argument according to which *a living organism, such as a cell, a virus or a human person, is composed by a very large number of extended constituents in conditions of continuous mutual entanglement of their wavepackets and, therefore, in continuous mutual communications.*

In view of the complexity and very large number of multi-valued internal communications, the best representation of the above conception of living organisms known to the authors, is given by two, hyperbimodular, Lie-admissible, H_v-structures, one for the representation of growth in time via hope, WASS and COW for ordered hypermodular hope to the right, and a second for the representation backward in time with hope, WASS and COW for ordered hypermodular hope to the left.

3.6 A specific hyperstructure formalism of living organisms.

As we present in section 3.3, in the transition from Lie theory to the covering Lie-admissible theory, we must specify an element S on the right and an element R to the left. In hyperstructure realization we can use as S and R, sets instead of elements. But in this case, we have hopes of constant length and the living organisms are not the case. Therefore, we suggest the use of a special case of the main e-construction to face the problem. Our construction equips the main product with an e-hope where the hyperproduct of two elements depend of those two elements. In fact, we keep the product and enlarge all the appropriate results.

Construction 3.1 *The Living Organism Construction.* In a set G equipped with several operations we take one product $(\cdot)$, where $(G,\cdot)$ is a group. Suppose that e is the unit, then we define in G, a large number of hopes $(\otimes)$ as follows:

$$e\otimes x = x\otimes e = x, \quad \forall x\in G,$$

$$x\otimes y = \{xy, g_{xy1}, g_{xy2},\ldots\}, \quad \forall x,y\in G-\{e\}, \text{ where } g_{xy1}, g_{xy1},\ldots \in G-\{e\}$$

$g_{xy1}, g_{xy2},\ldots$ depend on the pair (x,y). Then $(G,\otimes)$ becomes an H_v-group, because it contains the $(G,\cdot)$. The H_v-group $(G,\otimes)$ is an e-hypergroup. Moreover, if for each x,y such that $xy=e$, so we have $x\otimes y=xy$, then $(G,\otimes)$ becomes a strong e-hypergroup.

Remarks. **1.** In the H_v-group $(G,\otimes)$ the hope $(\otimes)$ is *WASS* and if the $(G,\cdot)$ is commutative, then the hope $(\otimes)$ is *COW.*
2. The Living Organism Construction can be used as S or R in forward genofields or backward genofields, respectively, according to section 3.3.

Recall that, according to the Schrödinger equation of quantum chemistry, living organisms are composed by a collection of isolated points. By contrast, according to the Schrödienger-Santilli genoequation of hadronic chemistry, living organisms are composed by the indicated large number of extended constituents in conditions of continuous entanglement and communication.

It is hoped that the proposed new conception of living organisms may allow new diagnostics, e.g., via the identification of possible miscommunications between different constituents, as well as new treatments, e.g., via the disruption of selected communications.

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ENTANGLEMENT IS REAL IN 3-D 'GAME OF LIE' STRAIGHT LINE GEOMETRIC ALGEBRA CELLULAR AUTOMATON

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Abstract

The revolutionary tenet in Marius Sophus Lie's 1871 Norwegian Ph.D. dissertation *Over en Classe Geometriske Transformationer* - and nowhere else or since so clearly expressed - is that while "Descartes…has chosen the point as the element of the geometry of the plane", its "geometrical transformation…can be perceived as consisting of a transition from a point to a straight line as element", and more particularly "the straight line of length equal to zero". In that and its exploration he stands forth in the history of mathematics as the true founder of linear algebra (together with Grassmann), differential equations, continuous transformation groups, spherical geometry, and indeed the standard model by Gell-Mann's per se mistaken Lie supermultiplet adoption; and thus reaching over all dimensions and numerical systems, fundamentally including the real. This is the *terra firma* of my back to the future return to Lie and the ordinary three-dimensional Euclidean space in Cartesian extension that he primarily inhabited from nil by spanning there a tangential "line-complex" system instead of a swarm of particle points. The resulting straight "curve-net" is the infinitesimal generator realization of the instantaneous phase transition "between the Plücker line-geometry and a geometry whose elements are the space's spheres" in the form of a universally extending isotropic vector matrix (IVM) lattice embedded and distributed in chaperoning Cartesian coordinate cages filling space by hierarchical piling of the direct structural hybrid $R^3 \times SO(3)$ wave-packets so constituted, and which by cellular automaton iterations in whole or parts exactly replicate the elementary particle, atomic and periodic table spectroscopy. Moreover, since the $R^3 \times SO(3)$ curve-net is timeless, all lines there, e.g. those occupied by coherently superposed photon pairs, are everlasting, hence solving all dilemmas and paradoxes of entanglement.

Introduction

Initially highly debated as such and analytically[1-3], entanglement with its from Schrödinger's *Verschrenkung*[3] imported bearings also on budding, branching, catenating, gluing etc., has over the years stood the tests to its now generally acknowledged designation as a special kind of nature's way of compounding physical reality and realization from the ground and onwards, namely, superposition. Elementary particles and waves, atoms, the periodic table, molecules and their combinations all submit to this fundamental both classical and quantum mechanical principle of joining together in matching constellations into larger states. These are then direct sum congregations of their irreducible ingredients and in subtraction thus release them in their accommodated postures so that, nothing else interfering, they still appear matched when measured along different paths after the separation.

But entangled states are coherent superpositions where each as a rule pair occupies one geodesics where it avoids forbidden collisions either by intertwining or more often diverging, with the counter-intuitive consequence of interdependence/identity of synchronized measurements between them over distances of any length; either according to the Einstein-Podolsky-Rosen (EPR) argument[1] by some shared internal mathematical algorithm ensuring equal evolution of the two, or according to the Copenhagen school[2] as a primary constitution of unknown kind.

As first resolved in the thought experiment Bell theorem in 1964[4], and 1999 in a laboratory replication by Aspect[5], and in both tested by a real world outcome, the evidence as accumulating in a legion of further investigations more and more favors the latter case. This has inspired a veritable panorama, not to say Pandora box of freely extrapolated macroworld realizations and applications which have reached cosmological dimensions with time travels, teleportation, wormholes, branes, multiverses and "connected spacetimes by entangling"[6] as almost everyday routine events, albeit so far totally unseen in practice. Yet, massive funding is directed to this novel utopia which of course has immense appeal and at least engenders a cornucopia of innovative mathematical, computer and animation techniques, theoretical insights and philosophical, epistemological, ontological and information science developments.

However, the alternative that there may be a an almost trivial, quite mundane combination of local gearing and remote coherence has not been investigated before but is an inherent feature of the three-dimensional space-filling lattice model in close compliance with Marius Sophus Lie's *geometriske transformationer*[7,8] here reported with special entanglement reference.

Methods and Results

Marius Sophus Lie's groundbreaking Ph.D. thesis *Over en classe geometriske transformationer*[1b.] from 1871 (and thus due for a most deserved 150-year anniversary) was in fact not well understood in the defence[9], and soon went down to oblivion in the faculty archives. When a hundred years later I as a budding cardiologist (see https://www.ncbi.nlm.nih.gov/pubmed/?term=trell+e) in search of clues on rotational transformations for electrocardiographic (ECG) applications obtained a photocopy of it from the Oslo University library I soon realized what a treasure it was; not directly to ECG of course but the very roots of linear algebra, differential equations and both the philosophical and structural "nature of Cartesian geometry".[7,8] With pivotal help and support from the inventor of Lie-admissible algebra[10,11], Professor Ruggero Maria Santilli, I made an English translation of it (now open access available)[8], expressing its crucial tenet that while "Descartes...has chosen the point as the element of the geometry of the plane", its "geometrical transformation...can be perceived as consisting of a transition from a point to a straight line as element", and more particularly "the straight line of length equal to zero".[7,8] Equally small as an infinitesimal point the difference is that as a linkable line it is in effect a partial derivative building unit filling both its space and path by its coherent steps as a virtual cellular automaton[12] (Figure 1) so as to crystalize a real form $R^3 \times SO(3)$

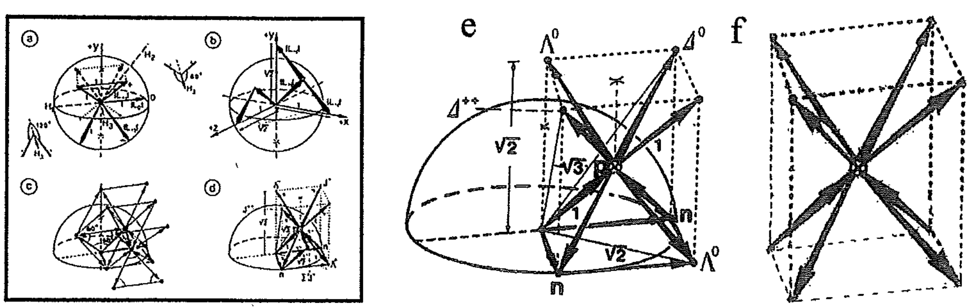


Figure 1. a-d) Killing A_3 root space diagram knits a hexagonal SO(3) infinitesimal generator lattice, e,f) globally distributed in space-filling parallelepiped R^3 chaperon enclosures to form a real space cellular automaton which in repeated steps of itself in parts or whole carries out a structural replication of the elementary particle, atomic and periodic table spectroscopies.

wave-packet compound which can structurally rewrite the elementary particle[13-21], atomic and periodic table spectroscopies. Previous reports on the latter[22-28] will here be complemented with the completion of the system with special emphasis on entanglement both in a classical and quantum mechanical classification, generator and computation regard.

Figure 2 summarizes how the charged *t* isospin root vectors can be linked to a coherent triple helix in a continuous cloverleaf 'singlet coil' sequence of 12 unit steps – equally many as the edges of the parallelepiped – into the only other space-filling regular solid convolution, namely, a complex of one octahedron and two tetrahedrons (Figure 2a), which in iteration (Figure 2b) forms an important nanotechnology structure, i.e. an ordinary space frame, alias

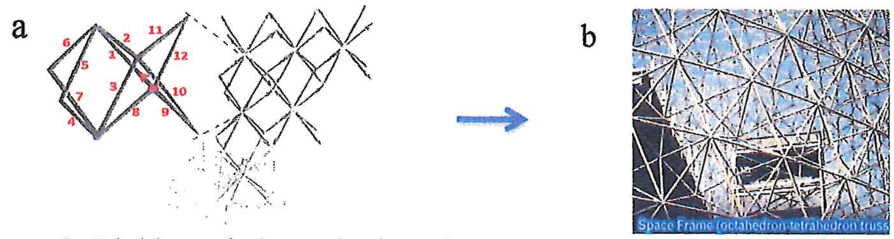


Figure 2. Linking of charged *t* isospin root vectors to an outwards connected lattice, iterating to space frame of isotropic vector matrix (IVM) constitution.

octahedron-tetrahedron, or octet truss[29] whose modular building blocks and their superposition can thus be realized from the ground of material organization. Figure 3 shows how the per se space-filling universal IVM lattice

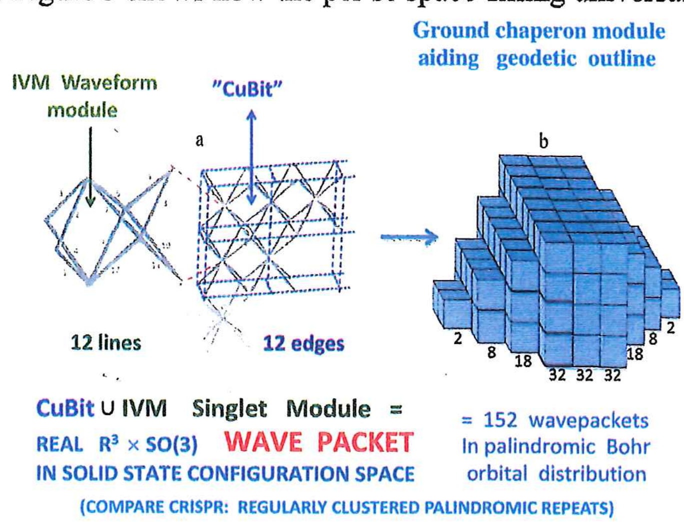


Figure 3. 2+8+18+32+32+32+18+8+2 = 152-step coherent chaperon sequence into a trapezoid bottom module of palindromic Bohr orbital distribution whose inner continuous 152×12-step SO(3) wave filament outlines the electron

is continuously distributed in Euclidean space by chaperoning R^3 encasements which in the first superposition period form a sideways tileable flat-bottomed neutron ground module which for vertical packing needs a reciprocal flat-roofed cap (Figure 4) so that each half of the combined 'transition apparatus' comprises 153×12 unit steps = 1836 = proton/electron relative inertial ratio. There is one square and one diagonal alternative of realization, of which the

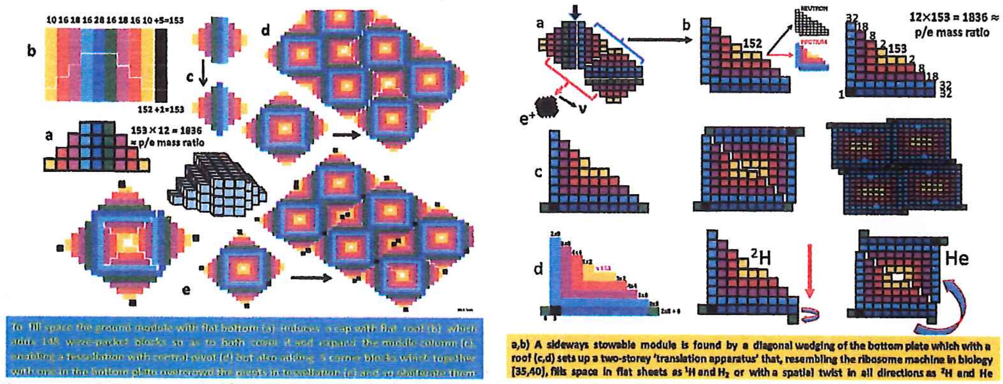


Figure 4. Hopefully self-explaining miniature illustrations (enlargeable in the screen) of square and twisted space-filling $R^3 \times SO(3)$ wave-packet module alternatives. *Colors refer to the noble gas in each period*

former obliterates space by its extra step but the latter (Figure 5) can continue the tessellation over it in exact replication of the atomic nucleosynthesis

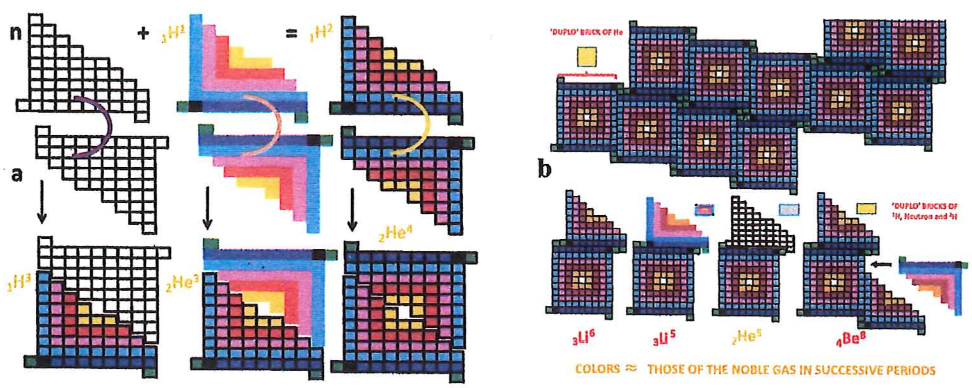


Figure 5. a) Nucleosynthesis of Deuterium from Neutron and Protium, and further to Tritium, $_2$Helium^3 and $_2$Helium^4. b) Same in Lithium to Beryllium.

succession including the isotope spectrum, subshell filling, neutron excess etc. in total correspondence with the periodic table. This will be exemplified by increasingly simplified graphical representation in the further periods. Figure 6 shows the second and third, each concluded by the respective saturated noble

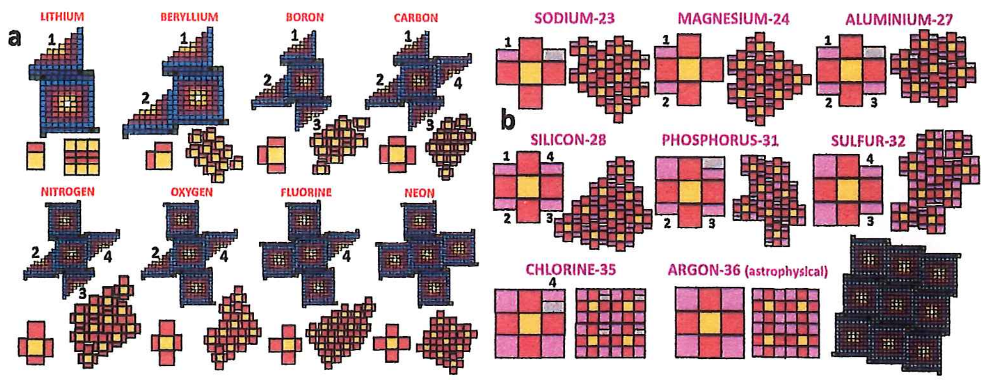
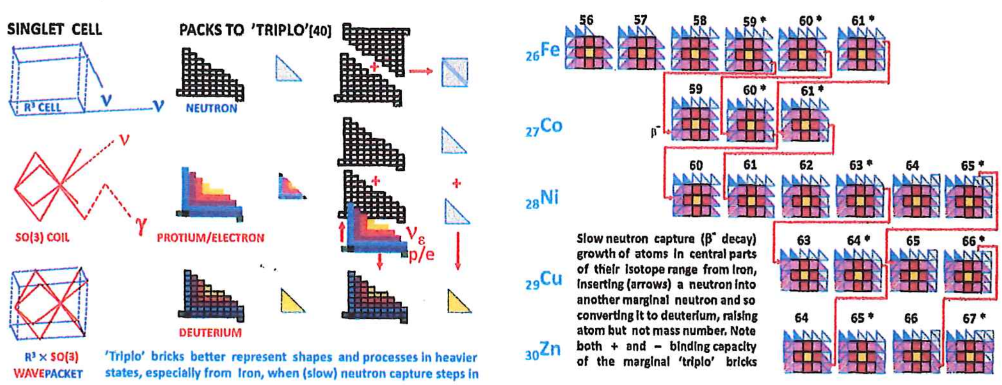


Figure 6. a) Fine-grained 'Lego' and coarse-grained 'Duplo'[30] representation of stable(st) second period atoms. b) Duplo representation of same in third period

gas with no remaining molecular binding sites. From the fourth period, when the neutron capture nucleosynthesis mechanism takes over in Iron and onwards, the 'triplo' representation (Figure 7a) is the most adequate for the marginal β decay superposition series demonstrated (Figure 7b) in the central isotopes of Iron to Zinc.



a b

Figure 7. a) Triplo representation (note generation of photon and neutrinos from wave-packet edges) b) Triplo representation of central isotopes with slow neutron capture β decays from Fe to Zn

Also in the heaviest atoms this marginal accretion applies, but increasingly by the rapid neutron capture process and sometimes downwards by α decay. All of this will be exemplified in the forthcoming periods, and it is notable and a further support of the model that in their subshell filling stage the lanthanides and actinides, too, can be continuously included in the respective sixth and seventh period. With brief comments showing parts of the periodic table series to its conclusion in Oganesson and onwards, the further aim is a review of entanglement as reproducible in the system in close compliance with reality.
In Figure 8a the last part of the fourth period is overviewed, and summed up in Figure 8b, where the fulfilment with α/He blocks is demonstrated.

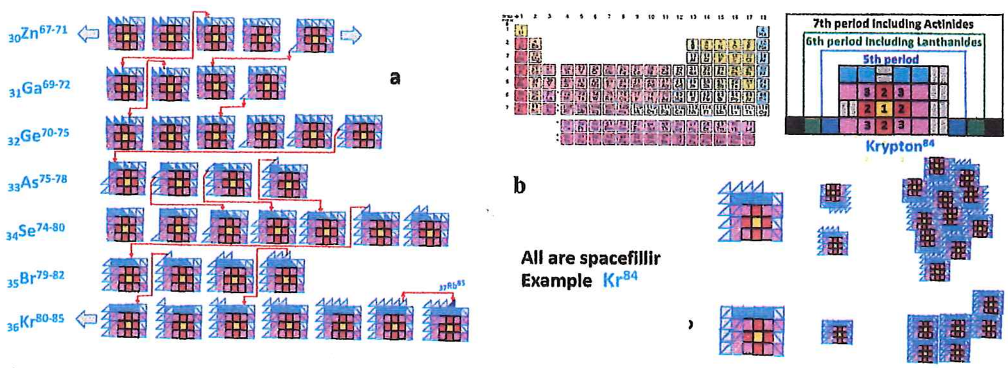


Figure 8. a) Zn to Kr. b) Summary of fourth period.

The conclusion of the fifth period from Rh to Xn is shown in Figure 9a, and the period is summarized in Figure 9b. Figure 10 comprises the same together with an example of spacefilling expansion in the sixth period.

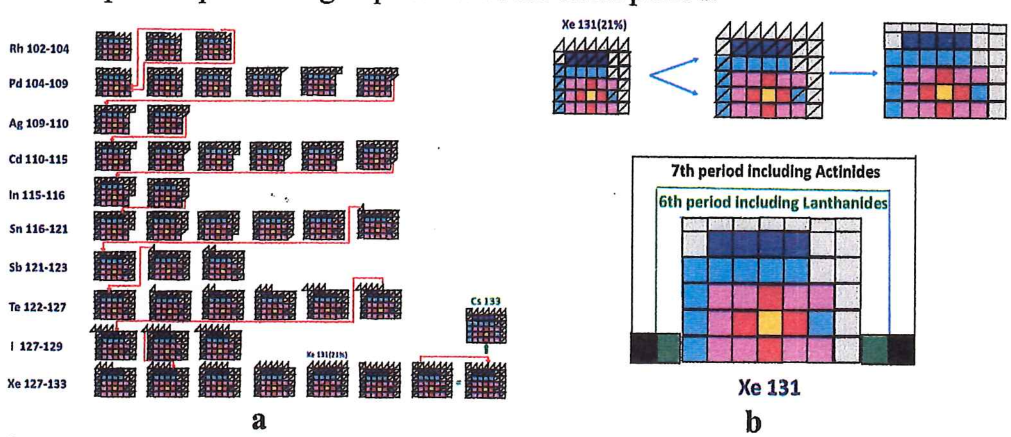


Figure 9. a) Rh to Xe. b) Summary of fifth period.

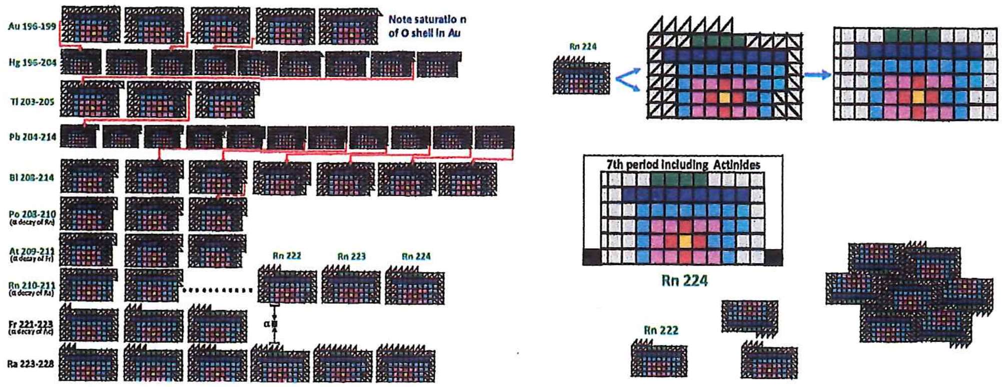


Figure 10. a) Conclusion of sixth period. Note saturation of subshells in same order as in reality and that all Lanthanides have been included in that subsurface saturation altering little of the binding sites in outer layer. Note also strong subshell filling and consequential stability in Au. b) Summary of sixth period.

Figures 11 and 12 show the initial and concluding stage of the seventh period.

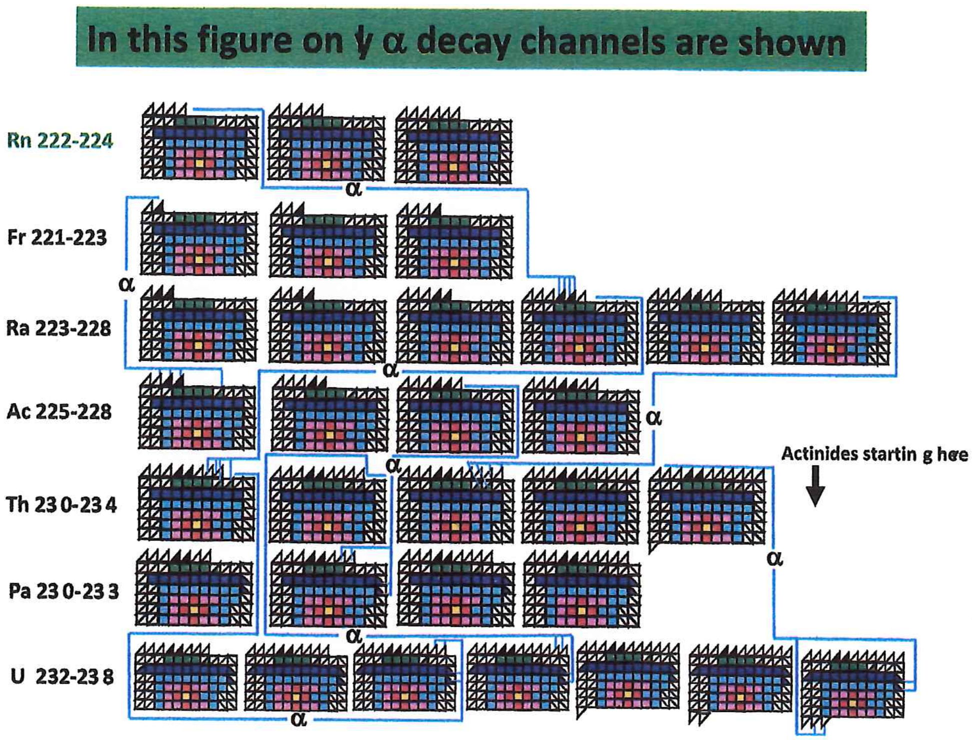


Figure 11. a) Start of seventh period. Here downward α decay nucleosynthesis is getting more frequent and is focused on in the figure.

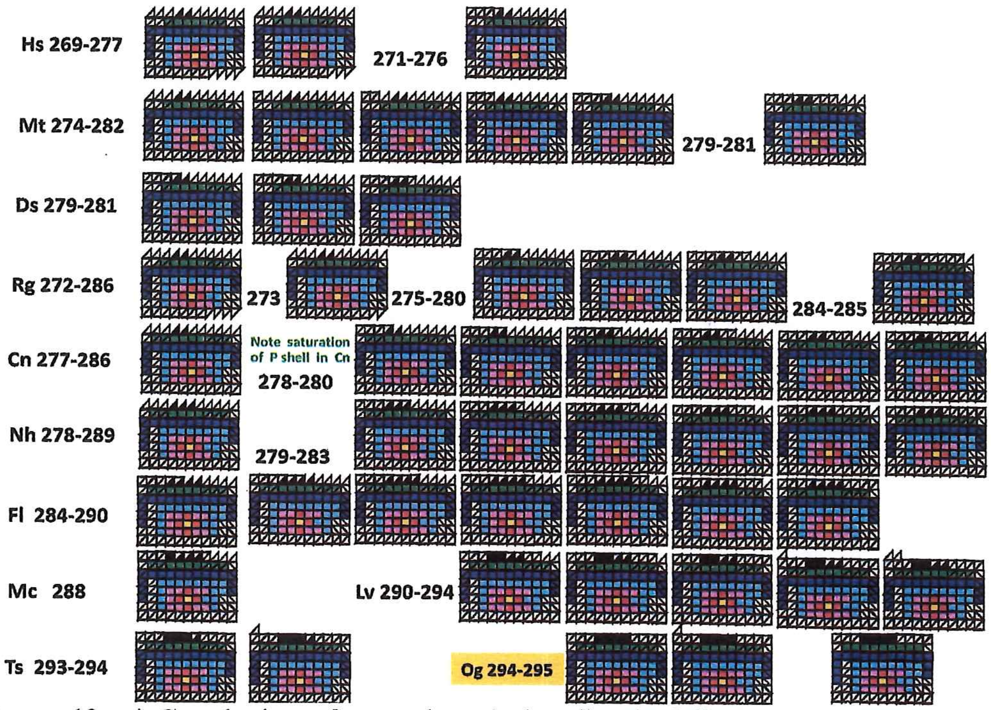
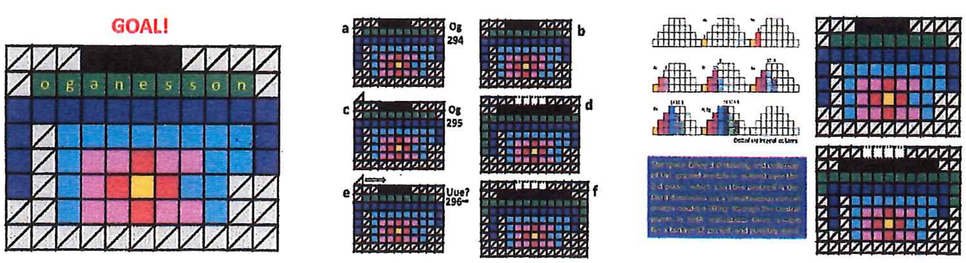


Figure 12. a) Conclusion of seventh period ending in full saturated Oganesson.

As demonstrated in Figure 13; a) Oganesson 294, which is the best documented isotope, is perfectly saturated and built in full α/He blocks with identical shell/subshell pattern and bolstering neutron excess as in reality, b) and in the 295 isotope providing first step into the eight period, for which, c) the model has good space in its tangential plane transferal of ground module comprisal.

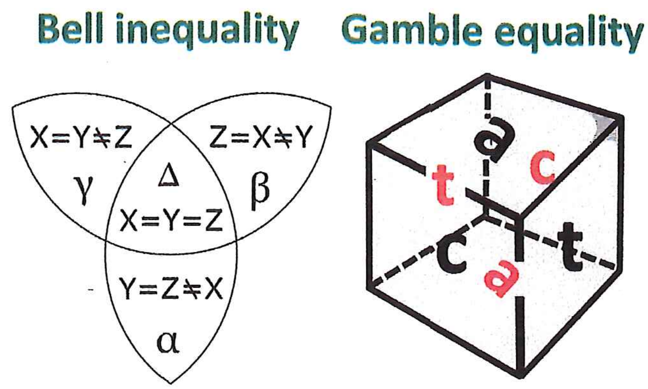


a b c

Figure 13. a) Oganesson 294. b) Isomers of Og294, Og 295, and possible Uue 296, and fulfilment of an eight period. c) Transferal of stowing pattern and potential in ground module to tangential plane distribution.

In conclusion so far, the reproducible descriptive results of the system should be sufficient evidence of its essential truth. It outlines, literally, a three-dimensional world which is atemporal but not ephemeral because its quantum second is eternity. In its permanence the entanglement phenomenon becomes natural and reciprocally a strong evidence of its framework. Can simultaneous action over distance have a local or global footing or both? The former is the EPR standpoint[1] and the latter is the Bell inequality analysis and its transferal to a still quantum mechanical instead of classical mechanism.[4]
It is important to realize that the search of inequality was to investigate and identify prospective quantum mechanical formulas that would enable locality, i.e., as surveyed in Figure 14, to endow a separating quantum superposition

In Bell's inequality re electron spin it is posed that X,Y,Z are the measurement variables with ± span in each. In reality this cannot evade that the entangled electrons must carry the spin with them; i.e. a carrier property before it can be measured as such. If e.g. an x is registered it must first be delivered. And when the outcome is ± there is in that minimal case a 2(x,-x, y,-y, z,-z) set of in- and outcomes created in the electron pair giving a 2/6 + 2/6 + 2/6 =1 total probability and no inequality. And if the only other option is a free mix between the variables there is the situation of two successive tosses of the CAT/cat dice with full equality



i	1	2	3	4	5	6	7	8
X_i	1	1	1	1	-1	-1	-1	-1
Y_i	1	1	-1	-1	1	1	-1	-1
Z_i	1	-1	1	-1	1	-1	1	-1
	Δ	γ	β	α	α	β	γ	Δ

$P(X=Y) + P(Y=Z) + P(Z=X) \geq 1$

c	c a t	c a t
a	c a t	c a t
t	c a t	c a t
c	c a t	c a t
a	c a t	c a t
t	c a t	c a t

σ(cat)(cat) =1

Figure 14. Comparing local quantum mechanical prediction and real gambling outcome of Bell thought experiment on electron spin, showing that local prediction exceeds real outcome and therefore rules out locality (but not reality)

with a kind of DNA formula determining equal further development of the two or more offshoots, and then to compare with real observation violating the quantum mechanical prediction. Finding just one such example would solve the dilemma, and Bell chose electron ± spin over x,y, and z coordinate axes, so the analogy of a both dead and alive - and spinning - cat could be trebled and emulated in the form of a correspondingly signed dice to throw the real fallout by.

However, experimental tests are needed to evaluate whether the quantum mechanical expectation computations match synchronous measurements at arbitrarily faraway spaced observation sites, customarily named Alice (originally Albert) and Bob (originally Niels) and thus support local reality, or are inequal from them and therefore must imply instant action over distance. The majority of such experiments use polarization of photon pairs in set-ups originally devised by Aspect[5] (Figure 15) and evaluated by a similar protocol as

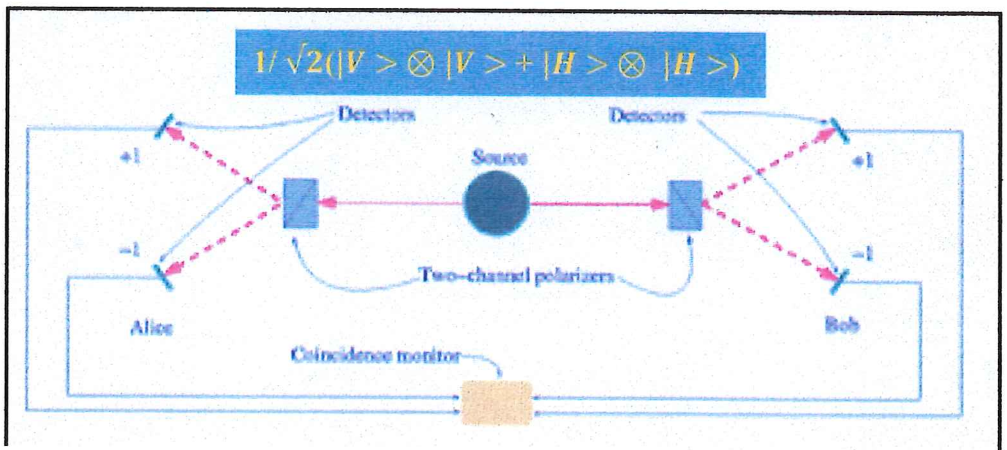


Figure 15. Experimental set-up of entanglement generation/analysis. Pairs of photons are discharged in opposite directions from the source through birefringent polarizers whose orientations are adjustable, and the split rays are individually detected, and + +, – –, + – and – + correlations under various angles counted in the coincidence monitor. Usually the A circuit is the local observer's, whereas B is arbitrarily far away. *V = vertical, H = horizontal*

Bell's, and overwhelmingly find that the real observations violate the predictions, no matter how long between each other they are, and by inference, then, over a beam between galaxies if so be. When using polarization of photons instead of spin of electrons, the quantum state of a pair of entangled photons is expressed as $1/\sqrt{2}(|V> \otimes |V> + |H> \otimes |H>)$, and is not the same as that of a spin-half particle, with different correspondence between angles and outcomes. Importantly, when the polarization of both photons is measured in equal direction both give identical, in 45° angle a random, and in 90° an anticorrelated outcome, but the general principle is the same. As mentioned the results excludes local quantum mechanical breeding of them, so the conclusion is that indeed instant interaction over distance is the case. However, there is one implication of its permanent observability at any site, namely that it must exist over the whole length of the trajectory, and this is the key to an entirely natural mechanism and explanation which is inherent in the present three-dimensional lattice system, as further explained in Figure 16. Every "geodetic curve"[7,8] there is lasting and in the quantum mechanical lingo could be likened to, let's say, a plank that lies extended, e.g. between the moon and asteroid B 612, weight-less and static in open space. But is it constant? Imagine that an astronaut tilts/measures it over any angle from whatever point along its span. By certainty; would that not instantly affect the whole plank and resolve whatever paradox and projection and exploitation of the EPR/Bell entanglement dilemma in the moment of a tilt? Well, now there is no such plank, but when instead there is a polarized, solely three-dimensional, i.e.

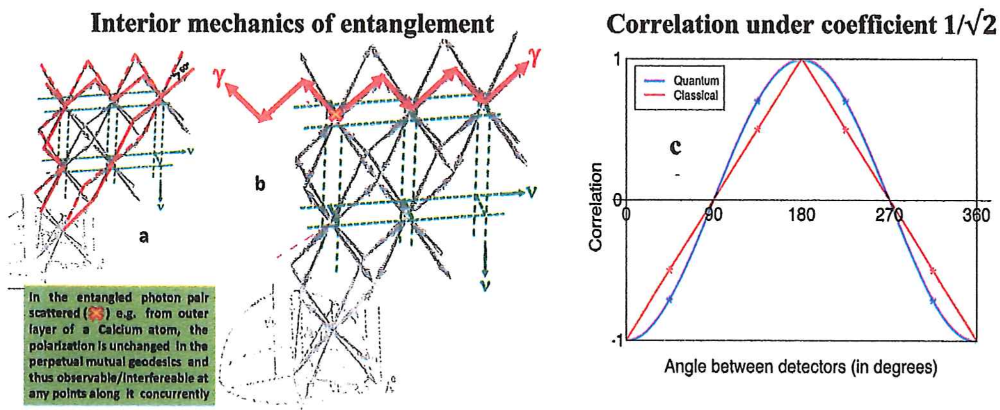


Figure 16. Interior mechanics of entanglement. a) In the SO(3) lattice there are both orthogonal and diagonal binary root vector sequences (red), all with a $\sqrt{0.5}/0.5 = \sqrt{2}/1$ vertical/horizontal span. b) when a photon pair is discharged from an intersection in the IVM lattice, it shares the same perpetual geodesics that lies there as a lopsided jagged plank which a polarizer at any point can by its inclination tilt or not tilt; in a single local moment turning the whole beam. c) When the QM correlation coefficient $1/\sqrt{2}$ is divided by the real $\sqrt{2}/1$ proportion the blue and red lines change place, supporting the classical case.

timeless, worldline in which the mediation is the measure, there is an identical situation, offering a strong support of the 'game of Lie' cellular automaton plan as well as being a direct consequence of it.

Discussion

At least I have no problem with the scheme and dare to meet any still facepalming audience of the pre-critically inflating standard model creed with it. Especially the vitally needed explication of the electron as not an amorphous point but of already described alternating transition matrix or wave function form[30] is a fact that undermines the ivory tower from the ground. But still it is sky-rocketing, with entanglement as one of its strongest spellbinding thrusts. However, the actual potentials are limited. With ± in various angles effectively being the only spin-off options, their tentative Morse patterns cannot more than project a structure. And to retrieve them an observer must still travel to the site of recording, and cannot do it faster than the speed of light, and the same applies to any deciphering or other communication broadcast to a receiver maybe hundreds of light-years out there. And the alternative to blindly fish after unaddressed planks in the endless ocean of the present is equally futile on both sides.

However, and from now naming these tilted quanta Plancks, the real advancement is in the field of information and communication. In a three-dimensional world there is a temporal arrow but no separate axis, and all apparent motion in every hierarchical subsystem formed by the timeless progression of successive moments in its track is relative to how this approximates to or deviates from other per se immobile trajectories. Pair-produced photons peeled off in the Aspect constellation[5] (Figure 15), from the outmost layer in one of a Calcium isotope's surface electron modules (Figure 16 b) or a laser, thus form a rigid beam swept over in various angles by the effector/detector plane, and if generated in larger bundles collective interaction emerges and sustains in consecutive pulses enabling correspondingly expanded information load for afferent and efferent messages amplified by both input and output coding and complexification gates etc. The Plancks glide away there, under the bridge and may by extra polarizers be directed into side-channels where some of them align with the walls so as to seemingly disappear, only to be redirected (but also possibly intercepted) and reappear in yet another polarization up- or downstream like light in a prism, streaming yet still…Well, excuse for the panegyric, but the opportunities are copious, and surely under intense study and exploration in many places.

When once installed in, for example, a spacecraft, and the link established, there is instant communication reaching as far as the flight goes but not yet to the next destination until it is there. And transferring the method to organisms or larger objects is not possible because that would require that every smallest molecule and all atoms in it be tilted to be sieved through the filter, and when some are not but in counterposition, explosion will happen, and the passage is still local. The only way is by proxy as far as the link goes and the proxy may print the chart but not the things and to reach an intended extraterrestrial location the link has first to be carried there which takes as many lightyears and more as the distance. But there might be that higher civilizations have developed the technique and are sending out signals in its band, for which a search should therefore be made which could instate an on-line exchange but no material impacts; and everything by classical means and mechanisms.
So, even if appearing in *Science* the more grandiose top-down visions of cosmic wormholes and teleportation and multiverses and that "it is possible to build up very generic connected spacetimes by entangling discrete noninteracting systems"[6] remain unseen and out of scope with the still very fascinating subject at hand, which has its roots in the bottom-up local scale. The electrons in atomic shells are entangled and so determine the time-less Aufbau of the periodic system, and for that essential operation EPR was right, too; quantum mechanics, in its most sophisticated perturbation and variation methods is incomplete, even in solving the electronic shell structure of the Helium atom and onwards.[32]

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EXTENDING MATHEMATICAL MODELS FROM NUMBERS TO H_v-NUMBERS

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Abstract

Hypergoup is a set equipped with a hyperoperation which is associative and reproductive. The fundamental relation β^* was introduced in 1970, which is the main tool in hyperstructures because it connects them with the corresponding classical structures. In 1990, Vougiouklis introduced the H_v-structures, by defining the *weak axioms* where the non-empty intersection replaces the equality. The quotient of a group by a partition is an H_v-group, so it is the largest class of hypergroups. The number of H_v-structures defined on a set is extremely greater than the number of the classical hyperstructures defined on the same set. Hyperstructures, especially the H_v-structures, have applications in many sciences including biomathematics, hadronic physics, lepton physics, and Santilli's iso-theory, to mention but a few. The hyperstructure theory is closely related to fuzzy theory; consequently, it can be applied in linguistic, sociology, industry and manugfacturing. In this paper, we focus on Lie-Santilli's theory especially on the *Hypernumbers or H_v-numbers* needed for the mathematical representation. The e-hyperfields, can be used as isofields, in such way to cover additional properties. Large classes of H_v-structures can be used in the Lie-Santilli theory especially when multivalued problems appeared, in finite or infinite case.

Key words: hyperstructures, hope, H_v-structures, H_v-fields, Lie-Santilli iso-theory.

1. HYPERSTRUCTURES

The largest class of hyperstructures, called *H_v-structures*, were introduced by the author in 1990 [19], [22]. These hyperstructures satisfy the *weak axioms* where the non-empty intersection replaces the equality. Some basic definitions are the following:

Definition 1.1 In a set H equipped with the hyperoperation (abbreviation: *hyperoperation*= ***hope***)

$$\cdot: H\times H \to P(H)-\{\varnothing\},$$

we abbreviate with *WASS* the *weak associativity*: $(xy)z\cap x(yz) \neq \varnothing$, $\forall x,y,z\in H$ and with *COW* the *weak commutativity*: $xy\cap yx \neq \varnothing$, $\forall x,y\in H$.

The hyperstructure $(H,\cdot)$ is called ***H_v-semigroup*** if it is *WASS*, it is called ***H_v-group*** if it is a reproductive H_v-semigroup, i.e. $xH = Hx = H$, $\forall x\in H$.

In the classical theory, the quotient of a group with respect to an invariant subgroup is a group. F. Marty stated in 1934 that, the quotient of a group with respect to any subgroup is a hypergroup. Finally, the quotient of a group with respect to any partition is an H_v-group [22].

The *powers* of an element $h\in H$ are: $h^1=\{h\}$, $h^2=h\cdot h$, …, $h^n=h^\circ\ldots{}^\circ h$, where $(^\circ)$ is the *n-ary circle hope*: the union of hyperproducts, n times, with all patterns of parentheses put on them. An H_v-semigroup $(H,\cdot)$ is a *cyclic of period s*, if there is a *generator* g, and a natural n, such that $H=h^1\cup\ldots\cup h^s$. If there is an h and s, such that $H=h^s$, then $(H,\cdot)$ is called *single-power cyclic of period s*.

In a similar way, more complicated hyperstructures can be defined:

Definitions 1.2 The $(\boldsymbol{R},+,\cdot)$ is an ***H_v-ring*** if (+) and (·) are *WASS*, the reproduction axiom is valid for (+) and (·) is *weak distributive* with respect to (+):

$$x(y+z)\cap(xy+xz)\neq\varnothing, \quad (x+y)z\cap(xz+yz)\neq\varnothing, \quad \forall x,y,z\in R.$$

Let $(\boldsymbol{R},+,\cdot)$ be H_v-ring, $(\boldsymbol{M},+)$ be COW H_v-group and let there exist an external hope

$$\cdot : R\times M \to P(M): (a,x) \to ax$$

such that, $\forall a,b\in \boldsymbol{R}$ and $\forall x,y\in \boldsymbol{M}$, then we have

$$a(x+y)\cap(ax+ay) \neq \varnothing, \quad (a+b)x\cap(ax+bx) \neq \varnothing, \quad (ab)x\cap a(bx) \neq \varnothing,$$

then $\boldsymbol{M}$ is an ***H_v-module*** over $\boldsymbol{F}$. In the case of an H_v-field $\boldsymbol{F}$ instead of an H_v-ring R, then the ***H_v-vector space*** is defined.

For more definitions and applications on H_v-structures one can see the books [1], [3], [4], [7], [9], [16], [22], [23], [28], [29], [32], [33].

Definition 1.3 The ***fundamental relations*** β^*, γ^* and ε^*, are defined, in H_v-groups, H_v-rings and H_v-vector spaces, respectively, as the smallest equivalences so that the quotient would be group, ring and vector spaces, respectively [19], [21], [22], [23], [31], [33].

The way to find the fundamental classes is given by the following:

Theorems 1.4 Let $(H,\cdot)$ be H_v-group and U be the set of finite products of elements of H. We define the relation β in H by setting $x\beta y$ iff $\{x,y\}\subset u$ where $u\in U$. Then β^* is the transitive closure of β.

Let $(R,+,\cdot)$ be H_v-ring and U the set of finite polynomials of elements of R. We define the relation γ in R by: $x\gamma y$ iff $\{x,y\}\subset u$ where $u\in U$. Then γ^* is the transitive closure of γ.

An element is called ***single*** if its fundamental class is singleton.

Definition 1.5 The fundamental relations are used for general definitions. Thus, an H_v-ring $(R,+,\cdot)$ is called ***H_v-field*** if R/γ^* is a field.

Definition 1.6 Let $(H,\cdot)$, $(H,*)$ be H_v-semigroups defined on the same set H. $(\cdot)$ is called ***smaller*** than $(*)$, and $(*)$ ***greater*** than $(\cdot)$, iff there exists an

$$f\in Aut(H,*) \quad \text{such that} \quad xy\subset f(x*y),\ \forall x,y\in H.$$

Then we say that $(H,*)$ *contains* $(H,\cdot)$. If $(H,\cdot)$ is a classical structure then it is called *basic structure* and $(H,*)$ is called *H_b-structure.*

The Little Theorem. Greater hopes than the ones which are WASS or COW, are WASS or COW, respectively.

This Theorem leads to a partial order, *posets*, on H_v-structures [28], [35], [9].

Definition 1.7 The H_v-semigroup $(H,\cdot)$ is called ***h/v-group*** if the quotient H/β^* is a group.

The h/v-groups are a generalization of the H_v-groups since in h/v-groups the *reproductivity of classes* is valid. This leads the quotient to be reproductive. In a similar way the *h/v-rings, h/v-fields, h/v-vector spaces* etc, are defined.

Definition 1.7 [18], [21], [22]. An H_v-structure is called ***very thin*** iff all hopes are operations except one, which has all hyperproducts singletons except only one, which is a subset of cardinality more than one.

Definition 1.8 [29], [32]. Let $(G,\cdot)$ be groupoid (resp., hypergroupoid) and $f:G\to G$ be a map. We define a hope (∂), called *theta-hope,* we write ***∂-hope***, on G as follows

$$x\partial y = \{f(x)\cdot y,\ x\cdot f(y)\},\ \ \forall x,y\in G. \ \ (\text{resp. } x\partial y= (f(x)\cdot y)\cup(x\cdot f(y),\ \forall x,y\in G)$$

If $(\cdot)$ is commutative then ∂ is commutative. If $(\cdot)$ is *COW*, then ∂ is *COW*.

Let $(G,\cdot)$ be groupoid (or hypergroupoid) and $f{:}G\to \boldsymbol{P}(G)-\{\emptyset\}$ be multivalued map. We define the (∂), on G as follows

$$x\partial y = (f(x)\cdot y)\cup(x\cdot f(y)),\ \ \forall x,y\in G.$$

Motivation for the theta-hope is the map *derivative* where only the multiplication of functions can be used. Basic property: if $(G,\cdot)$ is a semigroup then $\forall f$, the (∂) is *WASS*.

Definition 1.9 [17],[32]. Let $(G,\cdot)$ be groupoid, then $\forall P\subset G$, $P\neq\emptyset$ we define the following hopes called *P-hopes*: $\forall x,y\in G$

$$\underline{P}: x\underline{P}y= (xP)y\cup x(Py),\quad \underline{P}_r: x\underline{P}_r y= (xy)P\cup x(yP),\quad \underline{P}_l: x\underline{P}_l y= (Px)y\cup P(xy).$$

The $(G,\underline{P})$, $(G,\underline{P}_r)$ and $(G,\underline{P}_l)$ are called *P-hyperstructures*. If $(G,\cdot)$ is semigroup, then $x\underline{P}y=(xP)y\cup x(Py)=xPy$ and $(G,\underline{P})$ is a semihypergroup but we do not know about $(G,\underline{P}_r)$ and $(G,\underline{P}_l)$. In some cases, depending on the choice of P, the $(G,\underline{P}_r)$ and $(G,\underline{P}_l)$ can be associative or WASS.

A generalization of P-hopes is the following [6], [9]:

Construction 1.10 Let $(G,\cdot)$ be abelian group and P, subset of G. Define the hope $\times_P$ as follows:`

$$\begin{cases} x\times_P y = x\cdot P\cdot y = \{x\cdot h\cdot y \mid h\in P\} & \text{if } x\neq e \text{ and } c\neq e \\ x\cdot y & \text{if } x=e \text{ or } y=e \end{cases}$$

we call this hope P_e-hope. The hyperstructure $(G,\times_P)$ is an abelian H_v-group.

The general definition of an H_v-Lie algebra was given as follows [31], [9]:

Definition 1.11 Let $(\boldsymbol{L},+)$ H_v-vector space on $(\boldsymbol{F},+,\cdot)$, $\varphi{:}\boldsymbol{F}\to\boldsymbol{F}/\gamma^*$ canonical, $\omega_F=\{x\in\boldsymbol{F}{:}\varphi(x)=0\}$, where 0 is zero of $\boldsymbol{F}/\gamma^*$. Let ω_L the core of $\varphi'{:}\boldsymbol{L}\to\boldsymbol{L}/\varepsilon^*$ and denote 0 the zero of $\boldsymbol{L}/\varepsilon^*$. Consider the *bracket (commutator) hope*:

$$[\ ,\ ]: \boldsymbol{L}\times\boldsymbol{L} \to P(L): (x,y) \to [x,y]$$

then $\boldsymbol{L}$ is an *H_v-Lie algebra* over $\boldsymbol{F}$ if the following axioms are satisfied:

(L1) The bracket hope is bilinear, i.e.

$$[\lambda_1x_1+\lambda_2x_2,y]\cap(\lambda_1[x_1,y]+\lambda_2[x_2,y]) \neq \emptyset$$

$$[x,\lambda_1y_1+\lambda_2y_2]\cap(\lambda_1[x,y_1]+\lambda_2[x,y_2]) \neq \emptyset,\ \forall x,x_1,x_2,y,y_1,y_2\in L,\ \forall\lambda_1,\lambda_2\in\boldsymbol{F}$$

(L2) $[x,x]\cap\omega_L \neq \emptyset,\ \ \forall x\in\boldsymbol{L}$

(L3) $([x,[y,z]]+[y,[z,x]]+[z,[x,y]])\cap\omega_L \neq \emptyset,\ \ \forall x,y\in\boldsymbol{L}.$

The enlargement or reduction of hyperstructures are examined in the sense that an extra element appears in one result or we take out an element. In both directions most useful are those H_v-structures with the same fundamental structure [25], [27]:

Let (H,·) be H_v-semigroup and $v \notin H$. Extend (·) into the $\underline{H}=H\cup\{v\}$ as follows: $x\cdot v=v\cdot x=v$, $\forall x\in H$, and $v\cdot v=H$. The $(\underline{H},\cdot)$ is an h/v-group where $(\underline{H},\cdot)/\beta^*\cong Z_2$ and v is a single element. We call $(\underline{H},\cdot)$ the *attach h/v-group* of (H,·).

Theorem 1.12 Let (G,·) be semigroup and $v\notin G$ be an element appearing in a product ab, where $a,b\in G$, thus the result becomes a hyperproduct $a\otimes b=\{ab,v\}$. Then the minimal hope (⊗) extended in $G'=G\cup\{v\}$ such that (⊗) contains (·) in the restriction on G, and such that $(G',\otimes)$ is a minimal H_v-semigroup which has fundamental structure isomorphic to (G,·), is defined as follows:

$$a\otimes b=\{ab,v\},\quad x\otimes y=xy,\quad \forall(x,y)\in G^2-\{(a,b)\}$$

$$v\otimes v=abab,\quad x\otimes v=xab \quad\text{and}\quad v\otimes x=abx,\quad \forall x\in G.$$

$(G',\otimes)$ is very thin H_v-semigroup. If (G,·) is commutative then the $(G',\otimes)$ is strongly commutative.

Let (H,·) be hypergroupoid. We say that *remove* $h\in H$, if we consider the restriction of (·) on H-{h}. We say $\underline{h}\in H$ *absorbs* $h\in H$ if we replace h by $\underline{h}$. We say $\underline{h}\in H$ *merges* with $h\in H$, if we take as product of $x\in H$ by $\underline{h}$, the union of the results of x with both h, $\underline{h}$ and consider h and $\underline{h}$ as one class.

Now we present some 'small' h/v-fields.

Constructions 1.13 On the rings $(\boldsymbol{Z}_4,+,\cdot)$ and $(\boldsymbol{Z}_6,+,\cdot)$ we will define all the multiplicative h/v-fields which have non-degenerate fundamental field and, moreover they are,

(a) very thin minimal, (b) COW, (c) they have 0 and 1, scalars.

I. On $(\boldsymbol{Z}_4,+,\cdot)$ we have the isomorphic cases: $2\otimes3=\{0,2\}$ or $3\otimes2=\{0,2\}$. The fundamental classes are $[0]=\{0,2\}$, $[1]=\{1,3\}$ and we have $(\boldsymbol{Z}_4,+,\otimes)/\gamma^*\cong(\boldsymbol{Z}_2,+,\cdot)$. Thus it is isomorphic to $(\boldsymbol{Z}_2\times\boldsymbol{Z}_2,+)$. In this H_v-group there is only one unit and every element have a unique double inverse.

II. On $(\boldsymbol{Z}_6,+,\cdot)$, we have the only one hyperproduct,

(i) $2\otimes3=\{0,3\}$, $2\otimes4=\{2,5\}$, $3\otimes4=\{0,3\}$, $3\otimes5=\{0,3\}$, $4\otimes5=\{2,5\}$

Fundamental classes: $[0]=\{0,3\}$, $[1]=\{1,4\}$, $[2]=\{2,5\}$ and we have

$$(\boldsymbol{Z}_6,+,\otimes)/\gamma^*\cong(\boldsymbol{Z}_3,+,\cdot).$$

(ii) $2\otimes3=\{0,2\}$ or $2\otimes3=\{0,4\}$, $2\otimes4=\{0,2\}$ or $\{2,4\}$, $2\otimes5=\{0,4\}$ or $2\otimes5=\{2,4\}$, $3\otimes4=\{0,2\}$ or $\{0,4\}$, $3\otimes5=\{3,5\}$, $4\otimes5=\{0,2\}$ or $\{2,4\}$.

Fundamental classes: $[0]=\{0,2,4\}$, $[1]=\{1,3,5\}$ and we have

$$(\boldsymbol{Z}_6,+,\otimes)/\gamma^*\cong(\boldsymbol{Z}_2,+,\cdot).$$

Definition 1.14 The ***uniting elements*** method, introduced by Corsini & Vougiouklis in 1989 [2], is the following: Let G be algebraic structure and d be a property, which is not valid and described by a set of equations; then, consider the partition in G for which it is put together, in the same class, every pair that causes the non-validity of d. The quotient G/d is an h/v-structure. Then, quotient out the G/d by β^*, a stricter structure $(G/d)/\beta^*$ for which the property d is valid, is obtained.

A problem of the uniting elements occurs when more than one property is desired. The reason is that some of the properties lead straight to the classes than others. So, we apply the straightforward classes followed by the more complicated ones. The commutativity and reproductivity are easy applicable properties. One can do this because the following Theorem is valid [22], [25], [27].

Theorem 1.15 Let $(G,\cdot)$ be a groupoid, and $F=\{f_1,\ldots,f_m,f_{m+1},\ldots,f_{m+n}\}$ be system of equations on G consisting of subsystems $F_m=\{f_1,\ldots,f_m\}$, $F_n=\{f_{m+1},\ldots,f_{m+n}\}$. Let σ, σ_m be the equivalence relations defined by the uniting elements procedure on systems F and F_m, respectively, and let σ_n be the equivalence relation defined using the induced equations of F_n on the groupoid $G_m=(G/\sigma_m)/\beta^*$. Then

$$(G/\sigma)/\beta^*\cong(G_m/\sigma_n)/\beta^*.$$

i.e. the following diagram is commutative

$$\begin{array}{ccccc}
G & \xrightarrow{\rho_m} & G/\sigma_m & \xrightarrow{\varphi_m} & G_m \\
\rho\downarrow & & & & \downarrow\rho_n \\
G/\sigma & & & & G_m/\sigma_n \\
\varphi\downarrow & & \cong & & \downarrow\varphi_n \\
(G/\sigma)/\beta^* & & \longrightarrow & & (G_m/\sigma_n)/\beta^*.
\end{array}$$

Where all maps ρ, φ, ρ_m, φ_m, ρ_n, φ_n, are the canonicals.

The problem of enumeration and classification of hyperstructures, started from the beginning, it is complicate in H_v-structures because we have very great numbers [10]. For example, the number of H_v-groups with three elements, up to isomorphism, is 1.026.462. There are 7.926 abelian; the 1.013.598 are cyclic. The partial order in H_v-structures and the Little Theorem, transfers and restrict the problem in finding *the minimal, up to isomorphisms*, H_v-structures.

2. REPRESENTATIONS

Representations (we abbreviate with ***rep***) of H_v-groups, can be considered either by H_v-matrices [21], [22], [23], [24], [27] or by generalized permutations [20].

Definition 2.1 ***H_v-matrix*** (or *h/v-matrix*) is called a matrix with entries elements of an H_v-ring or H_v-field (or h/v-ring or h/v-field). The hyperproduct of H_v-matrices $\mathbf{A}=(a_{ij})$ and $\mathbf{B}=(b_{ij})$, of type m×n and n×r, respectively, is a set of m×r H_v-matrices, defined in a usual manner:

$$\mathbf{A}\cdot\mathbf{B} = (a_{ij})\cdot(b_{ij}) = \{\mathbf{C}=(c_{ij}) \mid c_{ij}\in\oplus\Sigma a_{ik}\cdot b_{kj}\},$$

where $(\oplus)$ is the *n-ary circle hope* on the hyperaddition.

The rep problem by H_v-matrices is the following:

Definition 2.2 Let $(H,\cdot)$ be H_v-group, $(R,+,\cdot)$ be H_v-ring and $\mathbf{M}_R=\{(a_{ij}) \mid a_{ij}\in R\}$, then any

$$\mathbf{T}: H\to\mathbf{M}_R: h\to\mathbf{T}(h) \text{ with } \mathbf{T}(h_1h_2)\cap\mathbf{T}(h_1)\mathbf{T}(h_2)\neq\varnothing,\ \forall h_1,h_2\in H,$$

is called H_v-matrix rep. If $T(h_1h_2)\subset T(h_1)T(h_2)$, then T is an inclusion rep, if $T(h_1h_2)=T(h_1)T(h_2)$, then T is a good rep an induced rep T* for the hypergroup algebra is obtained. If T is one to one and good, then it is a faithful rep.

Theorem 2.3 A necessary condition in order to have an inclusion rep T of an H_v-group $(H,\cdot)$ by n×n H_v-matrices over the H_v-ring $(R,+,\cdot)$ is the following: For all $\beta^*(x)$, $x\in H$ there must exist elements $a_{ij}\in H$, $i,j\in\{1,\dots,n\}$ such that

$$T(\beta^*(a)) \subset \{A = (a'_{ij}) \mid a'_{ij} \in \gamma^*(a_{ij}),\ i,j\in\{1,\dots,n\}\}$$

Thus, every inclusion rep $T: H\to M_R: a\mapsto T(a)=(a_{ij})$ induces a homomorphic rep T* of H/β^* over R/γ^* by setting $T^*(\beta^*(a))=[\gamma^*(a_{ij})]$, $\forall\beta^*(a)\in H/\beta^*$, where the element $\gamma^*(a_{ij})\in R/\gamma^*$ is the ij entry of the matrix $T^*(\beta^*(a))$.

The rep problem by *Generalized Permutations* (write ***gp***), is described as follows [20]:

Definitions 2.4 Let X be a set, then a map $f: X\to P(X)-\{\varnothing\}$, is a *gp of X* if it is reproductive:

$$\cup_{x\in X} f(x)=f(X)=X.$$

Denote by M_X the set of all gps on X. For an H_v-group (X,·) and $a\in X$, the gp f_a defined by $f_a(x)=ax$ is called *inner gp*. *Arrow* of f is any $(x,y)\in X^2$ with $y\in f(x)$. The $f_2\in M_X$ *contains* $f_1\in M_X$ or f_1 is a *sub-gp* of f_2, if $f_1(x)\subset f_2(x)$, $\forall x\in X$, then we write $f_1\subset f_2$. If, moreover, $f_1\neq f_2$, then f_1 is a *proper* sub-gp of f_2. An $f\in M_X$ is called *minimal* if it has no proper sub-gp. Denote $\underline{M}_X$ the set of all minimal gps of M_X. The *converse* of a gp f is the gp $\underline{f}$ defined by $\underline{f}(x)=\{z\in X: f(z)\ni x\}$, thus $\underline{f}$ is obtained by reversing arrows. We call *associated* to $f\in M_X$ the gp $f\circ\underline{f}$.

Theorem 2.5 Let $f\in M_X$, then $f\in\underline{M}_X$ iff, the following condition is valid:

If $a\neq b$ and $f(a)\cap f(b)\neq\emptyset$, then $f(a)=f(b)$ and $f(a)$ is singleton.

If $f\in\underline{M}_X$ then, $\underline{f}\in\underline{M}_X$. If $f\in\underline{M}_X$ then, $(f\circ\underline{f})(x)=\{y\in X:f(y)=f(x)\}$.

Several classes of H_v-structures can face special reps. Some of those classes are as follows [22], [24]:

Definition 2.6 Let $M=M_{m\times n}$, the set of m×n matrices on *R* and $P=\{P_i:i\in I\}\subseteq M$. We define, a kind of, P-hope $\underline{P}$ on *M* as follows

$$\underline{P}: M\times M\to P(M): (A,B)\ A\underline{P}B=\{AP^t_iB:\ i\in I\}\subseteq M$$

where P^t denotes the transpose of P. $\underline{P}$ is bilinear Rees' like operation where, instead of one sandwich matrix, a set is used. $\underline{P}$ is strong associative and the inclusion distributive to addition is valid:

$$A\underline{P}(B+C)\subseteq A\underline{P}B+A\underline{P}C,\ \forall A,B,C\in M$$

So $(M,+,\underline{P})$ defines a multiplicative hyperring on non-square matrices.

Let $M=M_{m\times n}$ be module of m×n matrices on R and take the sets

$$S=\{s_k:k\in K\}\subseteq R,\quad Q=\{Q_j:j\in J\}\subseteq M,\quad P=\{P_i:i\in I\}\subseteq M.$$

Define three hopes as follows

$$\underline{S}: R\times M\to P(M): (r,A)\to r\underline{S}A=\{(rs_k)A:\ k\in K\}\subseteq M$$

$$\underline{Q}_+: M\times M\to P(M): (A,B)\to A\underline{Q}_+B=\{A+Q_j+B:\ j\in J\}\subseteq M$$

$$\underline{P}: M\times M\to P(M): (A,B)\to A\underline{P}B=\{AP^t_iB:\ i\in I\}\subseteq M$$

Then $(M,\underline{S},\underline{Q}_+,\underline{P})$ is a hyperalgebra on *R* called *general matrix P-hyperalgebra*.

Hopes on any type of ordinary matrices can be defined [37], [8] they are called *helix hopes*.

Definition 2.7 Let $A=(a_{ij})\in\mathbf{M}_{m\times n}$ be matrix and $s,t\in N$, with $1\leq s\leq m$, $1\leq t\leq n$. The *helix-projection* is a map

$$\underline{st}: \mathbf{M}_{m\times n}\rightarrow \mathbf{M}_{s\times t}:\ A\rightarrow A\underline{st}=(\underline{a}_{ij}),$$

where $A\underline{st}$ has entries

$$\underline{a}_{ij} = \{a_{i+\kappa s, j+\lambda t} \mid 1\leq i\leq s,\ 1\leq j\leq t \text{ and } \kappa,\lambda\in N,\ i+\kappa s\leq m,\ j+\lambda t\leq n\}$$

Let $A=(a_{ij})\in\mathbf{M}_{m\times n}$, $B=(b_{ij})\in\mathbf{M}_{u\times v}$, $s=\min(m,u)$, $t=\min(n,v)$. We define a hyper-addition, called ***helix-sum***, by

$$\oplus : \mathbf{M}_{m\times n}\times\mathbf{M}_{u\times v}\rightarrow\mathbf{P}(\mathbf{M}_{s\times t}):(A,B)\rightarrow A\oplus B=A\underline{st}+B\underline{st}=(\underline{a}_{ij})+(\underline{b}_{ij})\subset\mathbf{M}_{s\times t}$$

where $(\underline{a}_{ij})+(\underline{b}_{ij})=\{(c_{ij})=(a_{ij}+b_{ij}) \mid a_{ij}\in\underline{a}_{ij} \text{ and } b_{ij}\in\underline{b}_{ij})\}$.
Let $A=(a_{ij})\in\mathbf{M}_{m\times n}$, $B=(b_{ij})\in\mathbf{M}_{u\times v}$ and $s=\min(n,u)$. Define the ***helix-product***, by

$$\otimes: \mathbf{M}_{m\times n}\times\mathbf{M}_{u\times v}\rightarrow\mathbf{P}(\mathbf{M}_{m\times v}): (A,B)\rightarrow A\otimes B=A\underline{ms}\cdot B\underline{sv}=(\underline{a}_{ij})\cdot(\underline{b}_{ij})\subset\mathbf{M}_{m\times v}$$

where $(\underline{a}_{ij})\cdot(\underline{b}_{ij})= \{(c_{ij})=(\sum a_{it}b_{tj}) \mid a_{ij}\in\underline{a}_{ij} \text{ and } b_{ij}\in\underline{b}_{ij})\}$.

The helix-sum is commutative, WASS, not associative. The helix-product is WASS, not associative and not distributive to the helix-addition.

We present a proof for the fundamental relation, analogous to Theorem 1.4, in the case of an H_v-module:

Theorem 2.8 Let $(\boldsymbol{M},+)$ be H_v-module over $\boldsymbol{R}$. Denote by U the set of expressions of finite hopes either on $\boldsymbol{R}$ and $\boldsymbol{M}$ or the external hope applied on finite sets. We define the relation ε in $\boldsymbol{M}$ by: $x\varepsilon y$ iff $\{x,y\}\subset u$, $u\in U$. Then the relation ε^* is the transitive closure of the relation ε.

Proof. Let $\underline{\varepsilon}$ be the transitive closure of ε, and denote by $\underline{\varepsilon}(x)$ the class of the element x. First we prove that the quotient set $\boldsymbol{M}/\underline{\varepsilon}$ is a module over $\boldsymbol{R}/\gamma^*$.
In $\boldsymbol{M}/\underline{\varepsilon}$ the sum ($\oplus$) and the external product ($\otimes$), using the γ^* classes in $\boldsymbol{R}$, are defined in the usual manner:

$$\underline{\varepsilon}(x)\oplus\underline{\varepsilon}(y) = \{\underline{\varepsilon}(z): z\in\underline{\varepsilon}(x)+\underline{\varepsilon}(y)\},\ \forall x,y\in\boldsymbol{M}.$$

$$\gamma^*(a)\otimes\underline{\varepsilon}(x) = \{\underline{\varepsilon}(z): z\in\gamma^*(a)\cdot\underline{\varepsilon}(x)\},\ \forall a\in\boldsymbol{R},\ \forall x,y\in\boldsymbol{M}.$$

Take $x'\in\underline{\varepsilon}(x)$, $y'\in\underline{\varepsilon}(y)$. Then $x'\underline{\varepsilon}x$ iff $\exists x_1,\ldots,x_{m+1}$ with $x_1=x'$, $x_{m+1}=x$ and $u_1,\ldots,u_m\in U$ such that $\{x_i, x_{i+1}\}\subset u_i$, $i=1,\ldots,m$ and $y'\underline{\varepsilon}y$ iff $\exists y_1,\ldots,y_{n+1}$ with $y_1=y'$, $y_{n+1}=y$ and $v_1,\ldots,v_n\in U$ such that $\{y_j,y_{j+1}\}\subset v_j, j=1,\ldots,n$.

From the above we obtain

$$\{x_i, x_{i+1}\}+y_1 \subset u_1+v_1,\ i=1,\ldots,m-1,\quad x_{m+1}+\{y_j, y_{j+1}\} \subset u_m+v_j,\ j=1,\ldots,n.$$

The $u_i+v_1=t_i$, $i=1,\ldots,m-1$, $u_m+v_j=t_{m+j-1}$, $j=1,\ldots,n$ are elements of $\boldsymbol{U}$, therefore, $t_k\in\boldsymbol{U}$, $\forall k\in\{1,\ldots,m+n-1\}$. Take elements $z_1,\ldots,z_{m+n}$ with $z_i\in x_i+y_1$, $i=1,\ldots,n$ and $z_{m+j}\in x_{m+1}+y_{j+1}$, $j=1,\ldots,n$, thus, $\{z_k,z_{k+1}\}\subset t_k$, $k=1,\ldots,m+n-1$.

Therefore, $\forall z_1\in x_1+y_1=x'+y'$ is $\underline{\boldsymbol{\varepsilon}}$ equivalent to $z_{m+n}\in x_{m+1}+y_{n+1}=x+y$. Thus $\underline{\boldsymbol{\varepsilon}}(x)\oplus\underline{\boldsymbol{\varepsilon}}(y)$ is a singleton so we can write, $\underline{\boldsymbol{\varepsilon}}(x)\oplus\underline{\boldsymbol{\varepsilon}}(y)=\underline{\boldsymbol{\varepsilon}}(z)$, $\forall z\in\underline{\boldsymbol{\varepsilon}}(x)+\underline{\boldsymbol{\varepsilon}}(y)$.

Similarly, using the properties of γ^* in $\boldsymbol{R}$, we prove that

$$\gamma^*(a)\otimes\underline{\boldsymbol{\varepsilon}}(x)=\underline{\boldsymbol{\varepsilon}}(z),\ \forall z\in\gamma^*(a)\cdot\underline{\boldsymbol{\varepsilon}}(x)$$

The WASS and the weak distributivity on $\boldsymbol{R}$ and $\boldsymbol{M}$ guarantee that the associativities and the distributivity are valid for $\boldsymbol{M}/\underline{\boldsymbol{\varepsilon}}$ over $\boldsymbol{R}/\gamma^*$. Therefore, $\boldsymbol{M}/\underline{\boldsymbol{\varepsilon}}$ is a module over $\boldsymbol{R}/\gamma^*$.

Now let σ equivalence relation in $\boldsymbol{M}$ such that $\boldsymbol{M}/\sigma$ is module on $\boldsymbol{R}/\gamma^*$. Denote $\sigma(x)$ the class of x. Then, $\sigma(x)\oplus\sigma(y)$ and $\gamma^*(a)\otimes\sigma(x)$ are singletons $\forall a\in\boldsymbol{R}$ and $x,y\in\boldsymbol{M}$, i.e.

$$\sigma(x)\oplus\sigma(y)=\sigma(z),\ \forall z\in\sigma(x)+\sigma(y),\ \gamma^*(a)\otimes\sigma(x)=\sigma(z),\ \forall z\in\gamma^*(a)\cdot\sigma(x).$$

Thus we write, $\forall a\in\boldsymbol{R}$, $x,y\in\boldsymbol{M}$ and $A\subset\gamma^*(a)$, $\boldsymbol{X}\subset\sigma(x)$, $\boldsymbol{Y}\subset\sigma(x)$

$$\sigma(x)\oplus\sigma(y)=\sigma(x+y)=\sigma(\boldsymbol{X}+\boldsymbol{Y}),\ \ \gamma^*(a)\otimes\sigma(x)=\sigma(ax)=\sigma(\boldsymbol{A}\cdot\boldsymbol{X}).$$

By induction, extend these relations on finite sums and products. Thus, $\forall u\in\boldsymbol{U}$, we have $\sigma(x)=\sigma(u)$, $\forall x\in u$. Consequently $x'\in\varepsilon(x)$ implies $x'\in\sigma(x)$, $\forall x\in\boldsymbol{M}$. But σ is transitively closed, so we obtain: $x'\in\underline{\boldsymbol{\varepsilon}}(x)$ implies $x'\in\sigma(x)$.

Therefore, $\underline{\boldsymbol{\varepsilon}}$ is the smallest equivalence on $\boldsymbol{M}$ such that $\boldsymbol{M}/\underline{\boldsymbol{\varepsilon}}$ is a module on $\boldsymbol{R}/\gamma^*$, i.e. $\underline{\boldsymbol{\varepsilon}}=\varepsilon^*$. ■

Recall that, an element is *single* if its fundamental class is singleton, so, in an H_v-group if s is single then $\beta^*(s)=\{s\}$. Denote S_H the set of singles. If $S_H\neq\varnothing$, then we can answer to the very hard problem, that is to find the fundamental classes. The following theorems are proved [22], [24], [35]:

Theorem 2.9 Let $(H,\cdot)$ be H_v-group and $s\in S_H\neq\varnothing$. Let $a\in H$, take an element $v\in H$ such that $s\in av$, then $\beta^*(a)=\{h\in H:hv=s\}$, and the core of H is $\omega_H=\{u\in H:us=s\}=\{u\in H: su=s\}$. Moreover, $sx=\beta^*(sx)$ and $xs=\beta^*(xs)$, $\forall x\in H$.

Important Conlusion. Two elements a,b are in the fundamental relation β if there are two elements x,y who bring a,b in the relation β. That means that the fundamental relation β^* 'depend' on the results. This fact leads to a special proof where we need to discover the 'reason' to have the results. Every relation needs even the last one result to characterize its classes. However, if there are special elements, as the singles, which are strictly formed and carry inside them the relation, then these elements form the fundamental classes.

3. LIE-SANTILLI HYPER-ADMISSIBILITY

Last decade hyperstructures have applications in mathematics and in other sciences. These applications include biomathematics – conchology and inheritance - hadronic physics, lepton physics, and Santilli's iso-theory, to mention but a few. The hyperstructure theory is closely related to fuzzy theory; consequently, it can be widely applied in linguistic, sociology, industry and production [1], [3], [4], [7], [9], [18], [28], [29], [31], [34], [36].

In [30], with '*The Santilli's theory 'invasion' in hyperstructures'*, there is a first description on how Santilli's theories effect in hyperstructures and how new theories in Mathematics appeared by Santilli's pioneer research. In 1996 Santilli & Vougiouklis [14], point out that in physics the most interesting hyperstructures are the one called e-hyperstructures. These hyperstructures contain a unique left ant right scalar unit, which is an important tool in Lie-Santilli theory. One can see the books and related papers for more definitions and results related topics: [5], [6], [9], [11], [12], [13], [14], [15], [26], [30].

Definition 3.1 A hyperstructure $(F,+,\cdot)$, where $(+)$ is an operation and $(\cdot)$ a hope, is called *e-hyperfield* if the following axioms are valid: $(F,+)$ is an abelian group with the additive unit 0, $(\cdot)$ is WASS, $(\cdot)$ is weak distributive with respect to $(+)$, 0 is absorbing element: $0\cdot x=x\cdot 0=0$, $\forall x\in F$, there exist a multiplicative scalar unit 1, i.e. $1\cdot x=x\cdot 1=x$, $\forall x\in F$, and $\forall x\in F$ there exists a unique inverse x^{-1}, such that

$$1\in x\cdot x^{-1}\cap x^{-1}\cdot x.$$

The elements of an e-hyperfield are called *e-hypernumbers*. If the relation: $1=x\cdot x^{-1}=x^{-1}\cdot x$, is valid, then we say that we have a *strong e-hyperfield.*

Definition 3.2 *The Main e-Construction.* Given a group $(G,\cdot)$, where e is the unit, then we define in G, a huge number of hopes $(\otimes)$ as follows:

$$x\otimes y=\{xy, g_1, g_2,\ldots\},\ \forall x,y\in G-\{e\},\text{ and } g_1, g_2,\ldots\in G-\{e\}.$$

The $(G,\otimes)$ is an e-hypergroup, which is an H_b-group because it contains $(G,\cdot)$. Moreover, if $\forall x,y$ with $xy=e$, so $x\otimes y=xy$, then $(G,\otimes)$ is strong e-hypergroup.

The proof is immediate since we enlarge the results of the group by putting elements from G and applying the Little Theorem. Moreover, the unit e is unique scalar and $\forall x\in G$, there exists a unique inverse x^{-1}, such that $e\in x\cdot x^{-1}\cap x^{-1}\cdot x$. Finally, if the last condition of the difinition is valid, then $e=x\cdot x^{1}=x^{-1}\cdot x$, so the $(G,\otimes)$ is a strong e-hypergroup.

Remark that the main e-construction gives an extremely large class of e-hopes. The most useful are the ones where only few products are enlarged and, even more, the extra elements are one or two.

Example 3.3 Consider the quaternion group $\boldsymbol{Q}=\{1,-1, i,-i, j,-j, k,-k\}$ with $i^2=j^2=k^2=-1$, $ij=k$, $jk=i$, $ki=j$. Denoting $\underline{i}=\{i,-i\}$, $\underline{j}=\{j,-j\}$, $\underline{k}=\{k,-k\}$ we may define a very large number (*) of hopes by enlarging only few products. For example, $(-1)*k=\underline{k}$, $k*i=\underline{j}$ and $i*j=\underline{k}$. Then the (Q,*) is a strong e-hypergroup.

An important new field in hypermathematics comes straightforward from Santilli's Admissibility. We can transfer Santilli's theory in admissibility for representations in two ways: using either, the ordinary matrices and a hope on them, or using hypermatrices and ordinary operations on them [11], [12], [13], [15], [30].

The general definition is the following:

Definition 3.4 Let $\boldsymbol{L}$ be H_v-vector space over the H_v-field $(\boldsymbol{F},+,\cdot)$, $\varphi:\boldsymbol{F}\to\boldsymbol{F}/\gamma^*$, the canonical map and $\omega_F=\{x\in\boldsymbol{F}:\varphi(x)=0\}$, where 0 is the zero of the fundamental field $\boldsymbol{F}/\gamma^*$. Let ω_L be the core of the canonical map $\varphi':\boldsymbol{L}\to\boldsymbol{L}/\varepsilon^*$ and denote by the same symbol 0 the zero of $\boldsymbol{L}/\varepsilon^*$. Take two subsets $\boldsymbol{R}, \boldsymbol{S}\subseteq\boldsymbol{L}$ then a ***Lie-Santilli admissible hyperalgebra*** is obtained by taking the Lie bracket, which is a hope:

$$[\ ,\ ]_{RS}:\boldsymbol{L}\times\boldsymbol{L}\to\boldsymbol{P}(\boldsymbol{L}):\ [x,y]_{RS}=x\boldsymbol{R}y-y\boldsymbol{S}x=\{xry-ysx \mid r\in\boldsymbol{R},\ s\in\boldsymbol{S}\}$$

On non square matrices we can define admissibility as follow:

Construction 3.5 Let $(\boldsymbol{L}=\boldsymbol{M}_{m\times n},+)$ be H_v-vector space of m×n hyper-matrices on the H_v-field $(\boldsymbol{F},+,\cdot)$, $\varphi:\boldsymbol{F}\to\boldsymbol{F}/\gamma^*$, canonical map and $\omega_F=\{x\in\boldsymbol{F}:\varphi(x)=0\}$, where 0 is the zero of $\boldsymbol{F}/\gamma^*$. Similarly, let ω_L be the core of $\varphi':\boldsymbol{L}\to\boldsymbol{L}/\varepsilon^*$ and denote by the same symbol 0 the zero of $\boldsymbol{L}/\varepsilon^*$. Take any two subsets $\boldsymbol{R},\boldsymbol{S}\subseteq\boldsymbol{L}$ then a *Santilli's Lie-admissible hyperalgebra* is obtained by taking the Lie bracket, which is a hope:

$$[\ ,\ ]_{RS}:\boldsymbol{L}\times\boldsymbol{L}\to\boldsymbol{P}(\boldsymbol{L}):\ [x,y]_{RS}=x\boldsymbol{R}^t y-y\boldsymbol{S}^t x.$$

Notice that $[x,y]_{RS}=x\boldsymbol{R}^t y-y\boldsymbol{S}^t x=\{xr^t y-ys^t x \mid r\in\boldsymbol{R} \text{ and } s\in\boldsymbol{S}\}$

Definition 3.6 According to Santilli's iso-theory [9], [13], [14], [15], [26], [30], on a field F=(F,+,·), *a general isofield* $\hat{\boldsymbol{F}}=\hat{\boldsymbol{F}}(\hat{a},\hat{+},\hat{\times})$ is defined to be a field with elements $\hat{a}=a\times\hat{1}$, called *isonumbers*, where $a\in F$, and $\hat{1}$ is a positive-defined element generally outside F, equipped with two operations $\hat{+}$ and $\hat{\times}$ where $\hat{+}$ is the sum with the conventional additive unit 0, and $\hat{\times}$ is a new multiplication

$$\hat{a}\,\hat{\times}\,\hat{b}:=\hat{a}\times\hat{T}\times\hat{b},\quad \text{with } \hat{1}=\hat{T}^{-1},\ \forall\hat{a},\hat{b}\in\hat{\boldsymbol{F}} \qquad \text{(i)}$$

called *iso-multiplication*, for which $\hat{1}$ is the left and right unit of $\hat{F}$,

$$\hat{1}\hat{\times}\hat{a}=\hat{a}\times\hat{1}=\hat{a}\ ,\ \forall\hat{a}\in\hat{\boldsymbol{F}} \qquad \text{(ii)}$$

called *iso-unit*. The rest properties of a field are reformulated analogously.

In order to transfer this theory into the hyperstructure case we generalize only the new multiplication $\hat{\times}$ from (i), by replacing with a hope including the old one. We introduce two general constructions on this direction as follows:

Construction 3.7 *General enlargement.* On a field $\boldsymbol{F}=(\boldsymbol{F},+,\cdot)$ and on the isofield $\hat{\boldsymbol{F}}=\hat{\boldsymbol{F}}(\hat{a},\hat{+},\hat{\times})$ we replace in the iso-product

$$\hat{a}\hat{\times}\hat{b}=\ \hat{a}\times\hat{T}\times\hat{b}, \quad \text{with } \hat{1}=\hat{T}^{-1}$$

the element $\hat{T}$ by a set of elements $\hat{H}_{ab}=\{\hat{T},\hat{x}_1,\hat{x}_2,\ldots\}$ where $\hat{x}_1,\hat{x}_2,\ldots\in\hat{\boldsymbol{F}}$, containing $\hat{T}$, for all $\hat{a}\hat{\times}\hat{b}$ for which $\hat{a},\hat{b}\notin\{\hat{0},\hat{1}\}$ and $\hat{x}_1,\hat{x}_2,\ldots\in\hat{\boldsymbol{F}}\text{-}\{\hat{0},\hat{1}\}$. If one of $\hat{a}$, $\hat{b}$, or both, is equal to $\hat{0}$ or $\hat{1}$, then $\hat{H}_{ab}=\{\hat{T}\}$. Therefore, the new iso-hope is

$$\hat{a}\hat{\times}\hat{b}=\hat{a}\times\hat{H}_{ab}\times\hat{b}=\hat{a}\times\{\hat{T},\hat{x}_1,\hat{x}_2,\ldots\}\times\hat{b},\ \forall\hat{a},\hat{b}\in\hat{\boldsymbol{F}} \qquad \text{(iii)}$$

$\hat{\boldsymbol{F}}=\hat{\boldsymbol{F}}(\hat{a},\hat{+},\hat{\times})$ is an *isoH_v-field.* The elements of $\boldsymbol{F}$ are called *isoH_v-numbers* or *isonumbers*.

Remarks 3.8 Important hopes of this construction are those where the result is enlarged only for few ordered pairs $(\hat{a},\hat{b})$, even more, the extra elements $\hat{x}_i$, are only few, preferable exactly one. Thus, this special case is if there exists only one pair $(\hat{a},\hat{b})$ for which

$$\hat{a}\hat{\times}\hat{b}=\ \hat{a}\times\{\hat{T},\hat{x}\}\times\hat{b},\ \ \forall\hat{a},\hat{b}\in\hat{\boldsymbol{F}}$$

and the rest are ordinary results, then we have a so called *very thin isoH_v-field.*

The assumption that $\hat{H}_{ab}=\{\hat{T}\}$, $\hat{a}$ or $\hat{b}$, is equal to $\hat{0}$ or $\hat{1}$, with that $\hat{x}_i$, are not $\hat{0}$ or $\hat{1}$, give that the isoH_v-field has one scalar absorbing $\hat{0}$, one scalar $\hat{1}$, and $\forall\hat{a}\in\hat{\boldsymbol{F}}$, has one inverse.

Construction 3.9 *The P-hope.* Consider an isofield $\hat{\boldsymbol{F}}=\hat{\boldsymbol{F}}(\hat{a},\hat{+},\hat{\times})$ with $\hat{a}=a\times\hat{1}$, the isonumbers, where $a\in F$, and $\hat{1}$ is a positive-defined element generally outside F, with two operations $\hat{+}$ and $\hat{\times}$, where $\hat{+}$ is the sum with the conventional unit 0, and $\hat{\times}$ is the iso-multiplication

$$\hat{a}\hat{\times}\hat{b}:=\ \hat{a}\times\hat{T}\times\hat{b}, \quad \text{with } \hat{1}=\hat{T}^{-1},\ \forall\hat{a},\hat{b}\in\hat{\boldsymbol{F}}$$

Take a set $\hat{P}=\{\hat{T},\hat{p}_1,\ldots,\hat{p}_s\}$, with $\hat{p}_1,\ldots,\hat{p}_s\in\hat{\boldsymbol{F}}\text{-}\{\hat{0},\hat{1}\}$, define the ***isoP-$H_v$-field***, $\hat{\boldsymbol{F}}=\hat{\boldsymbol{F}}(\hat{a},\hat{+},\hat{\times}_P)$, where the hope $\hat{\times}_P$ as follows:

$$\hat{a}\,\hat{\times}_P\,\hat{b} := \begin{cases} \hat{a}\times\hat{P}\times\hat{b} = \{\hat{a}\times\hat{h}\times\hat{b} \mid \hat{h}\in\hat{P}\} & \text{if } \hat{a}\neq\hat{1} \text{ and } \hat{b}\neq\hat{1} \\ \hat{a}\times\hat{T}\times\hat{b} & \text{if } \hat{a}=\hat{1} \text{ or } \hat{b}=\hat{1} \end{cases} \quad \text{(iv)}$$

The elements of $\hat{\boldsymbol{F}}$ are called *isoP-H$_v$-numbers*.

Remark. If $\hat{P}=\{\hat{T},\hat{p}\}$, that is that $\hat{P}$ contains only one $\hat{p}$ except $\hat{T}$. The inverses in isoP-H$_v$-fields, are not necessarily unique.

Consruction 3.10 The generalized P-construction can be applied on rings to obtain H$_v$-fields. Thus for, $\hat{\boldsymbol{Z}}_{10}=\boldsymbol{Z}_{10}(\underline{\hat{a}},\hat{+},\hat{\times})$, and if we take $\hat{P}=\{\underline{\hat{2}},\underline{\hat{7}}\}$, then we have the table

$\hat{\times}$	$\underline{\hat{\mathbf{0}}}$	$\underline{\hat{\mathbf{1}}}$	$\underline{\hat{\mathbf{2}}}$	$\underline{\hat{\mathbf{3}}}$	$\underline{\hat{\mathbf{4}}}$	$\underline{\hat{\mathbf{5}}}$	$\underline{\hat{\mathbf{6}}}$	$\underline{\hat{\mathbf{7}}}$	$\underline{\hat{\mathbf{8}}}$	$\underline{\hat{\mathbf{9}}}$
$\underline{\hat{\mathbf{0}}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$	$\underline{\hat{0}}$
$\underline{\hat{\mathbf{1}}}$	$\underline{\hat{0}}$	$\underline{\hat{1}}$	$\underline{\hat{2}}$	$\underline{\hat{3}}$	$\underline{\hat{4}}$	$\underline{\hat{5}}$	$\underline{\hat{6}}$	$\underline{\hat{7}}$	$\underline{\hat{8}}$	$\underline{\hat{9}}$
$\underline{\hat{\mathbf{2}}}$	$\underline{\hat{0}}$	$\underline{\hat{2}}$	$\underline{\hat{8}}$	$\underline{\hat{2}}$	$\underline{\hat{6}}$	$\underline{\hat{0}}$	$\underline{\hat{4}}$	$\underline{\hat{8}}$	$\underline{\hat{2}}$	$\underline{\hat{6}}$
$\underline{\hat{\mathbf{3}}}$	$\underline{\hat{0}}$	$\underline{\hat{3}}$	$\underline{\hat{2}}$	$\underline{\hat{3},\hat{8}}$	$\underline{\hat{4}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{6}}$	$\underline{\hat{2},\hat{7}}$	$\underline{\hat{8}}$	$\underline{\hat{4},\hat{9}}$
$\underline{\hat{\mathbf{4}}}$	$\underline{\hat{0}}$	$\underline{\hat{4}}$	$\underline{\hat{6}}$	$\underline{\hat{4}}$	$\underline{\hat{2}}$	$\underline{\hat{0}}$	$\underline{\hat{8}}$	$\underline{\hat{6}}$	$\underline{\hat{4}}$	$\underline{\hat{2}}$
$\underline{\hat{\mathbf{5}}}$	$\underline{\hat{0}}$	$\underline{\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{0}}$	$\underline{\hat{0},\hat{5}}$
$\underline{\hat{\mathbf{6}}}$	$\underline{\hat{0}}$	$\underline{\hat{6}}$	$\underline{\hat{4}}$	$\underline{\hat{6}}$	$\underline{\hat{8}}$	$\underline{\hat{0}}$	$\underline{\hat{2}}$	$\underline{\hat{4}}$	$\underline{\hat{6}}$	$\underline{\hat{8}}$
$\underline{\hat{\mathbf{7}}}$	$\underline{\hat{0}}$	$\underline{\hat{7}}$	$\underline{\hat{8}}$	$\underline{\hat{2},\hat{7}}$	$\underline{\hat{6}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{4}}$	$\underline{\hat{3},\hat{8}}$	$\underline{\hat{2}}$	$\underline{\hat{1},\hat{6}}$
$\underline{\hat{\mathbf{8}}}$	$\underline{\hat{0}}$	$\underline{\hat{8}}$	$\underline{\hat{2}}$	$\underline{\hat{8}}$	$\underline{\hat{4}}$	$\underline{\hat{0}}$	$\underline{\hat{6}}$	$\underline{\hat{2}}$	$\underline{\hat{8}}$	$\underline{\hat{4}}$
$\underline{\hat{\mathbf{9}}}$	$\underline{\hat{0}}$	$\underline{\hat{9}}$	$\underline{\hat{6}}$	$\underline{\hat{4},\hat{9}}$	$\underline{\hat{2}}$	$\underline{\hat{0},\hat{5}}$	$\underline{\hat{8}}$	$\underline{\hat{1},\hat{6}}$	$\underline{\hat{4}}$	$\underline{\hat{2},\hat{7}}$

Then the fundamental classes are

$$(0)=\{\underline{\hat{0}},\underline{\hat{5}}\},\quad (1)=\{\underline{\hat{1}},\underline{\hat{6}}\},\quad (2)=\{\underline{\hat{2}},\underline{\hat{7}}\},\quad (3)=\{\underline{\hat{3}},\underline{\hat{8}}\},\quad (4)=\{\underline{\hat{4}},\underline{\hat{9}}\},$$

and the multiplicative table is the following

×	**(0)**	**(1)**	**(2)**	**(3)**	**(4)**
(0)	(0)	(0)	(0)	(0)	(0)
(1)	(0)	(1),(2)	(2),(4)	(3),(1)	(4),(3)
(2)	(0)	(2),(4)	(3)	(2)	(1)
(3)	(0)	(3),(1)	(2)	(3)	(4)
(4)	(0)	(4),(3)	(1)	(4)	(2)

Consequently, $\hat{\boldsymbol{Z}}_{10}=\boldsymbol{Z}_{10}(\underline{\hat{a}},\hat{+},\hat{\times})$, is an H$_v$-field.

4. CONCUSIONS

Anytime applied sciences ask from mathematics to have a model to express a new theory, then mathematicians search for existing mathematics if there is an appropriate one. If there is none, then they try to create a new mathematics to represent the required axioms. The H_v-structures can offer to the Lie-Santilli theory some of the models needed, because they are multivalued and there is a huge number of H_v-structures defined on the same set. Moreover, the appropriate H_v-fields can sometimes answer the basic question of Santilli's theory: What are the hypernumbers on which the entire theory is constructed via mere compatibility arguments.

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EPR ARGUMENT AND MYSTERY OF THE REDUCED PLANCK'S CONSTANT

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Abstract

Abstract: According to Einstein, Podolsky and Rosen, 'quantum mechanics is incomplete' and it is popularly known as "EPR argument". R.M.Santilli is seriously working in this direction and trying to prove it. In this context, we would like to appeal that, when mass of any elementary particle is extremely small/negligible compared to macroscopic bodies, highly curved microscopic space-time can be addressed with large gravitational constants. Following this kind of approach, it is possible to show that, Reduced Planck's constant is a compactified coupling constant of electroweak gravity.

Keywords: Four gravitational constants; Electro weak Fermion; Reduced Planck's constant;

Nomenclature	
1) Newtonian gravitational constant $= G_N$	18) Neutron life time$= t_n$
2) Electromagnetic gravitational constant$= G_e$	19) Weak interaction string tension$= F_w$
3) Nuclear gravitational constant $= G_s$	20) Strong interaction string tension$= F_s$
4) Weak gravitational constant $= G_W$	21) Electromagnetic interaction string tension$= F_e$
5) Fermi's weak coupling constant $= G_F$	22) Gravitational interaction string tension$= F_g$
6) Strong coupling constant $= \alpha_s$	23) Weak interaction string potential$= E_w$
7) Mass of electroweak fermion $= M_w$	24) Strong interaction string potential $= E_s$
8) Reduced Planck's constant $= \hbar$	25) Electromagnetic interaction string potential $= E_e$
9) Speed of light $= c$	26) Gravitational interaction string potential $= E_g$
10) Elementary charge $= e$	27) Fine structure ratio $= \alpha$
11) Strong nuclear charge $= e_s$	28) Nuclear fine structure ratio $= \alpha_n$
12) Mass of proton $= m_p$	29) Mass of pions $= (m_\pi)^0, (m_\pi)^\pm$
13) Mass of neutron $= m_n$	30) Mass of weak bosons $= (m_z)^0, (m_w)^\pm$
14) Mass of electron $= m_e$	
15) Charge radius of nucleus$= R_0$	
16) Root mean square radius of proton $= R_p$	
17) Magnetic moment of proton $= \mu_p$	

1. Introduction

As it is well known, Albert Einstein did not accept quantum mechanical uncertainties as being final, for which reason he made his famous quote "God does not play dice with the universe." More particularly, Einstein believed that "quantum mechanics is not a complete theory," in the sense that it could be broadened into such a form to recover classical determinism at least under limit conditions. Einstein communicated his views to B. Podolsky and N. Rosen and they jointly published in 1935 the historical paper [1] that became known as the EPR argument. In view of the rather widespread belief that quantum mechanics is a final theory valid for all conceivable conditions existing in the universe, objections against the EPR argument have been voiced by numerous scholars, including by N. Bohr [2], J. S. Bell [3,4], J. von Neumann [5] and others (see Ref. [6] for a review and comprehensive literature). The field became known as local realism and included the dismissal of the EPR argument based on claims that quantum axioms do not admit hidden variables λ [7, 8]. In this context, R.M.Santilli is seriously working and proving the 'EPR' argument based on 'isosymmetries' [9-13].

Even though our approach is different, we would like to emphasize the point that, 'quantum mechanics' is certainly an incomplete theory because of its poor background associated with the mysterious origin of the 'reduced Planck's constant' and it is the root cause of failure of unification of 'quantum mechanics' and 'gravity'.

Subject of final unification is very interesting. But, unifying gravity and quantum mechanics (QM) is very much complicated and scientists are trying their level best in different ways. As gravitational effects are negligible at quantum level, standard model of particle physics attempts to explore the secrets of elementary particles. On the other hand, as quantum effects are negligible at macroscopic level, General theory of relativity (GTR) attempts to explore the secrets of the universe. The most complicated question to be answered is – If celestial objects are confirmed to be made up of various kinds of atoms, whether 'gravity' is causing the atoms to form into celestial spheres or quantum rules are causing the atoms to form into celestial spheres that show gravity?

Astrophysics point of view or 'Planck scale' point of view, there is a possibility of observing the combined effects of GTR and QM at intermediate energy scales. In between GTR and QM, there exist fascinating and most complicated astrophysical objects, i.e. Black holes. Even though their detection is a great mystery, one can see the best possibility of understanding QM and GTR at extreme energy scales. Here, we would like to emphasize the point that, astrophysical observations pertaining to Black holes and various other compact stellar objects just reveal the combined effects of GTR and QM but no way indicate the secrets of unification of QM and GTR. One most common point of QM and GTR is "mass". By understanding the massive origin of elementary particles, it may be possible to probe the secrets of QM and GTR.

The primary goal of quantum gravity is to join the laws of quantum mechanics with the laws of general relativity into a single mathematically consistent framework. Many scientists believe that, String theory [14,15,16] is one best candidate of quantum gravity. It is embedded with beautiful physical concepts like open strings, closed strings, string vibrations, string length, string tension and 'fermion-boson super symmetry'. Scientists strongly believe that, String theory is empowered with good mathematics and smartly fits gravity in unification program. Point to be noted is that, by considering the Planck length as characteristic amplitude associated with strings, String theory advances its ideological representation. Very unfortunate thing is that, even though, originally, String theory was proposed for understanding 'strong interaction, as Planck length is 20 orders of magnitude less than nuclear size, it is badly failing in explaining and predicting nuclear scale physical phenomena. Here we would like to stress the point that, the main reason for its fatal failure is – "implementation of the two famous physical constants H-bar and big G as-they-are". We would like to say that, without addressing the roots of H-Bar and big G, it is impossible to construct a workable model of final unification.

2. Three large atomic gravitational coupling constants

When mass of any elementary particle is extremely small/negligible compared to macroscopic bodies, highly curved microscopic space-time can be addressed with large gravitational constants and magnitude of elementary gravitational constant seems to increase with decreasing mass and increasing interaction range. Based on this logic, we consider the possibility of existence of three large gravitational constants assumed to be associated with the electromagnetic, strong and weak interactions [17-32]. Compared to multi-dimensions and unproved maths of any String theory model, our proposal can be given some positive consideration. Following the notion of string theory, compactification of un-observable spatial dimensions might be playing a key role in hiding the large magnitudes of the three atomic gravitational constants. If multi dimensional physics is having a real sense, then, compactification of large magnitudes of atomic gravitational constants can also be possible.

By following our idea, in analogy with Planck scale, as an immediate result, it seems possible to have three different string amplitudes corresponding to electromagnetic, strong and weak interactions. In this way, String theory can be shaped to a model of elementary particle physics associated with 3+1 dimensions. Another advantage is that, considering the combined effect of the three atomic gravitational constants, origins of H-Bar and big G can be understood. Including the Newtonian gravitational constant, as the subject under consideration deals with 4 different gravitational constants, our model can be called as 4G model of final unification or Microscopic Quantum Gravity. With further study, Planck scale and electroweak scale can be studied in a unified manner. During cosmic evolution, if one is willing to give equal importance to Higgs boson and Planck mass in understanding the massive origin of elementary particles, then it seems quite logical to expect a common relation between Planck scale and Electroweak scale.

3. Basic assumptions

1) There exists a characteristic electroweak fermion of rest energy, $M_w c^2 \cong 584.725$ GeV. It can be considered as the zygote of all elementary particles.
2) There exists a strong interaction elementary charge (e_s) in such a way that, its squared ratio with normal elementary charge is close to the reciprocal of the strong coupling constant.
3) Each atomic interaction is associated with a characteristic large gravitational coupling constant.

4. To validate assumption-1

To validate assumption-1, we argue with the following nuclear and particle level observations.

1) It is generally believed that, $(m_\pi)^0, (m_\pi)^\pm$ are the force carriers of strong interaction and $(m_z)^0, (m_w)^\pm$ are the force carriers of weak interaction. Considering Pions and electroweak bosons, to a great surprise, we noticed that,

$$\left(\frac{\sqrt{m_p m_n}}{M_w}\right) \cong 0.001606 \cong \left(\frac{\sqrt{\left(m_\pi c^2\right)^0 \left(m_\pi c^2\right)^\pm}}{\sqrt{\left(m_z c^2\right)^0 \left(m_w c^2\right)^\pm}}\right) \tag{1}$$
$$\cong \left(\frac{\sqrt{134.98 \times 139.57}\ \text{MeV}}{\sqrt{80379.0 \times 91187.6}\ \text{MeV}}\right) \cong 0.0016032.$$

2) It is also very interesting to note that,

$$\frac{\sqrt{m_p m_n}}{\sqrt{(m_\pi)^0 (m_\pi)^\pm}} \cong 6.84 \cong \frac{M_w}{\sqrt{(m_z)^0 (m_w)^\pm}} \cong 6.83. \tag{2}$$

3) As neutron's weak decay is directly responsible for nuclear stability associated with beta emission, based on the two numerical coincidences, i.e. 0.0016 and 6.83, existence of our assumed 584.725 GeV weak fermion can be confirmed and it is also possible to have a relation of the form,

$$M_w \cong \left(\frac{\sqrt{(m_z)^0 (m_w)^{\pm}}}{\sqrt{(m_\pi)^0 (m_\pi)^{\pm}}} \right) m_p \cong 585.244 \text{ GeV}/c^2 . \quad (3)$$

4) With reference to nucleons and pions, it is reasonable to argue that, if one is willing to consider $\left(m_z c^2\right)^0 \& \left(m_w c^2\right)^{\pm}$ as the force carriers of weak interaction [33,34,35,36], one should not ignore the possibility of considering the proposed weak fermion of rest energy 584.725 GeV as the characteristic field generator of weak interaction.
5) Weak force carriers cannot exist without the existence of their weak field generating fermion.
6) We would like to emphasize that, independent of the famous semi empirical mass formula (SEMF), and using the ratio 0.0016, nuclear stability and nuclear binding energy can be understood with four simple terms and one energy coefficient. See section 7.

5. Semi empirical derivations

This section has been divided into 4 sub sections [26,29]. Based on the proposed second and third assumptions, in section 5.1, relations (3), (7) and (11) have been defined.

In Section 5.2 important numerical fitting relations (13), (14), (15) and (16) have been proposed (pertaining to nuclear charge radius, Planck size and Fermi's weak coupling constant) and an attempt has been made to infer an expression for weak gravitational constant.

In Section 5.3, based on the results obtained from Section 5.1 and Section 5.2, an important inference i.e. relation (21) has been made.

Section 5.4 includes simplified relations pertaining to elementary mass ratios, Newtonian gravitational constant and strong coupling constant.

5.1 Defined basic relations and their consequences

5.1.1 Ratio of Newtonian and electromagnetic gravitational constants

Considering the similarities in between gravitational and electromagnetic interactions, relation (3) has been defined to understand the role and to estimate the approximate magnitudes of the electromagnetic and Newtonian gravitational constants [37].

$$\left.\begin{array}{l} m_p \cong \left(\dfrac{G_N}{G_e}\right)^{\frac{1}{6}} \sqrt{M_{pl} \times m_e} \cong \left(\dfrac{G_N}{G_e}\right)^{\frac{1}{6}} \left(\dfrac{\hbar c m_e^2}{G_N}\right)^{\frac{1}{4}} \\ \text{where, } M_{pl} \cong \sqrt{\dfrac{\hbar c}{G_N}} \cong \text{Planck mass} \end{array}\right\} \tag{4}$$

On rearranging relation (4),

$$M_{pl} \cong \sqrt{\frac{\hbar c}{G_N}} \cong \left(\frac{G_e}{G_N}\right)^{\frac{2}{3}} \left(\frac{m_p^2}{m_e}\right) \tag{5}$$

5.1.2 Proton – electron mass ratio

Pertaining to proton-electron mass ratio, relations (6) and (7) have been defined in the following way.

$$\frac{m_p}{m_e} \cong \left(\frac{G_e m_e^2}{\hbar c}\right)\left(\frac{G_s m_p^2}{\hbar c}\right) \tag{6}$$

$$\frac{m_p}{m_e} \cong \left(\frac{e_s^2}{4\pi\varepsilon_0 G_s m_p^2}\right) \div \left(\frac{e^2}{4\pi\varepsilon_0 G_e m_e^2}\right) \tag{7}$$

Based on the second assumption and relations (6) and (7),

$$\left(\frac{e_s}{e}\right)^2 \cong \frac{1}{\alpha_s} \cong \frac{G_s m_p^3}{G_e m_e^3} \cong \frac{G_s^2 m_p^4}{\hbar^2 c^2} \tag{8}$$

$$\left(\frac{e_s}{e}\right) \cong \sqrt{\frac{1}{\alpha_s}} \cong \sqrt{\frac{G_s m_p^3}{G_e m_e^3}} \cong \frac{G_s m_p^2}{\hbar c} \tag{9}$$

Based on relation (8), quantitatively, it can be inferred that,

$$\sqrt{\frac{e_s^2}{4\pi\varepsilon_0 G_s m_p m_e}} \cong 2\pi \tag{10}$$

Based on relation (8), substituting $e_s^2 \cong \left(\frac{G_s m_p^3}{G_e m_e^3}\right) e^2$ in relation (10),

$$\frac{m_p}{m_e} \cong 2\pi \sqrt{\frac{4\pi\varepsilon_0 G_e m_e^2}{e^2}} \tag{11}$$

Based on relation (4) and (6), on eliminating $\hbar c$,

$$\frac{m_p}{m_e} \cong \left(\frac{G_s}{G_e^{1/3} G_N^{2/3}}\right)^{1/7} \tag{12}$$

5.2 Numerical fits and their consequences pertaining to nuclear charge radius, Planck size and Fermi's weak coupling constant

With reference to nuclear gravitational constant, nuclear charge radius can be fitted with,

$$R_0 \cong \left(\frac{2G_s m_p}{c^2}\right) \cong 1.2393 \text{ fm} \tag{13}$$

With reference to Planck size, it has been noticed that,

$$\left(\frac{G_s m_p}{c^2}\right) \div \sqrt{\frac{G_N \hbar}{c^3}} \cong \left(\frac{m_p}{m_e}\right)^6 \tag{14}$$

Based on relation (13), Fermi's weak coupling constant can be fitted with,

$$G_F \cong \left(\frac{m_e}{m_p}\right)^2 \hbar c R_0^2 \cong \frac{4\hbar G_s^2 m_e^2}{c^3} \tag{15}$$
$$\cong 1.4402105\times10^{-62}\ \text{J.m}^3$$

Based on relations (13), (14) and (15),

$$G_F \cong \left(\frac{m_p}{m_e}\right)^{10} \frac{4\hbar^2 G_N}{c^2} \tag{16}$$

Based on the magnitude of weak gravitational constant proposed by Roberto Onofrio [21] and based on relation (16), it has been inferred that,

$$G_w \cong \left(\frac{m_p}{m_e}\right)^{10} G_N \tag{17}$$

Based on relations (16) and (17)

$$G_F \cong \frac{4\hbar^2 G_w}{c^2} \tag{18}$$

Based on relations (15) and (18),

$$\sqrt{\frac{G_F}{\hbar c}} \cong \frac{2G_s m_e}{c^2} \cong \sqrt{\frac{4G_w \hbar}{c^3}} \tag{19}$$

5.3 Important inference and its implications pertaining to first assumption

Based on the above relations (4) to (19), on eliminating the three proposed atomic gravitational constants, one can get the following relation.

$$\alpha \cong 4\pi^2 \left(\frac{m_e}{m_p}\right)^7 \sqrt{\frac{\hbar c}{G_N m_p^2}} \tag{20}$$

If one is willing infer that,

$$\hbar c \cong G_w M_w^2 \tag{21}$$

Based on relations (19) and (21),

$$G_s m_e \cong G_w M_w \tag{22}$$

Based on relations (20), (21) and (22), the following relations can be obtained.

$$m_p \cong \left(\frac{4\pi^2}{\alpha}\right)^{\frac{1}{3}} \left(M_w m_e^2\right)^{\frac{1}{3}} \tag{23}$$

$$G_F \cong G_w M_w^2 R_w^2 \quad \text{where, } R_w \cong \frac{2G_w M_w}{c^2} \tag{24}$$

$$m_e \cong \left(\frac{G_w}{G_s}\right) M_w \tag{25}$$

$$m_p \cong \left(\frac{G_s}{G_w}\right)\left(\frac{G_s}{G_e}\right) M_w \cong \left(\frac{G_s^2}{G_w G_e}\right) M_w \tag{26}$$

$$\frac{m_p}{m_e} \cong \frac{G_s^3}{G_w^2 G_e} \tag{27}$$

$$\hbar c \cong \left(\frac{G_e G_w}{G_s}\right) m_p m_e \cong G_s M_w m_e \tag{28}$$

5.4 Simplified relations for elementary mass ratios, Newtonian gravitational constant and strong coupling constant

On eliminating proton and electron rest masses, Newtonian gravitational constant and strong coupling constant take the following simplified forms.

$$G_N \cong \frac{G_w^{21} G_e^{10}}{G_s^{30}} \tag{29}$$

$$\frac{1}{\alpha_s} \cong \frac{G_s^{10}}{G_e^4 G_w^6} \tag{30}$$

Based on relations (29) and (30),

$$\alpha_s \cong \frac{G_N^{1/3} G_e^{2/3}}{G_w} \tag{31}$$

6. Characteristic unified relations pertaining to estimation of (G_e, G_s, G_w, G_N)

a) With the following relation, magnitude of G_e can be estimated.

$$G_e \cong \frac{e^2 m_p^2}{16\pi^3 \varepsilon_0 m_e^4} \tag{32}$$

b) After finding the value of G_e,with the following relation, magnitude of G_s can be estimated.

$$G_s \cong \frac{G_w^2 M_w^4}{G_e m_p m_e^3} \cong \frac{\hbar^2 c^2}{G_e m_p m_e^3} \tag{33}$$

c) After finding the value of G_s , weak gravitational constant can be estimated with a relation of the form,

$$G_w \cong \sqrt{\left(\frac{m_e}{m_p}\right)\frac{G_s^3}{G_e}} \tag{34}$$

d) Thus, quantitatively,

$$\left.\begin{array}{l} G_e \cong 2.374335\times10^{37}\ \mathrm{m^3kg^{-1}sec^{-2}} \\ G_s \cong 3.329561\times10^{28}\mathrm{m^3kg^{-1}sec^{-2}} \\ G_w \cong 2.909745\times10^{22}\ \mathrm{m^3kg^{-1}sec^{-2}} \\ G_N \cong 6.679855\times10^{-11}\ \mathrm{m^3kg^{-1}sec^{-2}} \\ G_F \cong 1.4402105\times10^{-62}\ \mathrm{J.m^3} \\ \alpha_s \cong 0.1151937 \text{ and } e_s \cong 2.9463591e \end{array}\right\}$$

e) Based on relation (25), we are working on developing procedures for estimating the magnitude of strong gravitational constant and weak gravitational constant independent of the reduced Planck's constant. Appropriate relations seem to be associated with the experimental values of strong coupling constant [38,39], nuclear charge radius [40,41,42,43], magnetic moment of proton and neutron life time [44].

$$\alpha_s \cong \frac{G_e m_e^3}{G_s m_p^3} \approx 0.1152 \tag{35}$$

$$R_0 \cong \frac{2G_s m_p}{c^2} \approx 1.24 \text{ fm.} \tag{36}$$

$$\mu_p \cong \frac{eG_s m_p}{2c} \approx 1.487\times10^{-26}\ \mathrm{J.Tesla^{-1}} \tag{37}$$

$$t_n \cong \left(\frac{G_e^2 m_n^2}{G_w\left(m_n - m_p\right)c^3}\right) \approx 874.94 \text{ sec} \tag{38}$$

7. Understanding nuclear binding energy with single energy coefficient and four simple terms

We would like to emphasize the fact that, physics and mathematics associated with fixing of the energy coefficients of semi empirical mass formula (SEMF) [45,46,47] are neither connected with residual strong nuclear force nor connected with strong coupling constant α_s. Since nuclear force is mediated via quarks and gluons, it is necessary and compulsory to study the nuclear binding energy scheme in terms of nuclear coupling constants. In this direction, N. Ghahramany and team members have taken a great initiative in exploring the secrets of nuclear binding energy and magic numbers [48,49] with reference to quarks. Very interesting point of their study is that - nuclear binding energy can be understood with two or three terms having single variable energy coefficient. In this direction, based on three unified assumptions connected with gravity and atomic interactions, in a semi empirical approach, we have developed a very simple formula for nuclear binding energy with single energy coefficient having four simple terms [27]. Corresponding relations can be expressed in the following way. Starting from Z=3 to 118,

$$A_s \cong 2Z + 0.0016(2Z)^2 \cong 2Z + 0.0064Z^2 \tag{39}$$

$\cong$ Estimated mass number close to proton-neutron mean stability line.

$$BE \cong \left\{ A - A_{fg} - A^{1/3} - \frac{(A_s - A)^2}{A_s} \right\} (B_0 \cong 10.1 \text{ MeV}) \tag{40}$$

$\cong$ Estimated nuclear binding energy

Here, we would like to appeal that,

1) $A_{fg} \cong \left(1 + 0.0019A\sqrt{ZN}\right)$ can be called as the geometric number of free or unbound nucleons.
2) $A^{1/3}$ can be called as radial factor associated with nucleons.
3) $\frac{(A_s - A)^2}{A_s}$ can be called as isotopic asymmetric term associated with mean stable mass number.
4) Binding energy coefficient, $B_0 \cong \frac{1}{\alpha_s}\left(\frac{e^2}{4\pi\varepsilon_0 R_0}\right) \cong 10.1 \text{ MeV}$ seems to be associated with nuclear radius, strong coupling constant and fine structure ratio.
5) Proceeding further, by considering electroweak interaction and eliminating the number 0.0019, it is possible to show that,

$$BE \cong \left(A - A^{1/3} - \frac{(A_s - A)^2}{A_s} \right) 10.1\,\text{MeV} - \left[1 + \left(0.00161 A \sqrt{ZN} \right) \right] 11.9\,\text{MeV} \qquad (41)$$

8. Estimating nuclear charge radii independent of quantum concepts

Without considering quantum concepts, nuclear charge radii can be estimated with the following expression. For medium and heavy atomic nuclides,

$$R_{(Z,N)} \cong \left[Z^{1/3} + \left(\sqrt{ZN} \right)^{1/3} \right] \left(\frac{G_s \sqrt{m_p m_n}}{c^2} \right) \qquad (42)$$

This relation can be compared the currently believed various relations pertaining to the estimation of nuclear charge radii [50,51]

9. Discussion

We would like to emphasise the following points:

1) Even though quantum mechanics is successful in understanding the quantum effects of microscopic systems, origin of the reduced Planck's constant is still a mystery at the microscopic level.
2) String theory is silent on the universal applicability of the reduced Planck's constant.
3) During cosmic evolution, if one is willing to give equal importance to Higgs Boson and Planck mass in understanding the massive origin of elementary particles, it seems quite logical to expect a common relation between Planck scale and Electroweak scale.
4) When microscopic space time is more curved than the macroscopic space time curvature, it is natural to assign a large value to microscopic gravitational constant.
5) Compared to particles having a structure, for point particles the magnitude of gravitational constant can be much higher.
6) Magnitude of the elementary gravitational constant seems to increase with the decreasing mass of the elementary particle under consideration.
7) According to the String theory, the real world is a compact manifold and out of 10 dimensions, 6 spatial dimensions get compressed and will not allow any observer to identify their existence. Applying this idea to our

proposal, compactification of 6 unobservable space dimensions might be playing a key role in hiding the large magnitudes of the three atomic gravitational constants.

8) Using the strong nuclear charge, proton magnetic moment $(e_s\hbar/2m_p)$, nuclear fine structure ratio $\alpha_n \cong (e_s^2/4\pi\varepsilon_0\hbar c)$, unified nuclear binding energy coefficient $B_0 \cong \frac{1}{2}\sqrt{\alpha \times \alpha_n}\,(m_p c^2)$ and Fermi gas model of nuclear potential $E_F \cong \sqrt{\alpha \times \alpha_n}\,(m_p c^2 + m_n c^2)$ can be fitted. Another interesting application is that, based on strong charge conservation, electromagnetic charge conservation and super symmetry, fractional charge quarks can be understood with generation of quark fermions and quark bosons [29,30,31,32].
9) 'String Tension' is a practical aspect of String Theory [52]. Considering the proposed three atomic gravitational constants and following the universal applicability of 'speed of light', approximate tensions associated with weak, strong, electromagnetic and gravitational interactions can be represented by,

$$\left\{\begin{array}{l} F_w \cong \left(\dfrac{c^4}{4G_w}\right) \cong 6.94\times10^{10}\,\text{N} \\ F_s \cong \left(\dfrac{c^4}{4G_s}\right) \cong 6.065\times10^{4}\,\text{N} \\ F_e \cong \left(\dfrac{c^4}{4G_e}\right) \cong 8.505\times10^{-5}\,\text{N} \\ F_g \cong \left(\dfrac{c^4}{4G_N}\right) \cong 3.026\times10^{43}\,\text{N} \end{array}\right\} \quad (43)$$

10) Following the universal applicability of 'elementary charge', approximate (operating) energy potentials associated with the above string tensions can be represented by,

$$\begin{Bmatrix} E_w \cong \sqrt{\frac{e^2}{4\pi\varepsilon_0}\left(\frac{c^4}{4G_w}\right)} \cong 25.0 \text{ GeV} \\ E_s \cong \sqrt{\frac{e^2}{4\pi\varepsilon_0}\left(\frac{c^4}{4G_s}\right)} \cong 23.3 \text{ MeV} \\ E_e \cong \sqrt{\frac{e^2}{4\pi\varepsilon_0}\left(\frac{c^4}{4G_e}\right)} \cong 874 \text{ eV} \\ E_g \cong \sqrt{\frac{e^2}{4\pi\varepsilon_0}\left(\frac{c^4}{4G_N}\right)} \cong 8.355\times 10^7 \text{J} \end{Bmatrix} \quad (44)$$

11) These estimated weak, strong and electromagnetic energy potentials seem to be close to experimental values.
12) Relation (21) needs in depth discussion at fundamental level.
13) With reference to the current experimental values of root mean square radius of proton, (0.833 ± 0.01) fm and $(0.831 \pm 0.007_{stat} \pm 0.012_{syst})$ fm, we noticed one interesting relation. It can be expressed as,

$$R_p \cong \sqrt{\left(\frac{4\pi\varepsilon_0\hbar^2}{e_s^2 m_p}\right)\left(\frac{\hbar}{m_p c}\right)} \cong \sqrt{\frac{4\pi\varepsilon_0\hbar^3}{e_s^2 m_p^2 c}} \cong 0.835 \text{ fm} \quad (45)$$

In this relation,

a) $\left(\frac{4\pi\varepsilon_0\hbar^2}{e_s^2 m_p}\right) \cong 3.32$ fm can be inferred as the Bohr's model of probable distance of finding proton in the nuclear well where the operating charge is $e_s \cong 2.946e$.

b) $\left(\frac{\hbar}{m_p c}\right) \cong 0.21$ fm can be considered as the reduced Compton wavelength of proton.

Based on relation (45) and assumption(2),.

$$\hbar \cong \left[\left(\frac{e_s^2}{4\pi\varepsilon_0 c}\right)\left(m_p R_p c\right)^2\right]^{\frac{1}{3}} \quad (46)$$

10. Conclusion

With reference to pions and electroweak bosons, it is possible to confirm the existence of $M_w c^2 \cong 584.725$ GeV. Proceeding further, based on the famous EPR argument and with further research, actual essence of final unification and mystery of the reduced Planck's constant can be understood.

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MEASUREMENTS OF THE POLARIZATION CORRELATION OF THE TWO-PHOTON SYSTEM PRODUCED IN POSITRON-ELECTRON ANNIHILATION

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Abstract

We present measurements of the polarization correlation of the two-photonsystem produced in positron-electron annihilation of ^{64}Cu that were conducted at Freiburg University 1976-1980. Our experiment was motivated by two contradicting results. An experiment conducted at Columbia University observed a quantum mechanical correlation whereas an experiment conducted in Catania, Sicily observed a correlation that was about 40% lower. The two back-to-back photons are either both right-handed or left-handed polarized. The polarization of each photon is measured with a Compton polarimeter on each side of the source consisting of a plastic scintillator as scatterer and a NaI detector, which records the Compton-scattered photons.The polarization correlation is measured by the difference in azimuth angle ϕ between the two polarimeters. We performed the measurements for different scatterer shapes, different scattering angles and different distances between the source and the polarimeters. We developed a detailed MonteCarlo program for simulating the quantum mechanical expectation for each measurement setup. All measurements agree rather well with quantum mechanics. We further reproduced the results of all conducted experiments with our simulation. The reduced Polarization correlation observed by the Sicilian experiment originated from a large fraction of double scattering in which the original polarization correlation is diminished. We further performed a test of Bells inequality with our results and those measured by other experiments.

1 Introduction

In the early 1970s two experiments were conducted that measured the polarization correlation of the two annihilation photons from electron-positron annihilation. The motivation came from the EPR article [1] and Bell's investigation into hidden variables [2, 3]. The experiment by Kasday-Ulman-Wu (Columbia U., 1975) [4] measured a polarization correlation that confirmed the quantum mechanical prediction. The experiment by Faraci et al. (Catania U., 1974) [5] measured a value that was 30% lower than the quantum mechanical expectation. The discrepancy could not be explained. Therefore, K. Meisenheimer and myself under supervision of professor Runge built a new experiment for measuring the polarization correlation of the two annihilation photons from at the University of Freiburg. The experiment was conducted like a modern high-energy physics experiment with respect to data taking, analysis and simulation.

2 Measurement Principle

Positrons from a ^{64}Cu source annihilate with electrons in the conduction band typically in an S=0 state into two nearly back-to-back photons (HWHM=5.9$\pm$ 0.1 mrad), which are either both left-handed or right-handed polarized. Thus, the state vector in terms of momentum $\vec{k}_i$ and polarization $\epsilon_i = R_i,\ L_i$ is

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big\{\,|k_1, R_1\rangle\,|k_2, R_2\rangle - |k_1, L_1\rangle\,|k_2, L_2\rangle\,\Big\}. \tag{1}$$

In terms of linear polarization we get

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big\{\,|k_1, X_1\rangle\,|k_2, Y_2\rangle - |k_1, Y_1\rangle\,|k_2, X_2\rangle\,\Big\}. \tag{2}$$

The polarization is detected via Compton scattering. The probability that a photon polarized in the X direction is scattered under the angles θ and ϕ is

$$dP_i = \frac{1}{2\pi} f(\theta_i)\Big[1 - m(\theta_i)\Big] \cos(2\phi_i) d\Omega \tag{3}$$

where $f(\theta_i)$ is the differential cross section for unpolarized photons $f(\theta_i)$

$$f(\theta_i) = \frac{1}{C_N}\left(\frac{k_i'}{k_i}\right)^2\left[\frac{k_i}{k_i'} + \frac{k_i'}{k_i} - \sin^2\theta\right] \tag{4}$$

and $m(\theta_i)$ is the θ_i-dependent amplitude that describes the strength of the ϕ_i dependence and, therefore, the polarization sensitivity,

$$m(\theta_i) = \sin^2\theta_i \left[\frac{k_i}{k_i'} + \frac{k_i'}{k_i} - \sin^2\theta\right]^{-1}. \tag{5}$$

The parameter C_N is a normalization constant. The maximum polarization sensitivity is obtained for $\theta = 82°$ yielding $m(82°) = 0.691$ while $f(82°)/f(0°) = 0.203$ for $\theta = 82°$. So, $f(\theta)$ is nearly at the minimum, For fixed θ, $dP/d\Omega$ becomes maximum if the initial polarization is orthogonal to the scattering plane ($\phi = 90°$) and minimum if it is in the scattering plane ($\phi = 0°$). The analyzing power of a polarimeter is defined by

$$\epsilon_P = \frac{P_{max} - P_{min}}{P_{max} + P_{min}} \tag{6}$$

where P_{max} is the maximum of $dP/d\Omega$ if $\vec{\epsilon}$ is parallel to the polarimeter axis and P_{min} is the minimum of $dP/d\Omega$ if $\vec{\epsilon}$ is orthogonal to the polarimeter axis. For a Compton polarimeter we get

$$\epsilon_{CP} = M(\theta, \Delta\theta) \cdot N(\Delta\phi) \tag{7}$$

with

$$M(\theta, \Delta\theta) = \frac{\int_{\theta-\Delta\theta}^{\theta+\Delta\theta} f(\theta) m(\theta) \sin\theta d\theta}{\int_{\theta-\Delta\theta}^{\theta+\Delta\theta} f(\theta) \sin\theta d\theta} \tag{8}$$

and

$$N(\Delta\phi) = \frac{\int_{\phi-\Delta\phi}^{\phi+\Delta\phi} \cos(2\phi) d\phi}{\int_{\phi-\Delta\phi}^{\phi+\Delta\phi} d\phi} = \frac{\sin(2\Delta\phi)}{2\Delta\phi} \tag{9}$$

where $\Delta\theta$ and $\Delta\phi$ are the polar angle and azimuth angle acceptances of the detector.

For example, for $\theta = 82°$, $\Delta\theta = 14°$ and $\Delta\phi = 10°$ the analyzing power is $\epsilon_{CP} = 0.648 \pm 0.002$, which is 94% of the maximum analyzing power ϵ_{CP}^{max}.

The detection efficiency of a polarimeter is defined by

$$\eta_P = \frac{\text{number of detected particles}}{\text{number of particles hitting the polarimeter}} \tag{10}$$

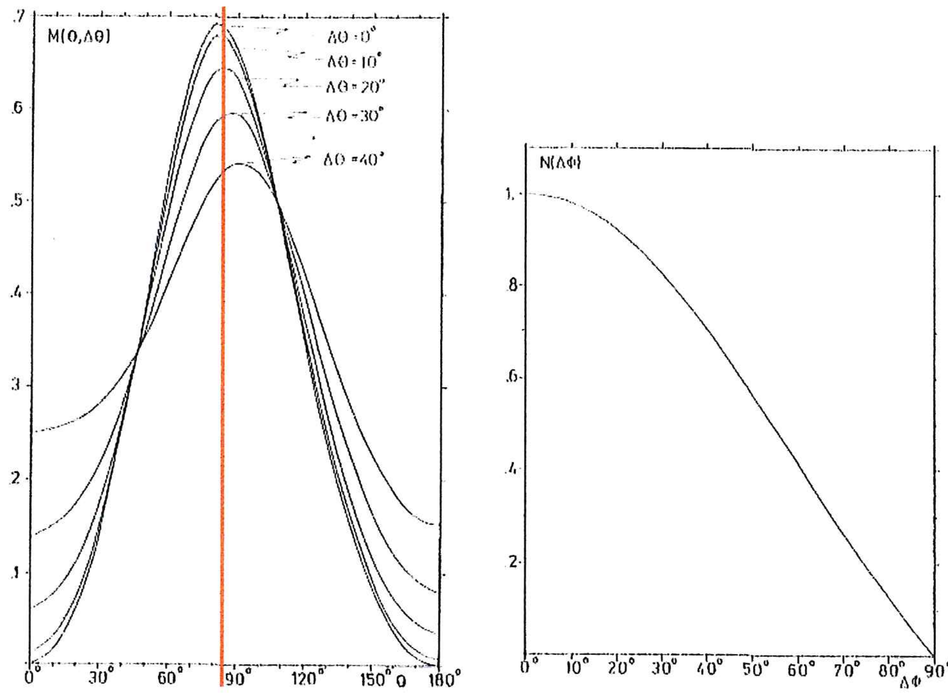

Figure 1: Left: The function $M(\theta, \Delta\theta)$ as a function of θ for different values of $\Delta\theta$. Right: The function $N(\Delta\phi)$ as a function of $\Delta\phi$.

For an ideal polarimeter $\eta_P = 1$. For a Compton polarimeter

$$\eta_{CP} = \eta_S \cdot \delta_S \cdot \eta_D \cdot \delta_D \cdot F(\theta, \Delta\Omega) \tag{11}$$

where η_S is the scattering probability in the scatterer, η_D is the probability to record a signal in the detector, δ_S is the detection limit of the scatterer, δ_D is the detection limit of the detector and $F(\theta, \Delta\Omega) = (1/2\pi) \int_{\Delta\Omega} f(\theta) d\Omega$. For the $\theta = 82°$, $\Delta\theta = 14°$ and $\Delta\phi = 10°$ we get $\eta_{CP} = (1.46 \pm 0.02) \times 10^{-3}$.

The probability that two photons represented by the state $|\psi\rangle$ are scattered under the polar angles θ_1 and θ_2 and azimuth angles ϕ_1 and ϕ_2 is given by

$$dP_{12} = \frac{1}{4\pi} f(\theta_1) f(\theta_2) \{1 - m(\theta_1) m(\theta_2)] \cos 2(\phi_1 - \phi_2) d\Omega_1 d\Omega_2\} \tag{12}$$

The flight directions of the two photons after scattering are correlated with respect to the azimuth angles with a strength of $m(\theta_1)m(\theta_2)$, which has a maximum value of 0.478. The two scattering angles are completely independent.

The product $f(\theta_1)f(\theta_2)$ denotes the probability for scattering two arbitrary polarized photons under the angles θ_1 and θ_2.

3 Experimental Setup

Figure 2 shows the experimental setup. In the center a ^{64}Cu is placed inside a collimator. The two Compton polarimeters are placed on both sides of the collimator. Each polarimeter consists of a plastic scintillator as scatterer and a NaI counter covered with a lead shield as detector. The plastic scintillator and the NaI crystal are both read out by a photomultiplier. For most measurements the NaI detector is placed at the optimal scattering angle of $\theta = 82°$. We measured four coincidence numbers: N_2 the coincidence between the two scatterers, N_{31}, N_{32} the coincidence between the two scatterers and detector 1, 2 and N_4 the coincidence between all four counters. The three- and four-fold coincidences have azimuth angle dependences.

Figure 3 (top) shows a photograph of the experiment. We chose the gravitational axis as symmetry axis. Figure 3 (bottom) shows a close up view of the Compton polarimeter. The lead shield in front of the detector permits to define the acceptance in $\Delta\theta$ and $\Delta\phi$.

4 Polarization Correlation

We define the polarization correlation $R(\theta_1, \theta_2)$ in terms of coincidence numbers to reduce systematic uncertainties from the source intensity, alignment and calibration issues. Thus

$$R(\theta_1, \theta_2) = \frac{N_4(\phi_1, \phi_2)/N_2}{N_{31}(\phi_1)/N_2 \cdot N_{32}(\phi_2)/N_2} = \frac{N_4(\phi_1, \phi_2) \cdot N_2}{N_{31}(\phi_1) \cdot N_{32}(\phi_2)} \tag{13}$$

In the laboratory system there is no preferred polarization direction. The 3-fold coincidences $N_{31}(\phi_1)$ and $N_{32}(\phi_2)$ are isotropic in ϕ_1 and ϕ_2. The angular dependence comes from $N_4(\phi_1, \phi_2)$, yielding

$$R(\theta_1, \theta_2) = A[1 - \beta \cos 2(\phi_1 - \phi_2)] \tag{14}$$

where $A = 1$ is the normalization and β is the polarization correlation parameter. Deviations from $A = 1$ may come from systematic effects like non-perfect

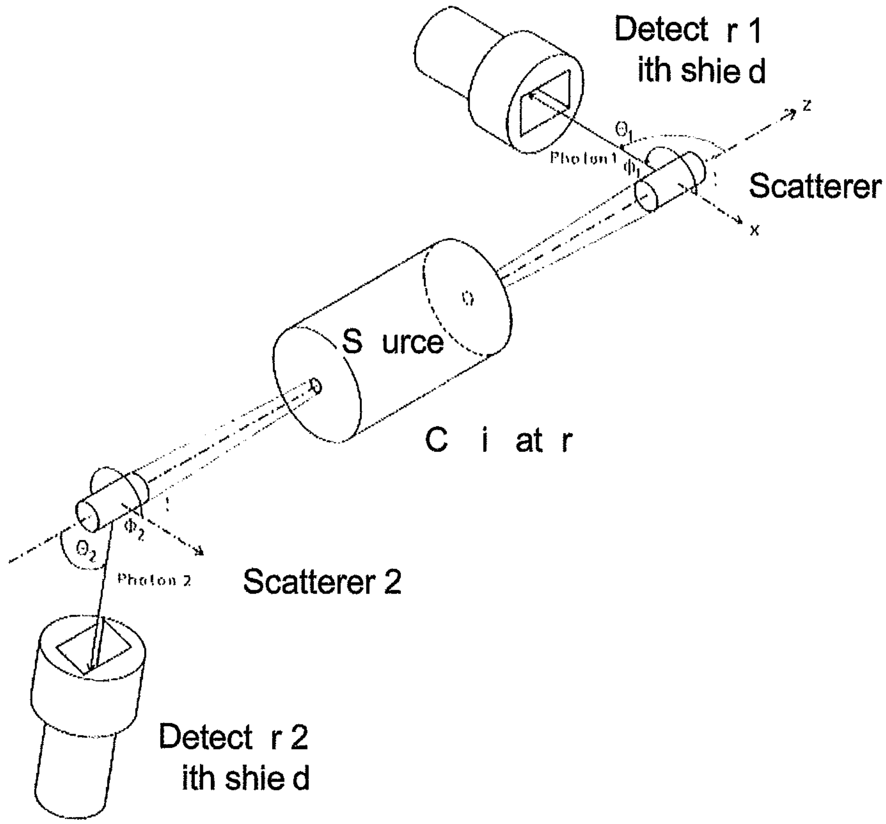

Figure 2: The experimental set up showing the source inside a collimator and the two Compton polarimeters, each consisting of a scatterer and a detector with a shield.

alignments. For uncorrelated photons $R(\phi_1, \phi_2) = 1$. In a quantum mechanical system, β is given by

$$\beta \leq m(\theta_1) \cdot m(\theta_2) \leq 0.478. \tag{15}$$

Several effects reduce the polarization correlation $R(\phi_1, \phi_2)$, such as the analyzing power of the Compton polarimeter, changes in the two-photon quantum state and backgrounds, which include accidental coincidences, uncorrelated coincident photons from a calibration source, secondary scattering in the

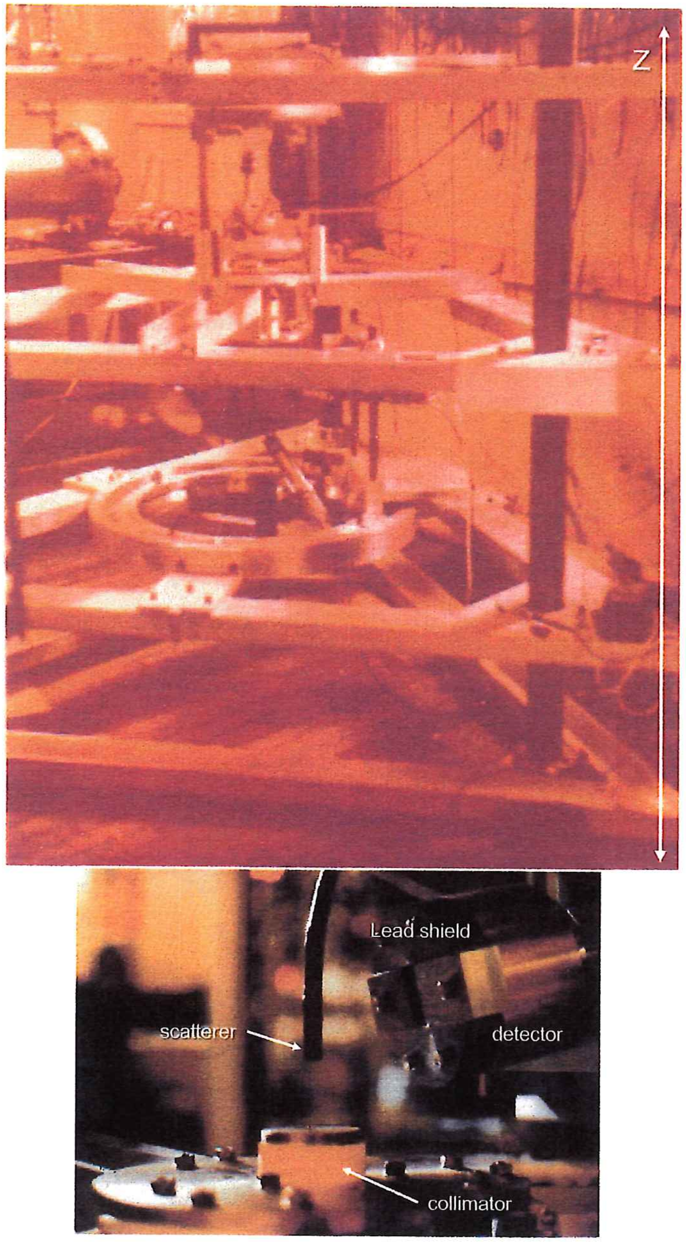

Figure 3: Top: Photograph of the experimental setup. Bottom: Photograph of a Compton polarimeter.

light guide and lead shield and double scattering in the scintillator. Furthermore, systematic effects like misalignment of the scatterers may change the analyzing power or backgrounds. We determined the polarization correlation parameter β_{exp} from $R(\phi_1, \phi_2)$ by measuring the $\phi_2 - \phi_1$ dependence in four data sets M_{i}.

1. Data set M_1: cylindrical shaped scatterers placed at 16 cm from the source for 82° scattering angles.
2. Data set M_2: conical shaped scatterers placed at 16 cm from the source for 82° scattering angles.
3. Data set M_3: conical shaped scatterers placed at 16 cm from the source for 68° scattering angles.
4. Data set M_4: conical shaped scatterers placed at 42 cm from the source for 82° scattering angles.

For all measurements the acceptances were set to $\Delta\theta = 13.5°$ and $\Delta\phi = 9.5°$. For each measurement we recorded the following observables:

- Coincidence numbers N_2, $N_{31}(\phi_1)$, $N_{32}(\phi_2)$ and $N_4(\phi_1, \phi_2)$ and the corresponding accidental coincidences A_2, $A_{31}(\phi_1)$, $A_{32}(\phi_2)$, and $A_4(\phi_1, \phi_2)$.
- Single rates in the scatterers and detectors, N_{S1}, N_{S2}, N_{D1} and N_{D2}.
- Response times T_{S1}, T_{S2}, T_{D1} and T_{D2} and the TDC spectra for all three-fold and four-fold coincidences.
- The measured energies E_{S1}, E_{S2}, E_{D1} and E_{D2} for all three-fold and four-fold coincidences.

For a four-fold coincidence the response times have to be consistent with expectations within errors and the energies of the scattered photon E_γ and the recoil electron E_{e} have to satisfy $E_{\text{e}} + E_\gamma = E_{\text{S}} + E_{\text{D}} = 511$ keV within errors, where E_{S} and E_{D} are the energies measured in a scatterer and corresponding detector, respectively.

5 Simulation of the Experiment

The goal of the Monte Carlo simulation is to determine the quantum mechanical expectation value for the polarization correlation parameter β_{QM} for the four data sets. To obtain precise predictions we have simulated the experiment in very great detail, including the form and position of the source, the effect of the collimator, the shape and position of the scatterers, the opening of the lead slits and the absorption in the NaI detectors. The simulations allowed us to determine the dependence of $\beta_{\rm QM}$ on the scattering angle θ and the angular acceptances $\Delta\theta$ and $\Delta\phi$ and to study the effect of multiple scattering in scatterers on $\beta_{\rm QM}$. We further determined the efficiencies of scatterers and detectors, energy distributions in the scatterers and detectors and ratios of coincidence numbers.

Note that due to the event-by-event simulation, $\beta_{\rm QM}$ cannot be determined from dP_{12} directly because for the generation of Compton scattering we need to make concrete assumption about the polarization state of the two-photon system. The pure state $|\psi >$, represented by the density matrix

$$\rho_{\rm QM} = |\psi\rangle\langle\psi| = \frac{1}{2}\Big(|x_1y_2\rangle\langle y_2x_1| + |y_1x_2\rangle\langle x_2y_1| + |x_1y_2\rangle\langle x_2y_1| + |y_1x_2\rangle\langle y_2x_1| \Big) \tag{16}$$

cannot be factorized into the individual polarization states, *i.e.* dP_{12} cannot be written as a product of dP_1 and dP_2, We have two possibilities to determine the quantum mechanical expectation value, either determine the analyzing power of each Compton polarimeter and then multiply their values or determine the expectation value for a symmetric mixture.

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\Big(|x_1y_2\rangle + |y_1x_2\rangle \Big) \tag{17}$$

for which we get

$$dP_{12}^{\rm sm} = \frac{1}{4\pi} f(\theta_1 0 f(\theta_2)\Big[1 - \frac{1}{2}m(\theta_1)m(\theta_2)\Big]\cos 2(\phi_2 - \phi_1). \tag{18}$$

This has the same form as dP_{12} except for the extra factor $1/2$. We use both methods to simulate $\beta_{\rm QM}$ for the four data sets, yielding either $\beta_{\rm QM} = \epsilon_1^{\rm CP}\cdot\epsilon_2^{\rm CP}$ or $\beta_{\rm QM} = 2\beta^{\rm sm}$ for the symmetric mixture.

6 Analysis Requirements

Figure 4 shows the measured energy spectrum $E/m_e c^2$ in the NaI detector. The signal region is marked by A. Region B results from events that are scattered in the scatterer and scattered again in the lead shield, while region C

results from double scattered events. Region D is caused by the Sn calibration source. To reduce backgrounds from accidental coincidences and multiple scattering we apply different selection criteria and define four event categories.

- Category 0: measured coincidences corrected for accidental coincidences.
- Category T: add time requirements.
  - $t_{S2} - t_{S1} \leq 1.6$ ns for N_2.
  - $t_{D1} - t_{S1} \leq 4.6$ ns for N_{31}.
  - $t_{D2} - t_{S2} \leq 3.6$ ns for N_{32}.
- Category S: add energy sums.
  - $370 \leq E_{S1} + E_{D1} \leq 650$ keV.
  - $385 \leq E_{S2} + E_{D2} \leq 635$ keV.
- Category E: add detector energy constraints.
  - $200 \leq E_{D1} \leq 325$ keV.
  - $195 \leq E_{D2} \leq 320$ keV.

These selection criteria are applied successively.

7 Quantum Mechanical Expectations

We have simulated the quantum mechanical expectation value β_{QM} under different assumptions. Figure 5 (left) shows β_{QM} for single-scattered events for category 0 and category E as a function of the scattering angle for $\Delta\theta = 14.5°$ and $\Delta\phi = 10°$. The peak value is at $\theta = 82°$ and is not very different for the two categories. The solid curve shows the theoretical calculation of the polarization correlation for an ideal Compton polarimeter by Snyder *et al.* [6]. Our simulations confirm that the polarization correlation parameter for our setup is rather close to that of an ideal Compton polarimeter. Figure 5 (right) shows β_{QM} for single and multiple scattered events. The quantum mechanical expectation value β_{QM} is lowered also for category E even though the energy selection removes some multiple-scattered events. So, compared to the prediction by Snyder *et al.* the measured polarization correlation parameter for category E is reduced by about 10%. Figures 6 (left, right) respectively show

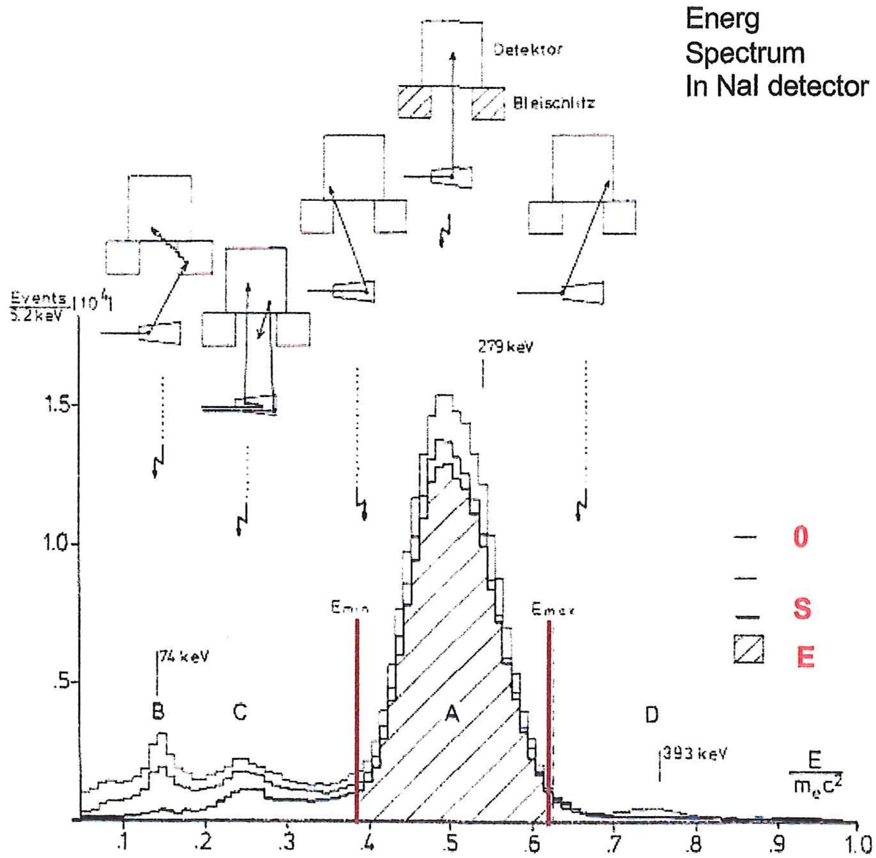

Figure 4: Energy spectrum in the NaI detector.

β_{QM} for single-scattered and single- plus multiple-scattered events as a function of $\Delta\theta$ for a scattering angle of $\theta = 82°$ for three values of $\Delta\phi$. Note that multiple scattering and large acceptances $\Delta\theta$ and $\Delta\phi$ reduce the correlation parameter β_{QM} significantly.

8 Systematic Effects

Multiple scattering in the scatterer affects β_{exp}. In our setup the amount of double-scattered four-fold coincidences is 17.8% for category 0 and 10% for category E. The effect of accidental coincidences on β_{exp} is rather small.

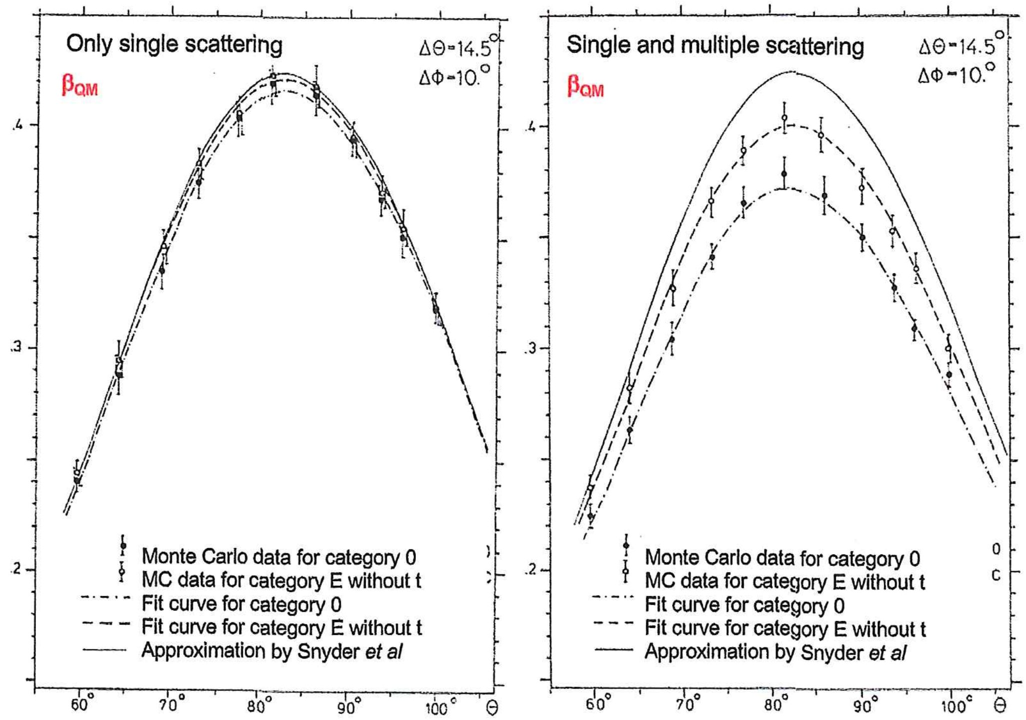

Figure 5: Left: Quantum mechanical expectation value $\beta_{\rm QM}$ as a function of θ for $\Delta\theta = 14.5^\circ$ and $\Delta\phi = 10^\circ$ for categories 0 and E. The curves show fits to the data. Right: The corresponding plots for single- and multiple-scattered events. The solid lines show the theoretical calculation of the polarization correlation parameter for an ideal Compton polarimeter by Snyder *et al.*

For category E the fraction of accidental four-fold coincidences is less 0.15%. Coincidences with a 1.34 MeV photon from ^{64}Cu source are negligible and events having secondary scatterings in other parts of the detector are removed by criteria S. The accuracy in the azimuth angle setting is better than 1° leading to a systematic error on $\beta_{\rm exp}$ less than 1%. Furthermore, deviations from rotation symmetry yields a negligible error on $\beta_{\rm exp}$.

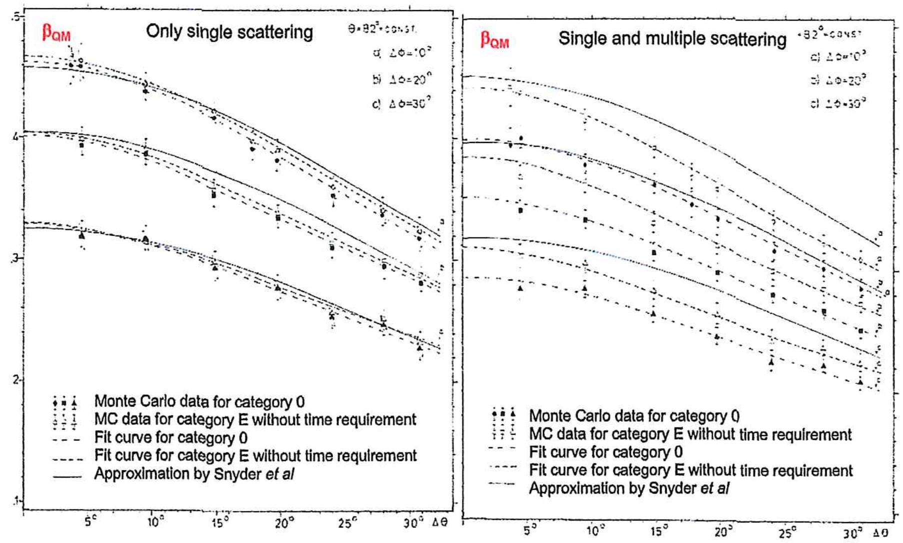

Figure 6: Left: Quantum mechanical expectation value β_{QM} as a function of $\Delta\theta$ for $\theta = 82°$ and three values of $\Delta\phi = 10°$ for categories 0 and E. The curves show fits to the data. Right: The corresponding plot for single- and multiple-scattered events. The solid lines show the theoretical calculation of the polarization correlation parameter for an ideal Compton polarimeter by Snyder *et al.*

9 Results

Figure 7 shows the measured polarization correlation $R_{12}(\phi_1, \phi_2)$ as a function of the azimuth angle $\phi = \phi_2 - \phi_1$ for data set II for categories 0 (top left), T (bottom left), S (top right) and E (bottom right). We fit each distributions to the function $A[1+\beta_{\text{exp}}^{\text{cat}} \cos 2(\phi_2 - \phi_1)]$ and extract A and $\beta_{\text{exp}}^{\text{cat}}$ for each category. We apply the same procedure to the other three data sets. Table 1 summarizes our measurements for the four data sets evaluated for each category. In addition, we list the quantum mechanical expectation value, the ratio of observed-to-expected polarization correlation parameters and the scaled value $\hat{\beta} = \beta_{\text{exp}}/\beta_{\text{QM}} \cdot \beta_{\text{QM}}^{\text{ideal}}$ where $\beta_{\text{QM}}^{\text{ideal}} = 0.478$ is the polarization correlation parameter for ideal Compton polarimeters expected for the representation of the two-photon system by the quantum mechanical state vector $|\psi\rangle$. The latter value will be used for comparison with other experiments. For all data sets

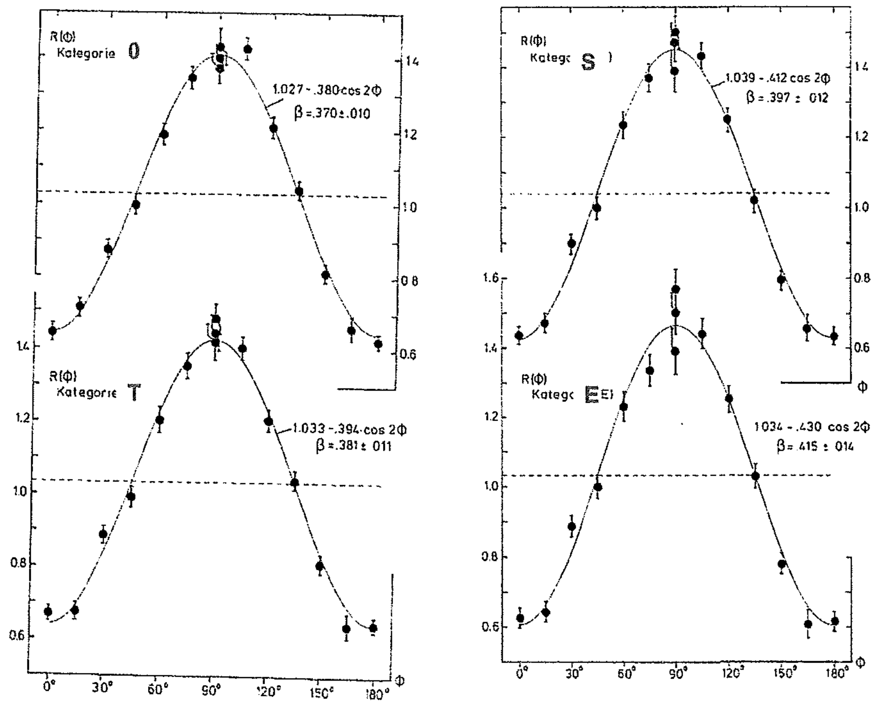

Figure 7: :Simulated polarization correlation for data set II. Top left: For category 0. Top right: For category T. Bottom left: For category S. Bottom right: For category E.

and categories the quantum mechanical correlation parameter is determined by $\beta^{\mathrm{QM}} = \epsilon_1 \cdot \epsilon_2 \cdot q_{\mathrm{corr}}$ where $q_{\mathrm{corr}} = 1.026 \pm 0.023$ is a correction factor. For data set II β_{QM} is evaluated also with method 2. The two methods yield the same results. Note that for category E $\beta_{\mathrm{exp}}/\beta_{\mathrm{QM}}$ is one. So for category E we use $\hat{\beta}^{\mathrm{E}} = \beta^{\mathrm{E}}_{\mathrm{exp}}/\beta^{\mathrm{E}}_{\mathrm{QM}} \cdot 0.478$, while for all other categories k we normalize to $\beta^{\mathrm{T}}_{\mathrm{QM}}$ yielding $\hat{\beta}^{\mathrm{k}} = \beta^{\mathrm{k}}_{\mathrm{exp}}/\beta^{\mathrm{T}}_{\mathrm{QM}} \cdot 0.478$.

Figure 8 shows our results for the four data sets and four categories scaled to the polarization correlation parameter of an ideal Compton polarimeter. For mixing of the first kind $\hat{\beta} = 0.239$ and for uncorrelated photons it is zero. The average over the four data sets yields $\langle\hat{\beta}^0\rangle = 0.454 \pm 0.009$, $\langle\hat{\beta}^{\mathrm{T}}\rangle = 0.463 \pm 0.009$, $\langle\hat{\beta}^{\mathrm{T}}\rangle = 0.483 \pm 0.010$ and $\langle\hat{\beta}^{\mathrm{E}}\rangle = 0.479 \pm 0.011$. For categories S and E, the measurements are in excellent agreement with the quantum mechanical

Table 1: Measured polarization correlation parameter β_{exp}, the expected polarization correlation parameter β_{QM}, their ratio and the computed value for an ideal Compton polarimeter $\hat{\beta}$ for all four data sets and all categories.

data set	Category	β_{exp}	β_{QM}	$\beta_{\text{exp}}/\beta_{\text{QM}}$	$\hat{\beta}$
I	0	0.374 ± 0.009		0.959 ± 0.03	0.458 ± 0.014
	T	0.382 ± 0.01	0.390 ± 0.008	0.979 ± 0.033	0.468 ± 0.016
	S	0.395 ± 0.011		1.013 ± 0.035	0.484 ± 0.017
	E	0.418 ± 0.012	0.416 ± 0.009	1.005 ± 0.036	0.480 ± 0.017
II	0	0.370 ± 0.01		0.941 ± 0.03	0.450 ± 0.015
	T	0.381 ± 0.011	0.393 ± 0.007	0.969 ± 0.033	0.463 ± 0.016
	S	0.397 ± 0.012		1.01 ± 0.035	0.483 ± 0.017
	E	0.415 ± 0.014	0.413 ± 0.009	1.005 ± 0.04	0.480 ± 0.19
III	0	0.315 ± 0.01		0.940 ± 0.041	0.449 ± 0.02
	T	0.316 ± 0.011	0.335 ± 0.01	0.943 ± 0.043	0.451 ± 0.021
	S	0.337 ± 0.013		1.006 ± 0.049	0.481 ± 0.023
	E	0.368 ± 0.015	0.369 ± 0.011	0.997 ± 0.052	0.477 ± 0.025
IV	0	0.373 ± 0.019		0.969 ± 0.052	0.463 ± 0.025
	T	0.377 ± 0.02	0.385 ± 0.008	0.979 ± 0.055	0.468 ± 0.026
	S	0.387 ± 0.023		1.005 ± 0.062	0.480 ± 0.03
	E	0.405 ± 0.026	0.405 ± 0.009	1.00 ± 0.068	0.478 ± 0.032

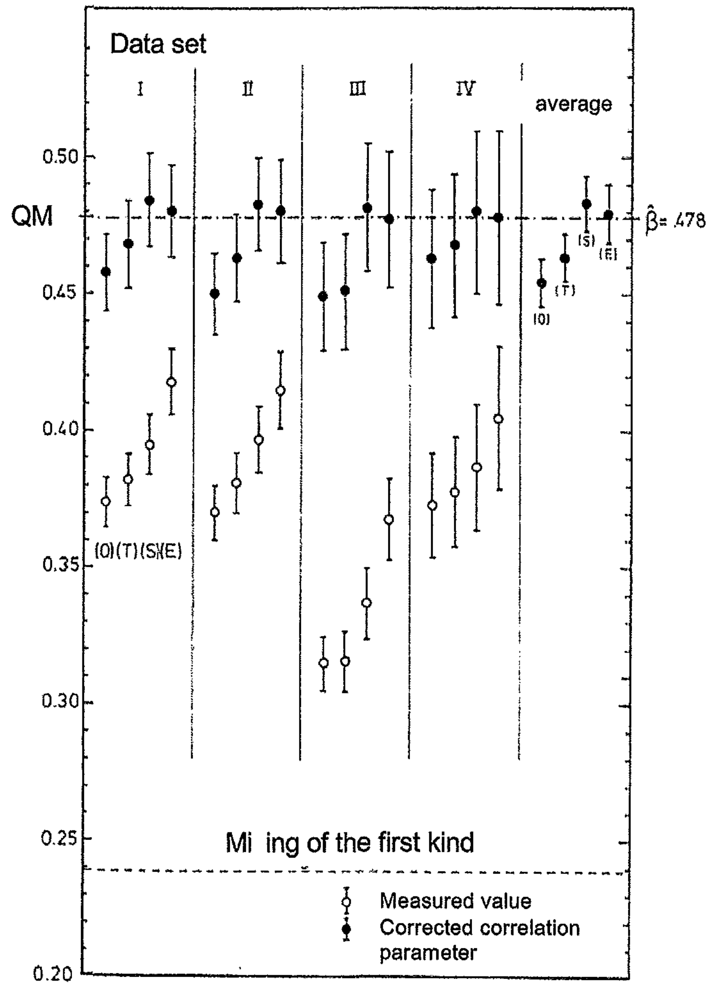

Figure 8: Measured polarization correlation parameters for the four data sets and four categories scaled to ideal Compton polarimeters. The last column shows the average values.

expectation.

The average values $\langle\hat{\beta}^0\rangle$ and $\langle\hat{\beta}^{\mathrm{T}}\rangle$ respectively are $5.0\pm1.9\%$ and $3.2\pm2.0\%$ below the quantum mechanical expectation value. This systematic deviation is caused by events that are scattered in other parts of the detector, which are removed by the selection criteria in category S. Assuming the azimuth angle distribution for such events to be similar to that of multiple scattered events we can use the estimate

$$\left\langle \beta^{\mathrm{T}}_{\mathrm{app}} \right\rangle \simeq \left(1 + 2\tilde{n}_{\mathrm{m}}\left[1 - \epsilon^{0}_{\mathrm{m}}/\epsilon^{0}_{\mathrm{s}}\right]\right)\langle \beta^{0}_{\beta_{\mathrm{s}}} \rangle, \tag{19}$$

where $\tilde{n}_{\mathrm{m}}$ is the amount of photons that are scattered a second time outside the scatterer ($\tilde{n}_{\mathrm{m}} = 41\%$), $\langle \beta_{\mathrm{s}} \rangle = 0.483$ is the quantum mechanical polarization correlation parameter for single-scattered events and $\epsilon^{0}_{\mathrm{m}}/\epsilon^{0}_{\mathrm{s}} = 0.37$ is the ratio of analyzing powers for multiple-scattered and single-scattered events. We get $\left\langle \beta^{\mathrm{T}}_{\mathrm{app}} \right\rangle = 0.459 \pm 0.006$, which is consistent with the average values $\langle \hat{\beta}^{0} \rangle$ and $\langle \hat{\beta}^{\mathrm{T}} \rangle$. The selection criteria in categories S and E remove most of this background. Note that for categories S and E the deviation from the quantum mechanical expectation value are $1.0 \pm 2.0\%$ and $0.2 \pm 2.3\%$, respectively. Thus, mixing of the first kind [7] is clearly ruled out as well as a value of $0.478/\sqrt{2}$ [3].

10 Comparison with other Experiments

Though the discrepancy between the results of Kasday [4] and Faraci [5] initiated this work, we found two other experiments by Langhoff [9] and Bruno [8]. Table 2 list the parameters of the four experiments. Using the information provided in the publications we simulated the polarization correlation in the four experiments. Figure 10 shows the results in comparison to our results. We have sorted them according to our four selection categories. The Faraci experiments has not incorporated any timing requirements and energy selections except for a loose time requirement on four-fold coincidences whereas the Kasday experiment applied similar selections as we did. The Bruno experiment did not give any time requirements but just imposed requirements on the energy sums, which varied slightly for the different data sets. The Langhoff experiment applied time constraints and requirements of the energy detected in the NaI detectors.

The final measurements of the Langhoff, Bruno and Kasday experiments and the those adjusted by our simulations agree well with quantum mechanics for categories S and E. With our simulation the results of the Faraci experiment are consistent with quantum mechanics while their own corrections yield values consistent with $\beta_{\mathrm{QM}}/\sqrt{2}$. Note that their corrections did not account for multiple scattering effects.

Table 2: Parameters used in other polarization correlation experiments.

Parameter/Experiment	Faraci [5]	Kasday [4]	Bruno [8]	Langhoff [9]
β^+ source	^{22}Na	^{64}Cu	^{22}Na	^{22}Na, ^{64}Cu
Absorber	plexiglass	brass	Cu-plexiglass	plexiglass-aluminum
collimator $[\Omega/4\pi]$	-	3.13×10^{-4}	1.54×10^{-4}	3.28×10^{-4}
Scatterer:	cylindrical	conical	cylindrical	cylindrical
Radius [cm]	1.27	0.319/0.956	1.0	1.0
Length [cm]	2.55	0.383	3.0	2.0
Opening angle	-	9.46°	-	-
Detector: shield depth	-	1.39	1,2	-
Nai crystal ⊘ [cm]	5.1	5.1	7.0	3.82
Length [cm]	5.1	5.1	7.0	2.55
Source-scatterer [cm]	5.5/5.5	15.8/15.8	10.0/10.0	25.0/25.0
Scatterer-detector [cm]	20.0	5.1	10	10
Average scattering angle	60°, 80°	82°	60°, 82°, 98°	82°
θ acceptance [FWHM]	5.6°	23.8°	11.7°	8.5°
ϕ acceptance [FWHM]	6.0°	9.6°	11.4°	8.4°
Coincidence times t_2 [ns]	-	21	-	5
t_{31}, t_{32} [ns]	-	95	-	-
t_4 [ns]	30	-	-	30
S selection [keV]	-	427, 595	434, 588	-
	-	-	427, 595	-
	-	-	413, 609	-
E selection [keV]	-	256, 307	-	205, 320
	-	307, 358	-	-

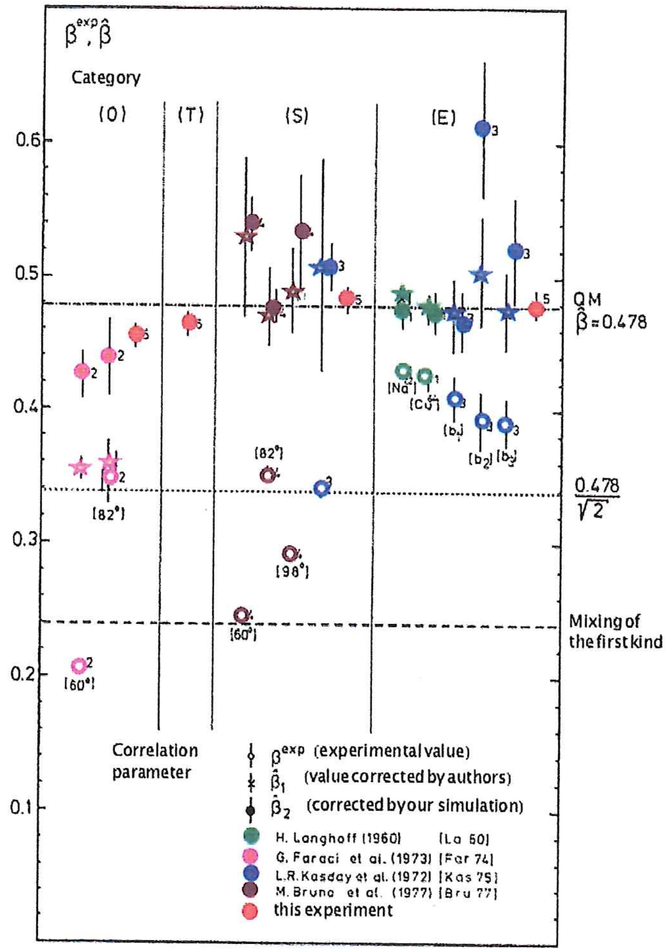

Figure 9: Measured polarization correlation parameters scaled to ideal Compton polarimeters of four experiments, Faraci (magenta points) [5], Kasday (blue points) [4], Bruno (brown point) [8] and Langhoff (green points) [9] in comparison to our results (red points). Open points show the measured values without correction, stars show the results after corrections by the authors and solid points depict the results with our corrections.

11 Test of Bell's Inequality

Bell's inequality states that

$$F(a,b,a',b') = |P(a,b) \mp P(a,b')| + |P(a',b) \mp P(a',b')| \leq 2 \qquad (20)$$

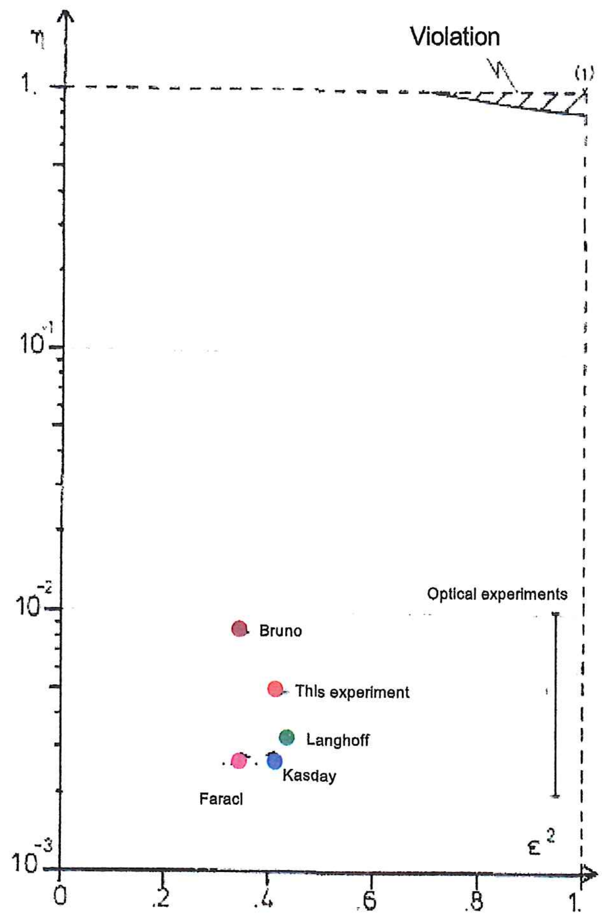

Figure 10: Graphical presentation of the violation of Bell's inequality in the $\eta - \epsilon^2$ plane. In addition, the detection efficiencies and squares of analyzing power are shown for the five experiments.

where $P(a,b) = \langle A(a) \cdot B(b) \rangle$. For a real experiment the quantum mechanical correlation is

$$P_{\rm QM}(\phi_1, \phi_2) = 1 - \frac{\Delta\Omega}{4\pi} p_2 \eta \{2 - \eta[1 - \epsilon^2 \cos 2(\phi_2 - \phi_1)]\} \tag{21}$$

where $p_2 = N_2/N_0 \frac{2\pi}{\Delta\Omega}$, ϵ is the analyzing power of a Compton polarimeter and η is the detection efficiency. Using the values $2\phi_1 = \pi/4$, $2\phi_1' = 3\pi/4$, $2\phi_2 = 0$

and $2\phi_2' = -\pi/2$ yields the inequality

$$F_{\rm QM} - 2 = 2\frac{\Delta\Omega}{2\pi} p_2 \eta \{\eta[1 + \epsilon^2\sqrt{2}] - 2\} \leq 2 \tag{22}$$

So, we find violation if $\eta[1 + \epsilon^2\sqrt{2}] > 2$. However, for present experiments $\eta[1 + \epsilon^2\sqrt{2}] \leq 1.68\eta \ll 2$. Thus, there is no violation of Bell's inequality since both the analyzing power and efficiency of present experiments are too small.

12 Conclusions

We measured the polarization correlation of the two-photon system from positron-electron annihilation. We varied the shape of the scatterer, scattering angle and distance source-scatterer. We simulated the quantum mechanical expectation value with a very sophisticated Monte Carlo program. All results are in good agreement with the quantum mechanical expectation value. Furthermore, we simulated the QM expectation value of four other experiment. The measurements either agree rather well or are consistent with the QM expectation value. The discrepancy of the Faraci with the QM expectation value is due to the neglect of multiple scattering, which reduces the polarization correlation parameter and needs to be corrected for.

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